Distortion of Information under Adverse Selection and Moral Hazard

Shinji Kobayashi

1. Introduction

In many principal-agent relationships, agents often have incentives to distort their private information. For instance, regulated firms might report on their revenue and costs with some manipulation to the authorities concerned. This paper examines the optimal contract in a principal-agent model with both moral hazard and adverse selection in which the agent can choose a costly signal to falsify her private information. We consider a setting between a government (principal) and a firm (agent) in which the government contracts with the firm for procuring a product. The firm is assumed to have private information about her production costs. We also assume that the firm can signal her type by incurring costs and exert an effort to increase the probability of being the efficient type.

Specifically, we consider a contract game in which first, the government offers a contract to the firm, and then the firm accepts or rejects the contract. If the firm accepts the contract, she can choose an effort to increase the probability of being the efficient type and then the state is realized. Finally, transfers take place. We analyze this contracting game to examine the effects of the firm's costly signal and efforts on the production. We demonstrate that the optimal contract exhibits different regimes depending on the costs of signals and those of exerting efforts, and that the firm sends the government a costly signal regarding the production costs.

Our paper is related to a number of papers regarding information distortion. Maggi and Rodriguez-Clare (1995a) examine the falsification of information in a principal-agent model. They show that costly information distortion can arise at the equilibrium. Lacker and Weinberg (1989) examine the problem of costly falsification in a model of an exchange economy in which a risk averse agent can misrepresent her own endowment and characterize the optimal contract with no falsification. Dye (1988) considers an overlapping generation model under falsification without costs and show that information distortion may be optimal. Crocker and Morgan (1998) show that optimal insurance contracts induce falsification in an environment in which insureds can falsify actual losses. Laffont and Martimort (2002) examine costly signals in a principal-agent model with adverse selection. Our paper is different from those papers in that we consider a principal-agent model in the presence of both adverse selection and moral hazard.

Crocker and Slemrod (2007) examine the optimal contract with costly falsification in a model with private information and hidden action. We focus on the effects of limited liability constraints on the optimal contract, while they do not consider limited liability constraints. In addition, their setting is based on the relationship between shareholders and a manager, while we consider a procurement model between a government and a firm. Furthermore, we discuss a case in which the firm's fixed costs depend upon her type, that is, upon the asymmetric information parameter. We show that it is possible that countervailing incentives arise at equilibrium. For the issue of countervailing incentives, see for instance, Lewis and Sappington (1989) and Maggi and Rodriguez-Clare (1995b).

The paper is organized as follows. Section 2 describes the basic model. In Section 3, we characterize the optimal contract. In Section 4, we consider a case in which fixed production costs also depend on the agent's private information and discuss the effects of signaling costs and fixed costs on the possibility of countervailing incentives. Section 5 concludes.

2. The Model

Let us consider a setting in which a risk-neutral firm (agent) supplies a product to a risk-neutral government (principal). Let q denote the quantity of the product. The firm is assumed to have private information on the constant marginal cost of production, β, either low (efficient) β_L or high (inefficient) β_H, with respective probabilities λ and $1-\lambda$, and $0<\beta_L<\beta_H$. We also assume that these probabilities depend on the firm's effort or action e, that is, $\lambda(e)$ and $1-\lambda(e)$, and that $\lambda'(e)>0$ and $\lambda''(e)<0$. Thus the firm can exert an effort to increase the probability of being efficient. Let $\varphi(e)$ denote the disutility of exerting an effort. We further assume that the firm can signal her type by incurring costs. Let $c(\beta, \theta)$ denote the cost of a signal, where θ denotes a signal by the firm. We assume that $c_\theta(\beta, \theta)>0$, where $c_\theta(\beta, \theta)$ is a partial derivative with respect to θ. Let $B(q)$ denote social benefit and satisfy $B'(q)>0$ and $B''(q)<0$. Let t denote the payment from the government to the firm. Then the firm's payoff U is given by $U=t-\beta \cdot q-c(\beta, \theta)-\varphi(e)$. The government's payoff π is given by $\pi=B(q)-t$.

The timing of the contracting game is as follows. First, the government offers a contract to the firm. Second, the firm exerts an effort to increase the probability of being efficient. Third, the efficiency parameter is realized and only the firm learns it. Fourth, the firm chooses a costly signal. Finally, transfers between the government and the firm take place.

3. Characterization of Optimal Contracts

In this section, we derive the optimal contract under both asymmetric information and moral hazard. First let us consider a benchmark case in which marginal cost β and the firm's effort e are known to both the government and the firm.

