Working Paper Series

No. 14-01

March 2014

Money, Inflation, and Currency Choice: A Primer

Tetsuya Saito

Research Institute of Economic Science

College of Economics, Nihon University

Money, Inflation, and Currency Choice: A Primer[1]

Tetsuya Saito
Nihon University College of Economics
1-3-2, Misakicho, Chiyoda-Ku, Tokyo, Japan 101-8360
saito.tetsuya@nihon-u.ac.jp

March 5, 2014

[1] I am thankful to Jai-Won Ryou and all participants of my session at the Korean International Economic Association Annual Meetings 2013 and Hayama Meetings for Further Development of Dynamics Economic Research (Chuo University). This study was supported by a grant-in-aid from Zengin Foundation for Studies on Economics and Finance (#1312).

1 Introduction

The search-theoretic model of money developed by Kiyotaki and Wright [6, 7], Trejos and Wright [14], and Lagos and Wright [8] has had several impacts in the study of the function of money. I believe that the basic idea of the search-theoretic approach is absolutely right. However, there is an annoying factor in the model when we deal with inflation. Such an outcome emanates from the risk-sharing behavior of agents in the bargaining process: higher storage costs of money imposed on buyers are compensated by sellers. Even the third generation model (Lagos and Wright, *op. cit.*) is also not free from this problem so long as bargaining process is included in the model.

In addition to the behavior of the money-search model, as stated above, we have a more fundamental problem in dealing with inflation. In particular, as it has long been argued, the use of additively-separable form refrains us from dealing with intertemporal risks; hence, agents in such models are intertemporal risk-neutral even if they are intratemporal risk-averse. In order to resolve this kind of problem, for example, Epstein and Zin [4] considers an aggregation using a CES form and Wakai [15] considers axiomatizing intertemporal preference for smoothing in a general framework employing typical von Neumann-Morgenstern utility.[1] When we consider intertemporal consumptions, we *at least* want to express the value function (V) using expected value (E) and variance (Σ^2) as $V = E - \gamma\Sigma^2$, where γ is a risk preference parameter, as broadly applied in finance literatures.

This paper considers an extension of the second generation money-search model (Trejos and Wright, *op. cit.*) to tackle the intertemporal risk problem focusing on the relationship between value of money and inflation. Fortunately, in the money-search model, we do not face state-dependent difficulty, as discussed in Wakai [16]. In addition, the utility is already decomposed into two components in the Trejos-Wright model: one is the expected loss of purchasing power, say *expected value effect*, and the other is the risk, say *variance effect*, in the matching process. The intertemporal effect is then computed as a residual in the value function. This structure further simplifies our analysis.

The extension employs a similar framework of Li [9] that studies taxation to design inflation in this paper. In particular, inflation is supposed to confiscate money from money holders stochastically to endow non money holders with the confiscated money. We can then find that inflation devalues money via the variance effect instead of the expected value effect. This result also indicates that the intertemporal risks are important factors to use the search-theoretic model to study money. In addition, the search framework can naturally be translated into experimental frameworks, as Brown [1] and Duffy and Ochs [2, 3]. In the experiment, we may have to control the intertemporal problem, as actual players may not be ideal. The framework developed in this study can be applied to further experimental investigations to see behaviors of agents in a monetary economy during inflation.

The extended model is further extended to a dual-currency environment. In the dual-currency model, we consider a good money and a bad money to find the Gresham's Law, where the good money has zero inflation rate while the bad money has a positive inflation rate. The law is conditionally observed and the conditions are given *vis-à-vis* the expected value and the variance effects of the two currencies. The result is not surprising, but it will be more important to provide a testable hypothesis in the experimental study.

The discussion is developed as follows. The basic single-currency framework is given by Section 2.1. The equilibria and their characterizations are provided in Section 2.2. By extending the single-currency model, Section 3 develops a dual-currency model. The basic framework of the dual-currency model is provided in Section 3.1 and it is simplified to study a dual-currency environment of a fiat money and a commodity money in Section 3.2 to apply for the Gresham's Law in Section 3.3. Section 4 concludes the study.

