# Network Externalities, Compatibility, and Product Differentiation

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## 1. Introduction

Standardization and product compatibility in network industries where network externalities are significant have been topics of growing interest in the industrial organization literature. It is said that network externalities are present when consumers' valuations of a good or service increases with the size of the network related to that good. This paper studies firms' product compatibility decisions and price competition in a differentiated duopoly with network externalities.

Standardization and product compatibility in the presence of network externalities have been extensively analyzed in the literature. Katz and Shapiro (1985) analyzed the private and social incentives for firms to achieve compatibility. They found that firms with small networks tend to favor product compatibility. Farrell and Saloner (1985) examined the possibility that benefits of standardization hinder the collective shift from an inferior standard to an efficient standard. They showed that there can be inefficient inertia under incomplete information. Farrell and Saloner (1986) analyzed the private and social incentives for adoption of a new technology incompatible with the installed base. Katz and Shapiro (1986a) examined the relationship between standardization and innovation. Farrell and Saloner (1992) analyzed the relationship between the equilibrium adoption of conversion technology and its optimality. Katz and Shapiro (1986b) further examined this issue in a dynamic context. Bental and Spiegel (1995) studied market coverage in the presence of network externalities. They showed that, under free entry, market coverage is larger with a single industry standard. de Palma and Leruth (1996) examined compatibility choices by firms in a two-stage game and showed that firms prefer compatibility when they have an equal probability of becoming the largest under incompatibility. Baake and Boom (2001) examined firms' compatibility decisions in a vertical differentiation model with network externality and showed that an adapter is provided in equilibrium.

The literature concerning network externalities, however, has focused only on positive externalities. We explicitly consider both positive and negative network externalities. Moreover, in contrast to the papers above, we examine asymmetric as well as symmetric network externalities.

We analyze games in which, at the first stage, duopolistic firms determine product compatibility choices and, at the second stage, undertake price competition in a vertically differentiated industry with network externalities. We derive subgame perfect equilibrium for the games. We demonstrate that when the two products become more differentiated, equilibrium prices become higher and that the level of the network externality is larger, the equilibrium prices and profits become higher. We also show that at subgame perfect equilibrium, the firm producing a higher-quality product chooses the lowest compatibility level when negative network externalities exist.

Moreover we examine the case in which network externalities are symmetric and show that if the level of the network externality is larger, the firm producing the higher-quality product will receive a larger profit.

The paper is organized as follows. Section 2 presents the model. In Section 3, we derive subgame perfect equilibrium of the games. In Section 4, we analyze the case of symmetric externalities. Section 5 concludes.

## 2. The Model

Let us consider an industry in which two firms produce vertically differentiated products.[1] Let the two firms be labeled firm 1 and 2. Let $c_i$ denote firm $i$'s constant unit cost, $i=1,2$. Suppose that the two products are differentiated in terms of product quality. Let $S_i$ denote the quality of product $i$. We assume that product 1 is of higher quality than product 2, that is, $S_1 > S_2$. Let $P_i$ denote the price of product $i$. For simplicity, we assume that consumers are uniformly distributed on the set $\Theta = [0,1] \subset R$. Let $\theta \in \Theta$. We assume the density function $f(\theta) = 1$ on $\Theta$. Let $x_i$ denote the quantity of product $i$.

Suppose that each consumer purchases one of the two products. Let each consumer's utility be given by

$$U = \begin{cases} \theta S_1 + \alpha x_1 + \beta x_2 - p_1, \text{ if he buys product 1} \\ \theta S_2 + \gamma x_2 + \delta x_1 - p_2, \text{ if he buys product 2} \\ 0, \text{ otherwise,} \end{cases} \tag{1}$$

where $\alpha > 0$, $\gamma > 0$, $\alpha > \beta$, and $\gamma > \delta$.
Parameters $\alpha$ and $\gamma$ represent the strength of network externality. Parameters $\beta$ and $\delta$ represent the effects of network externality in regard to the rival good. Note that $\beta$ and $\delta$ can be negative. We assume $\beta \in [\underline{\beta}, \overline{\beta}]$ and $\delta \in [\underline{\delta}, \overline{\delta}]$.

We suppose that each firm decides whether it produces a product compatible with its rival product. To model the firms' compatibility choices, we consider two scenarios. First, we consider the case in which firm 1 determines the compatibility of its own product, which results in the magnitude of externality effect $\delta$. Similarly, firm 2 determines the compatibility of its own product, which results in the magnitude of externality effect $\beta$. For the second scenario, we suppose that firm 1 determines the compatibility of its own product, which results in the magnitude of externality effect $\beta$. Similarly, firm 2 determines the compatibility of its own product, which results in the magnitude of externality effect $\delta$.