We assume that the firm is protected by the following limited liability constraints: For the efficient type,

$$U_L = t_L-\beta_L \cdot q_L-c(\beta_L, \theta_L) \geqq 0 \tag{1}$$

and for the inefficient type,

$$U_H = t_H - \beta_H \cdot q_H - c(\beta_H, \theta_H) \geqq 0. \tag{2}$$

Then the government's problem under complete information is

$$\max_{t,q,e} \quad \lambda(e)\{B(q_L) - t_L\} + \{1 - \lambda(e)\}\{B(q_H) - t_H\}$$
$$\text{subject to (1) and (2).}$$

Thus, the firm's outputs under complete information are given by

$$B'(q_L^{FB}) = \beta_L$$

and

$$B'(q_H^{FB}) = \beta_H.$$

The firm chooses the first best output for each of the two states.

Next we examine the optimal contract under both asymmetric information and moral hazard. Suppose that both β and e are known only to the firm. Then the optimal contract has to satisfy the following incentive compatibility constraints. For the efficient type, the incentive compatibility constraint is

$$\begin{aligned} U_L &= t_L - \beta_L \cdot q_L - c(\beta_L, \theta_L) \\ &\geqq t_H - \beta_L \cdot q_H - c(\beta_L, \theta_H) \\ &= U_H + (\beta_H - \beta_L)\, q_H + c(\beta_H, \theta_H) - c(\beta_L, \theta_H). \end{aligned} \tag{3}$$

For the inefficient type, the incentive compatibility constraint is

$$\begin{aligned} U_H &= t_H - \beta_H \cdot q_H - c(\beta_H, \theta_H) \\ &\geqq t_L - \beta_H \cdot q_L - c(\beta_H, \theta_L) \\ &= U_L - (\beta_H - \beta_L)\, q_L + c(\beta_L, \theta_L) - c(\beta_H, \theta_L). \end{aligned} \tag{4}$$

In the model we assume that the firm can exert efforts to increase the probability of being efficient. Then the firm chooses her effort level to maximize her expected payoff. Thus the moral hazard incentive constraint is given by

$$\max_{e} \lambda(e)\, U_L + \{1 - \lambda(e)\}\, U_H - \varphi(e). \tag{5}$$

The first order condition with respect to e is

$$\lambda'(e)\,(U_L - U_H) - \varphi'(e) = 0. \tag{6}$$

Since the contract is offered before the agent's types are chosen, the ex ante participation constraint is given by

$$\lambda(e)\, U_L + \{1 - \lambda(e)\}\, U_H - \varphi(e) \geqq 0. \tag{7}$$

Hence when the cost state β and the firm's effort e are known only to the firm, the government's problem becomes

$$\max_{t,q,e} \quad \lambda(e)\{B(q_L)-t_L\}+\{1-\lambda(e)\}\{B(q_H)-t_H\}$$
$$\text{subject to (1), (2), (3), (4), (6) and (7).}$$

Depending on which constraints are binding, we need to distinguish the following two cases.

Case 1: Suppose that constraints (2), (3) and (6) are binding. Since the limited liability constraint in (2) is binding, we have

$$t_H=\beta_H \cdot q_H+c(\beta_H,\theta_H).$$

When the incentive compatibility in (3) is binding, we have

$$\begin{aligned} U_L &= t_L-\beta_L \cdot q_L-c(\beta_L,\theta_L) \\ &= (\beta_H-\beta_L)q_H+c(\beta_H,\theta_H)-c(\beta_L,\theta_H). \end{aligned}$$

It follows that

$$t_L=(\beta_H-\beta_L)q_H+\beta_L \cdot q_L+c(\beta_H,\theta_H)+c(\beta_L,\theta_L)-c(\beta_L,\theta_H).$$

Hence the government's problem can be rewritten as

$$\begin{aligned} &\max_{q,e,\theta} \lambda(e)\,[B(q_L)-t_L]+\{1-\lambda(e)\}[(B(q_H)-t_H] \\ =&\lambda(e)\,[B(q_L)-\{(\beta_H-\beta_L)q_H+\beta_L \cdot q_L+c(\beta_H,\theta_H)+c(\beta_L,\theta_L)-c(\beta_L,\theta_H)\}] \\ &+\{1-\lambda(e)\}[B(q_H)-\{\beta_H \cdot q_H+c(\beta_H,\theta_H)\}]. \end{aligned} \tag{8}$$

The first order condition with respect to q_L is

$$B'(q_L)=\beta_L.$$

Thus, for the efficient agent, the second best output q_L^{SB} is given by

$$q_L^{SB}=q_L^{FB}.$$

The optimal output of the efficient agent is the same as that under the full information case.