2 The Single Currency Model

2.1 Basic Framework

We consider an extension of the Trejos-Wright model (Trejos and Wright [14]), where money is indivisible but goods are perfectly divisible. In the extended model, inflation is considered by using a similar notion as Li [9] uses to consider optimum taxation. In Li [*op. cit.*], government agents are in the matching process and players that are paired with these agents pay tax (money is thus randomly confiscated to redistribute it). In my model, I suppose that inflation confiscates money from money holders to redistribute it for non money holders. In other words, inflation punishes money holders and it rewards non money holders. The flowchart for the matching process is depicted in Figure 1.

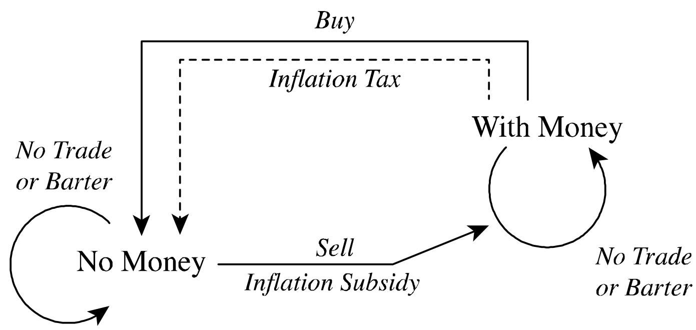

Figure 1: Flowchart for the single-currency model

Let us consider a matching process conforming to a Poisson process. There are sufficient number of infinitely lived agents in this process. In the model, $\mu \times 100\%$ of agents have money and we call them *buyers*. The rest of $(1-\mu) \times 100\%$ of agents do not hold money and we call them *sellers*. Each agent is capable of producing a certain of product. The production technology of each type is commonly represented by $c(q)$ such that $c(q) \geq 0$ with equality at $q = 0$, $c'(q) > 0$, and $c''(q) \geq 0$. They derive utility from products of other agents of a particular type using a common utility function $u(q)$ such that $u(q) \geq 0$ with equality at $q = 0$, $u'(q)$, and $u''(q) < 0$. In addition, we assume $u'(0) > c'(0)$ and $u'(q) < c'(q)$ for $q \to +\infty$.

For simplicity, with no loss of generality, we assume a single coincidence of want is *ad hoc*, which means that agents change their preference every time of meeting. The probability of a single coincidence is then given by $\sigma \in (0, 1]$. As the single coincidence of want is *ad hoc*, the double coincidence of wants is also *ad hoc* and its probability is given by δ. The frequency of meeting an agent is represented by arrival rate $\lambda > 0$ and the length of each period is given by $\tau > 0$. Without loss of generality, we can take τ being small enough to set $\lambda \leq 1$.

The impacts of inflation are decomposed into the expected value effect and the variance effect. The first effect is represented by the probability of confiscation of money π and the second one by the overall risks in the future given by γ_0 for a seller and γ_1 for a buyer. The variance effect in our model is an analogue of storage cost of money, which is a utility cost imposed only on buyers, in the standard Trejos-Wright model. The confiscated money is redistributed to non money holders at a certain probability π'. Since $\mu\pi$ of money is confiscated and there are $1-\mu$ of sellers, π' is computed as

$$\pi' = \frac{\mu\pi}{1-\mu}. \tag{1}$$

Since the number of sellers who are subsidized by inflation does not exceed the population of sellers, we have $\mu\pi < 1-\mu$, so that, $\pi' < 1$.

We let $V_0(t)$ be the value being a seller in period t. Similarly, we let $V_1(t)$ be the value being a buyer then. When a common future discounting rate is $r > 0$, the present values satisfy Bellman equations such as

$$\begin{aligned}(1+\tau r)V_0(t) &= \pi' \begin{pmatrix} \tau\lambda\sigma(1-\mu)\{u(q)+V_0(t+\tau)\} \\ +\tau\lambda\delta\{u(\bar{q})-c(\bar{q})+V_1(t+\tau)\} \\ +\{1-\tau\lambda\{\sigma(1-\mu)+\delta\}\}V_1(t+\tau)+\tau\gamma_1 \end{pmatrix} \\ &\quad +(1-\pi')\begin{pmatrix} \tau\lambda\sigma\mu\{V_1(t+\tau)-c(q)\} \\ +\tau\lambda\delta\{u(\bar{q})-c(\bar{q})+V_0(t+\tau)\} \\ +\{1-\tau\lambda(\sigma\mu+\delta)\}V_0(t+\tau)+\tau\gamma_0 \end{pmatrix}+o_0(\tau), \end{aligned} \tag{2}$$