The games we consider below have two stages. At the first stage, each firm decides on the compatibility of its product. At the second stage, the two firms compete on price. We assume that the entire market is served.[2] A consumer $\hat{\theta}$ who is indifferent between purchasing product 1 and product 2 is given by

$$\hat{\theta} = \frac{p_1 - p_2 + (\delta - \alpha)x_1 + (\gamma - \beta)x_2}{S_1 - S_2}.$$

Then demand for product 1 is given by $x_1 = 1 - \hat{\theta}$ and demand for product 2 by $x_2 = \hat{\theta}$. The segment $[\hat{\theta}, 1]$ of consumers purchases product 1 and the segment $[0, \hat{\theta}]$ of consumers buys product 2. Thus the marginal consumer $\hat{\theta}$ is rewritten as

$$\hat{\theta} = \frac{p_1 - p_2 + (\delta - \alpha)}{S_1 - S_2 + (\delta - \alpha) + (\beta - \gamma)}. \tag{2}$$

Note that if a quality difference becomes larger, then the demand for product 1 will increase.

## 3. Equilibrium Analysis

To derive a subgame perfect equilibrium, we first analyze the second stage of the entire game. Firm $i$'s profit is $\Pi_i = (p_i - c_i)x_i$, $i = 1, 2$. Then the first-order condition of firm 1's profit maximization is

$$\frac{\partial \Pi_1}{\partial p_1} = 1 - \frac{2p_1 - p_2 - (\delta - \alpha) - c_1}{S_1 - S_2 + (\delta - \alpha) + (\beta - \gamma)} = 0.$$

Hence firm 1's reaction function is given by

$$p_1 = \frac{p_2 + c_1 + (S_1 - S_2) + (\beta - \gamma)}{2}. \tag{3}$$

Similarly, the first-order condition of firm 2's profit maximization is

$$\frac{\partial \Pi_2}{\partial p_2} = \frac{p_1 - 2p_2 + c_2 + (\delta - \alpha)}{S_1 - S_2 + (\delta - \alpha) + (\beta - \gamma)} = 0.$$

Thus firm 2's reaction function is given by

$$p_2 = \frac{p_1 + c_2 + (\delta - \alpha)}{2}. \tag{4}$$

From (3) and (4), we obtain the two firms' equilibrium prices as follows.

$$p_1^* = \frac{2c_1 + c_2 + 2(S_1 - S_2) + (\delta - \alpha) + 2(\beta - \gamma)}{3} \tag{5}$$

and

$$p_2^* = \frac{c_1 + 2c_2 + (S_1 - S_2) + 2(\delta - \alpha) + (\beta - \gamma)}{3}. \tag{6}$$

Note that if $\alpha$ or $\gamma$ increases, then equilibrium prices will decrease and that if $\beta$ or $\delta$ increases, then equilibrium prices will increase. From (2), (5) and (6), the marginal consumer $\hat{\theta}$ at the equilibrium is given by

$$\hat{\theta} = \frac{(S_1 - S_2) + c_1 - c_2 + 2(\delta - \alpha) + (\beta - \gamma)}{3[(S_1 - S_2) + (\delta - \alpha) + (\beta - \gamma)]}. \tag{7}$$

Hence we have

$$1 - \hat{\theta} = \frac{2(S_1 - S_2) - c_1 + c_2 + (\delta - \alpha) + 2(\beta - \gamma)}{3[(S_1 - S_2) + (\delta - \alpha) + (\beta - \gamma)]}.$$

Thus the two firms' profits at the equilibrium are given as

$$\Pi_1^* = \frac{[2(S_1 - S_2) - c_1 + c_2 + (\delta - \alpha) + 2(\beta - \gamma)]^2}{9[(S_1 - S_2) + (\delta - \alpha) + (\beta - \gamma)]}$$

and

$$\Pi_2^* = \frac{[(S_1 - S_2) + c_1 - c_2 + 2(\delta - \alpha) + (\beta - \gamma)]^2}{9[(S_1 - S_2) + (\delta - \alpha) + (\beta - \gamma)]}$$

To simplify the analysis, we suppose that $c_1 = c_2 = c$. Then the equilibrium profits are given in the following lemma.