The first order condition with respect to q_H is

$$\lambda(e)[-(\beta_H-\beta_L)]+\{1-\lambda(e)\}[B'(q_H)-\beta_H]=0.$$

Thus, for the inefficient agent, the second best output q_H^{SB} satisfies

$$B'(q_H^{SB})=\beta_H+\frac{\lambda(e)}{1-\lambda(e)}(\beta_H-\beta_L).$$

Therefore, we have

$$q_H^{SB}<q_H^{FB}.$$

For the inefficient agent, the second best output level is lower than that of the complete information case.

For optimal efforts, from (6), we have

$$\lambda'(e)\{(\beta_H - \beta_L)\, q_H + c(\beta_H, \theta_H) - c(\beta_L, \theta_H)\} = \varphi'(e).$$

Next we examine an optimal signal θ^{SB}. For the efficient agent, by differentiating (8) with respect to θ_L, we have

$$\lambda(e)[-c_{\theta_L}(\beta_L, \theta_L)] < 0$$

since $c_{\theta_L}(\beta_L, \theta_L) > 0$. Thus $\theta_L^{SB} = 0$ and the efficient agent does not send signals.

For the inefficient agent, by differentiating (8) with respect to θ_H, we have

$$\lambda(e)\, c_{\theta_H}(\beta_L, \theta_H) - c_{\theta_H}(\beta_H, \theta_H) \gtreqless 0.$$

Therefore if $\lambda(e)\, c_{\theta_H}(\beta_L, \theta_H) - c_{\theta_H}(\beta_H, \theta_H) > 0$, then we have $\theta_H^{SB} > 0$ provided that there exists an upper bound $\bar{\theta}$ on θ.

Case 2: Suppose that constraints (2) and (6) are binding. Then we have

$$\lambda'(e)[U_L - U_H] = \varphi'(e).$$

Hence the government's problem can be written as

$$\begin{aligned} &\max_{q,e,\theta} \lambda(e)[B(q_L) - t_L] + \{1 - \lambda(e)\}[(B(q_H) - t_H] \\ &= \lambda(e)[B(q_L) - \{\beta_L \cdot q_L + c(\beta_L, \theta_L)\} - U_L] \\ &\quad + \{1 - \lambda(e)\}[B(q_H) - \{\beta_H \cdot q_H + c(\beta_H, \theta_H)\}]. \end{aligned} \tag{9}$$

Then the first order condition with respect to q_L is

$$B'(q_L) = \beta_L.$$

Thus for the efficient agent, the second best output q_L^{SB} is given by

$$q_L^{SB} = q_L^{FB}.$$

The first order condition with respect to q_H is

$$B'(q_H) = \beta_H.$$

Thus for the inefficient agent, the second best output q_H^{SB} is given by

$$q_H^{SB} = q_H^{FB}.$$

The second best outputs in case 2 are the same as those under the full information case. Next we derive optimal signals. For the efficient agent, by differentiating (9) with respect to θ_L, we have

$$\lambda(e)[-c_{\theta_L}(\beta_L, \theta_L)] < 0.$$

Thus we have $\theta_L^{SB} = 0$. For the inefficient agent, by differentiating (9) with respect to θ_H, we have

$$[1 - \lambda(e)][-c_{\theta_H}(\beta_H, \theta_H)] < 0.$$

Thus we also have $\theta_H^{SB} = 0$. Therefore the optimal contract induces no falsification. We summarize these results in the following proposition.

Proposition 1 *The optimal contract has the following features. If binding constraints are* (2), (3) *and* (6), *then we have, for the efficient agent,*

$$q_L^{SB} = q_L^{FB},$$

and for the inefficient agent,

$$q_H^{SB} < q_H^{FB}.$$

Moreover the inefficient agent chooses costly signals provided that $\lambda(e)c_{\theta_H}(\beta_L, \theta_H) - c_{\theta_H}(\beta_H, \theta_H) > 0$. *If binding constraints are* (2) *and* (6), *then we have, for the efficient agent,*

$$q_L^{SB} = q_L^{FB},$$

and for the inefficient agent,

$$q_H^{SB} = q_H^{FB}.$$

The agent will not send signals.

4. Discussion

In the previous section, we examined optimal contracts when the firm has private information on the costs of production and can send a costly signal to the government. We showed that the efficient type of the agent obtains information rents and that there exists a case in which the firm sends a costly signal to the government. In this section, we analyze a setting in which the firm's production costs are given by $\beta \cdot q + C(\beta)$, where $C(\beta)$ denotes fixed costs, which depend upon the agent's type. In this case, the limited liability constraints become

$$U_L = t_L - \beta_L \cdot q_L - c(\beta_L, \theta_L) - C(\beta_L) \geqq 0 \tag{10}$$

and

$$U_H = t_H - \beta_H \cdot q_H - c(\beta_H, \theta_H) - C(\beta_H) \geqq 0. \tag{11}$$