and

$$\begin{aligned}(1+\tau r)V_1(t) &= (1-\pi)\begin{pmatrix} \tau\lambda\sigma(1-\mu)\{u(q)+V_0(t+\tau)\} \\ +\tau\lambda\delta\{u(\bar{q})-c(\bar{q})+V_1(t+\tau)\} \\ +\{1-\tau\lambda\{\sigma(1-\mu)+\delta\}\}V_1(t+\tau)+\tau\gamma_1 \end{pmatrix} \\ &\quad +\pi\begin{pmatrix} \tau\lambda\sigma\mu\{V_1(t+\tau)-c(q)\} \\ +\tau\lambda\delta\{u(\bar{q})-c(\bar{q})+V_0(t+\tau)\} \\ +\{1-\tau\lambda(\sigma\mu+\delta)\}V_0(t+\tau)+\tau\gamma_0 \end{pmatrix}+o_1(\tau), \end{aligned} \tag{3}$$

where $o_i(\tau)$ represents counting losses such that $o_i(\tau)/\tau = 0$ for $\tau \to 0$, and $i \in \{0,1\}$ the amount of money that an agent currently stores. In addition, in the above two equations, $\bar{q}$ denotes the quantity of trade when there is a double-coincidence of wants.[2] The Bellman equations are then arranged for $\tau \to 0$ to get

$$\begin{aligned} r(V_1-V_0) &= (1-\hat{\pi})\lambda\sigma[(1-\mu)\{u(q)+V_0-V_1\}-\mu\{V_1-V_0-c(q)\}] \\ &\quad +(1-\hat{\pi})(\gamma_1-\gamma_0)-\hat{\pi}\left(\lim_{\tau\to 0}\frac{V_1(t+\tau)-V_0(t+\tau)}{\tau}\right) \\ &\quad +\dot{V}_1(t)-\dot{V}_0(t), \end{aligned} \tag{4}$$

where

$$\lim_{\tau\to 0}\frac{V_1(t+\tau)-V_0(t+\tau)}{\tau} = \lim_{\tau\to 0}\underbrace{\frac{V_1(t+\tau)-V_0(t+\tau)}{M(t+\tau)-M(t)}}_{\text{Marginal Benefit of Money}} \times \frac{M(t+\tau)-M(t)}{\tau}, \tag{5}$$

and

$$\hat{\pi} \equiv \pi+\pi' = \frac{\pi}{1-\mu}. \tag{6}$$

By definition, we have

$$\frac{M(t+\tau)-M(t)}{\tau} = 1. \tag{7}$$

Thus, the marginal benefit of money ϕ is alternatively given by

$$\phi = \lim_{\tau\to 0}\frac{V_1(t+\tau)-V_0(t+\tau)}{\tau}, \tag{8}$$

and then

$$r\left(V_1 - V_0\right) = \left(1-\hat{\pi}\right)\lambda\sigma\left[\left(1-\mu\right)\left\{u\left(q\right)+V_0-V_1\right\}-\mu\left\{V_1-V_0-c\left(q\right)\right\}\right] + \left(1-\hat{\pi}\right)\left(\gamma_1-\gamma_0\right)-\hat{\pi}\phi+\dot{V}_1-\dot{V}_0. \tag{9}$$

Let $\theta \in [0,1]$ be buyer's bargaining power. The splitting rule is then applied to get the bargaining solution rule as

$$\frac{u\left(q^*\right)-V_1+V_0}{V_1-V_0-c\left(q^*\right)} = \frac{\theta}{1-\theta} \quad \Rightarrow \quad V_1-V_0 = \left(1-\theta\right)u\left(q^*\right)+\theta c\left(q^*\right), \tag{10}$$

which is substituted into (9) to get

$$\dot{V}_1-\dot{V}_0 = \left\{r\left(1-\theta\right)-\left(1-\hat{\pi}\right)\lambda\sigma\left(\theta-\mu\right)\right\}u\left(q^*\right)+\left\{r\theta+\left(1-\hat{\pi}\right)\lambda\sigma\left(\theta-\mu\right)\right\}c\left(q^*\right) + \left(1-\hat{\pi}\right)\left(\gamma_0-\gamma_1\right)+\hat{\pi}\phi. \tag{11}$$