**Lemma 1.** *The equilibrium profits are*

$$\Pi_1^{**} = \frac{[2(S_1 - S_2) + (\delta - \alpha) + 2(\beta - \gamma)]^2}{9[(S_1 - S_2) + (\delta - \alpha) + (\beta - \gamma)]} \tag{8}$$

*and*

$$\Pi_2^{**} = \frac{[(S_1 - S_2) + 2(\delta - \alpha) + (\beta - \gamma)]^2}{9[(S_1 - S_2) + (\delta - \alpha) + (\beta - \gamma)]}. \tag{9}$$

By (8) and (9), we obtain the following propositions.

**Proposition 1.** *If $\alpha$ increases, then firm1's profit will increase, while firm 2' profit will decrease.*

**Proposition 2.** *If $\beta$ increases, then both firms' profits will increase.*

Next we analyze the second stage of the game, in which each of the two firms chooses the compatibility of its product. Let $X_i$ denote firm $i$ 's product compatibility choice. We consider two scenarios regarding the firms' product compatibility choices. For the first scenario, we consider the case in which firm 1 determine the compatibility of its own good in such a way that it chooses the level of $\delta$. For the second scenario, we suppose that firm 1 determines the compatibility of its own good in such a way that it chooses the level of $\beta$.

For the first scenario, by (8), we can conclude that the firm producing a higher-quality product will choose $\underline{\delta}$. Similarly, we can conclude that the firm producing a lower-quality product will choose $\overline{\beta}$. Thus the two firms' compatibility choices are $(X_1, X_2) = (\underline{\delta}, \overline{\beta})$. Therefore, for the first scenario of compatibility choices, the subgame perfect equilibrium of the game consists of $(p_1^*, p_2^*)$ and $(\underline{\delta}, \overline{\beta})$.

**Proposition 3.** *At the subgame perfect equilibrium, the firm producing a higher-quality product will choose $\underline{\delta}$ and the firm producing a lower-quality product will choose $\overline{\beta}$.*

For the second scenario, by (8), we can conclude that the firm producing a higher-quality product will choose $\bar{\delta}$. Similarly, we can conclude that the firm producing a lower-quality product will choose $\bar{\beta}$. Thus two firms' compatibility choices are $(X_1, X_2)=(\bar{\beta}, \bar{\delta})$. Therefore, for the second scenario of compatibility choices, the subgame perfect equilibrium consists of $(p_1^*, p_2^*)$ and $(\bar{\beta}, \bar{\delta})$.

**Proposition 4.** *At subgame perfect equilibrium, the firm producing a higher-quality product will choose* $\bar{\beta}$ *and the firm producing a lower-quality product will choose* $\bar{\delta}$.

## 4. Symmetric Externalities

In the previous section, we considered the case in which there can be negative externalities. In this section, we examine two games with symmetric positive externalities. Suppose that each consumer has the following utility function :

$$U=\begin{cases}\theta S_i+\alpha x_i+\beta x_j-p_i, \text{ if he buys product} i,\\ 0, \text{ otherwise,}\end{cases}$$

where $\alpha>\beta\geq 0$.

The first game we consider in this section has two stages. At the first stage, two firms decide whether they coordinate compatibility of products, that is, either $\beta>0$ or $\beta=0$. At the second stage, the firms undertake price competition.

A consumer $\hat{\theta}$ who is indifferent between purchasing product 1 and product 2 is given by

$$\hat{\theta}=\frac{p_1-p_2+(\beta-\alpha)(x_1-x_2)}{S_1-S_2}.$$

Since $x_1=1-\hat{\theta}$ and $x_2=\hat{\theta}$, we have

$$\hat{\theta}=\frac{p_1-p_2+(\beta-\alpha)}{S_1-S_2+2(\beta-\alpha)}.$$

From the first-order conditions of the two firms' profit maximization, the equilibrium prices are given by

$$p_1^{S*}=\frac{2c_1+c_2+2(S_1-S_2)+3(\beta-\alpha)}{3}$$

and

$$p_2^{S*}=\frac{c_1+2c_2+(S_1-S_2)+3(\beta-\alpha)}{3}.$$

Thus the marginal consumer is given by

$$\hat{\theta}=\frac{c_1-c_2+(S_1-S_2)+3(\beta-\alpha)}{3[(S_1-S_2)+2(\beta-\alpha)]}.$$

For simplicity, we assume that $c_1 = c_2 = c$. Then the two firms' equilibrium profits are

$$\Pi_1^{S*} = \frac{[2(S_1 - S_2) + 3(\beta - \alpha)]^2}{9[(S_1 - S_2) + 2(\beta - \alpha)]}$$

and

$$\Pi_2^{S*} = \frac{[(S_1 - S_2) + 3(\beta - \alpha)]^2}{9[(S_1 - S_2) + 2(\beta - \alpha)]}.$$

**Proposition 5.** *If $\alpha$ increases, then firm 1's profit will increase and firm 2's profit will decrease.*

**Proposition 6.** *If $\beta$ increases, then each firms' profit will increase.*

Therefore the firms choose product compatibility $\beta > 0$ at the second stage of the game.