The ex ante participation constraint is given by

$$\lambda(e)\}U_H + \{1 - \lambda(e)\}U_L - \varphi(e) \geqq 0. \tag{12}$$

The incentive compatibility constraints are given by, for the efficient type,

$$\begin{aligned} U_L &= t_L - \beta_L \cdot q_L - C(\beta_L) - c(\beta_L, \theta_L) \\ &\geqq t_H - \beta_L \cdot q_H - C(\beta_L) - c(\beta_L, \theta_H) \end{aligned} \tag{13}$$

$$\begin{aligned}&= U_H + (\beta_H - \beta_L)\, q_H + C(\beta_H) - C(\beta_L)\\&\quad + c(\beta_H, \theta_H) - c(\beta_L, \theta_H)\end{aligned}$$

and for the inefficient type,

$$\begin{aligned}U_H &= t_H - \beta_H \cdot q_H - C(\beta_H) - c(\beta_H, \theta_H)\\&\geqq t_L - \beta_H \cdot q_L - C(\beta_H) - c(\beta_H, \theta_L)\\&= U_L - (\beta_H - \beta_L)\, q_L - C(\beta_H) + C(\beta_L)\\&\quad + c(\beta_L, \theta_L) - c(\beta_H, \theta_L).\end{aligned} \tag{14}$$

The moral hazard incentive constraint is given by

$$\lambda'(e)\{U_H - U_L\} - \varphi'(e) = 0. \tag{15}$$

Thus the government's problem can be written as

$$\begin{aligned}\max_{q,e,\theta}\quad & \lambda(e)\{B(q_L) - t_L\} + \{1 - \lambda(e)\}\{B(q_H) - t_H\}\\&\text{subject to (10), (11), (12), (13), (14), and (15).}\end{aligned} \tag{16}$$

The optimal contract in this case can depend on $C(\beta)$ and $c(\beta,\ \theta)$. Therefore it is possible that countervailing incentives arise in this case. The efficient agent's second best output can be larger than the full information level of output.

We look at the case in which constraints (10) and (14) are binding. By (10), we have $t_L = \beta_L \cdot q_L + c(\beta_L, \theta_L) + C(\beta_L)$. Then, from (14), we have

$$t_H = \beta_H \cdot q_H - (\beta_H - \beta_L)\, q_L + C(\beta_L) + c(\beta_L, \theta_L) - c(\beta_H, \theta_L) + c(\beta_H, \theta_H).$$

Substituting these transfers into (16), we have

$$\begin{aligned}&\lambda(e)\{B(q_L) - \beta_L \cdot q_L - c(\beta_L, \theta_L) - C(\beta_L)\}\\&+ \{1 - \lambda(e)\}\{B(q_H) - \beta_H \cdot q_H + (\beta_H - \beta_L)\, q_L\\&- C(\beta_L) - c(\beta_L, \theta_L) + c(\beta_H, \theta_L) - c(\beta_H, \theta_H).\end{aligned} \tag{17}$$

Differentiating (17) with respect to q_L and q_H, we have

$$B'(q_L) = \beta_L - \frac{\lambda(e)}{1 - \lambda(e)}(\beta_H - \beta_L)$$

and

$$B'(q_H) = \beta_H.$$

Therefore we can conclude that in this case,

$$q_L^{SB} > q_L^{FB} \text{ and } q_H^{SB} = q_H^{FB}.$$

For optimal signals, we have $\theta_L > 0$ if $\{1 - \lambda(e)\} c_{\theta_L}(\beta_H, \theta_L) - c_{\theta_L}(\beta_L, \theta_H) > 0$ for the efficient type and θ_H

= 0 for the inefficient type. We summarize these results in the following proposition.

Proposition 2 *Suppose that binding constraints are* (10) *and* (14). *Then, we have, for the efficient agent*,

$$q_L^{SB} > q_L^{FB},$$

and for the inefficient agent,

$$q_H^{SB} = q_H^{FB}.$$

Moreover the efficient agent chooses costly signals provided that $\{1-\lambda(e)\}c_{\theta_L}(\beta_H, \theta_L) - c_{\theta_L}(\beta_L, \theta_H) > 0$.

5. Conclusion

This paper has studied optimal contracts between the firm and the government in the presence of both moral hazard and adverse selection in a model in which the firm can choose a costly signal to falsify her private information. We have shown that the optimal contract exhibits different regimes depending on the costs of signals and those of exerting efforts, and that there exists a case in which the firm sends a costly signal to the government. Furthermore, we have examined a setting in which the firm's production costs are composed of a variable cost and a fixed cost, and the latter also depends upon the firm's type. We have shown that there may exist a case in which countervailing incentives and a costly signal arise at equilibrium.

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