Since $M\left(t+\tau\right)-M\left(t\right)=1$, we find $\phi = V_1 - V_0$ and ϕ is computed as

$$\phi = \left(1-\theta\right)u\left(q^*\right)+\theta c\left(q^*\right), \tag{12}$$

which is substituted into (11) to eventually get

$$\dot{V}_1-\dot{V}_0 = \left\{\left(1-\theta\right)\left(r+\hat{\pi}\right)-\left(1-\hat{\pi}\right)\lambda\sigma\left(\theta-\mu\right)\right\}u\left(q^*\right) + \left\{\theta\left(r+\hat{\pi}\right)+\left(1-\hat{\pi}\right)\lambda\sigma\left(\theta-\mu\right)\right\}c\left(q^*\right)+\left(1-\hat{\pi}\right)\eta, \tag{13}$$

where η is defined by

$$\eta \equiv \gamma_0 - \gamma_1. \tag{14}$$

In (14), unless agents have risk-loving preferences, $\gamma_i \leq 0$ is held; hence, $|\gamma_0|$ and $|\gamma_1|$ are interpreted as risks being a seller and a buyer, respectively, and the variance effect is represented by $\eta \equiv |\gamma_1| - |\gamma_0|$. Thus, η hereafter is said the relative risk of being a buyer for $\eta > 0$, as $|\gamma_1| > |\gamma_0|$, and of being a seller for $\eta < 0$, as $|\gamma_0| > |\gamma_1|$. It should also be noted here that (13) coincides with the corresponding equation in the standard Trejos-Wright model when $\gamma_0 = 0$ and $\pi = 0$.

2.2 The Steady State Equilibria

In a steady state equilibrium, we find

$$\dot{V}_i\left(t\right)-\dot{V}_0\left(t\right)=0, \quad \forall t. \tag{15}$$

The bargaining solution rule (10) further provides

$$\dot{V}_1-\dot{V}_0 = \dot{q}^*\left\{\left(1-\theta\right)u'\left(q^*\right)+\theta c'\left(q^*\right)\right\}, \tag{16}$$

and that implies

$$\dot{q}^* \gtreqless 0 \quad \Leftrightarrow \quad \dot{V}_1-\dot{V}_0 \gtreqless 0. \tag{17}$$

Table 1: Summary of Proposition 2

	$\theta > \mu$	$\theta < \mu$
$\hat{\pi} < 1$	1. $\cap$-shaped (Figure 2) 2. Buyer's Risk > Seller's Risk ($\eta > 0$) 3. Relative increase in buyer's risk	1. $\cup$-shaped (Figure 3) 2. Seller's Risk > Buyer's Risk ($\eta < 0$) 3. Relative increase in buyer's risk
$\hat{\pi} > 1$	1. $\cup$-shaped (Figure 3) 2. Buyer's Risk > Seller's Risk ($\eta > 0$) 3. Relative increase in seller's risk	1. $\cap$-shaped (Figure 2) 2. Seller's Risk > Buyer's Risk ($\eta < 0$) 3. Relative increase in seller's risk

1. Shape of RHS and figure to refer
2. Relative risks of seller's and buyer's
3. Cause of devaluation by the variance effect

Thus, from (13) and (17), we obtain the following condition.

$$\dot{q}^* \gtreqless 0 \quad \Leftrightarrow \quad (1-\theta)\, u(q^*) + \theta c(q^*) \gtreqless \frac{(1-\hat{\pi})\left[\lambda\sigma(\theta-\mu)\{u(q^*)-c(q^*)\}-\eta\right]}{r+\hat{\pi}} \tag{18}$$

The stable and instable equilibria are then determined as shown in Figures 2 and 3, where LHS and RHS denote the left-hand-side and the right-hand-side of (18). The feasible range for q^* is given by the individual rationality condition $q^* \in \{q^* | u(q^*) - c(q^*) \geq 0\}$. Characterizations of the stable equilibrium are provided in the following two propositions for expected value and variance effects, respectively.