Thus far we have examined the cases of the full market coverage. Next we consider the game in which the market is not covered. To focus on the effects of the magnitudes of network externalities on firms' profits, we assume $\alpha = \beta$ and $c_1 = c_2 = 0$. Let $\hat{\theta}$ be the marginal consumer who is indifferent between purchasing product 1 and product 2. Let $\underline{\theta}$ be the marginal consumer who is indifferent between purchasing product 2 and nothing. Then the demand for product 1 is given by $x_1 = 1 - \hat{\theta}$ and the demand for product 2 by $x_2 = \hat{\theta} - \underline{\theta}$. The segment $[\hat{\theta}, 1]$ of consumers purchases product 1 and the segment $[\underline{\theta}, \hat{\theta}]$ of consumers buys product 2. Then $\hat{\theta}$ and $\underline{\theta}$ are given by

$$\hat{\theta} = \frac{p_1 - p_2}{S_1 - S_2}$$

and

$$\underline{\theta} = \frac{p_2 - \alpha}{S_2 - \alpha}.$$

Firm 1's profit is $\Pi_1 = p_1(1 - \hat{\theta})$. The first-order condition of firm 1's profit maximization is

$$\frac{\partial \Pi_1}{\partial p_1} = 1 - \frac{2p_1 - p_2}{S_1 - S_2} = 0.$$

Hence firm 1's reaction function is given by

$$p_1 = \frac{p_2 + S_1 - S_2}{2}.$$

Firm 2's profit is $\Pi_2 = p_2(\hat{\theta} - \underline{\theta})$. Then the first-order condition of firm 2's profit maximization is

$$\frac{\partial \Pi_2}{\partial p_2} = \frac{p_1 - 2p_2}{S_1 - S_2} - \frac{2p_2 - \alpha}{S_2 - \alpha} = 0.$$

Thus firm 2's reaction function is given by

$$p_2 = \frac{p_1(S_2 - \alpha) + \alpha(S_1 - S_2)}{2(S_1 - \alpha)}.$$

The equilibrium prices are given by

$$p_1^{S**} = \frac{(S_1 - S_2)(2S_1 - \alpha)}{4S_1 - S_2 - 3\alpha} \tag{10}$$

and

$$p_2^{S**} = \frac{(S_1 - S_2)(S_2 + \alpha)}{4S_1 - S_2 - 3\alpha}. \tag{11}$$

The effects of a change in $\alpha$ on the prices are $\frac{\partial p_1^{S**}}{\partial \alpha} > 0$ and $\frac{\partial p_2^{S**}}{\partial \alpha} > 0$.

The equilibrium profits are

$$\Pi_1^{S**} = \frac{(S_1 - S_2)(2S_1 - \alpha)^2}{(4S_1 - S_2 - 3\alpha)^2}$$

and

$$\Pi_2^{S**} = \frac{(S_1 - S_2)(S_1 - \alpha)(S_2 + \alpha)^2}{(4S_1 - S_2 - 3\alpha)^2 (S_2 - \alpha)}.$$

Therefore we have the following result.

**Proposition 7.** $\frac{\partial \Pi_2^{S**}}{\partial \alpha} > 0$.

Note that contrary to Proposition 1, if α increases, then firm 1 will receive a larger profit. Note also that the sign of $\frac{\partial \Pi_2^{S**}}{\partial \alpha}$ is indeterminate.

## 5. Conclusion

This paper has studied firms' product compatibility decisions and price competition in a differentiated duopoly with network externalities. In contrast to existing literature concerning network externalities, we have explicitly considered both positive and negative network externalities. We have examined compatibility choices by firms in two-stage games and have shown that firms prefer incompatibility when there are negative externalities. Moreover we have analyzed the case of symmetric network externalities and demonstrated that firms choose product compatibility.

**Notes**

1 For vertical product differentiation models, see, for instance, Gabszewicz and Thisse (1979, 1980), Shaked and

Sutton (1982, 1983), and Tirole (1988).

2　In Section 4, we briefly consider the case in which the market is not covered.

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