Proposition 1 (Expected Value Effect) *We consider the stable equilibrium. The purchasing power of money goes up at the stable equilibrium as π or μ goes up, but it goes down as λ or σ goes up.*

Proof. As $\hat{\pi} = \pi/(1-\mu)$, it is easily found that $(1-\hat{\pi})/(r+\hat{\pi})$ is decreasing in π and μ by taking partial derivatives with respect to π and μ, respectively. An increase in π or μ then induces a downward shift of RHS for $RHS \geq 0$, as indicated by Figures 2 and 3, which implies that the value of the stable equilibrium (purchasing power of money) goes up at the stable equilibrium. Similarly, an increase in λ or σ induces an upward shift of RHS for $RHS > 0$, which implies the value of stable equilibrium goes down. ■

Proposition 2 (Variance Effect) *Suppose there is a stable state. At the stable steady state, an increase in the buyer's risk relative to the seller's devalues money when $\hat{\pi} < 1$. In contrast, an increase in the seller's risk relative to the buyer's devalues money when $\hat{\pi} > 1$.*

Proof. Figure 2 for $(1-\hat{\pi})(\theta-\mu) > 0$ and Figure 3 for $(1-\hat{\pi})(\theta-\mu) < 0$ are referred to find respective equilibria. We have to have $(1-\hat{\pi})\eta < 0$ for $(1-\hat{\pi})(\theta-\mu) > 0$ and $(1-\hat{\pi})\eta < 0$ for $(1-\hat{\pi})(\theta-\mu) > 0$ to find a stable equilibrium in each case. At the stable equilibrium for both cases, an increase in the intercept implies a reduction of the value of money. Therefore, at the stable equilibrium, the relative risk of being a buyer $\mu > 0$ devalues money for $\hat{\pi} < 1$ and $\theta > \mu$; the relative risk of being a seller $\mu < 0$ does for $\hat{\pi} > 1$ and $\theta < \mu$; the relative risk of being a buyer $\mu > 0$ does for $\hat{\pi} < 1$ and $\theta < \mu$; and the relative risk of being a seller $\mu < 0$ does for $\hat{\pi} > 1$ and $\theta > \mu$. ■

The statements for μ, λ, and σ in Proposition 1 are well-known in the standard Trejos-Wright model. This proposition suggests that the expected value effects of λ and σ that increase the frequency of trade devalue money to reduce the price level, which is a quite plausible observation. However, it also suggests that the expected value effect of "inflation" represented by π that does not devalue money, as the expected value effect of μ does (another candidate for inflation), as we have annoyed when we apply the search-theoretic

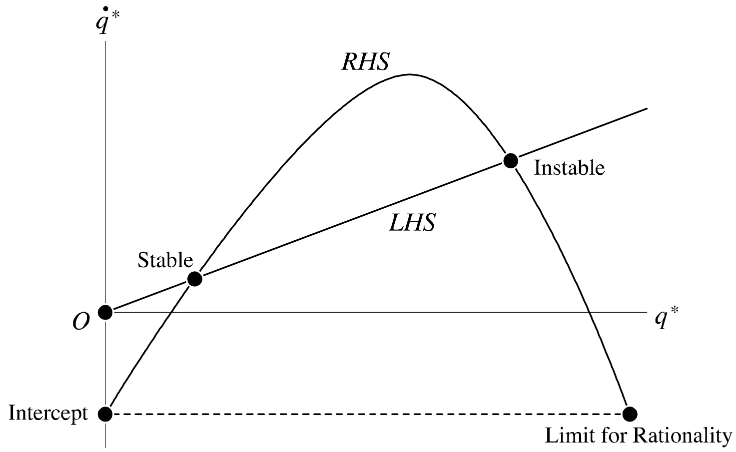

Figure 2: Determination of equilibria for $(1-\hat{\pi})(\theta-\mu)>0$

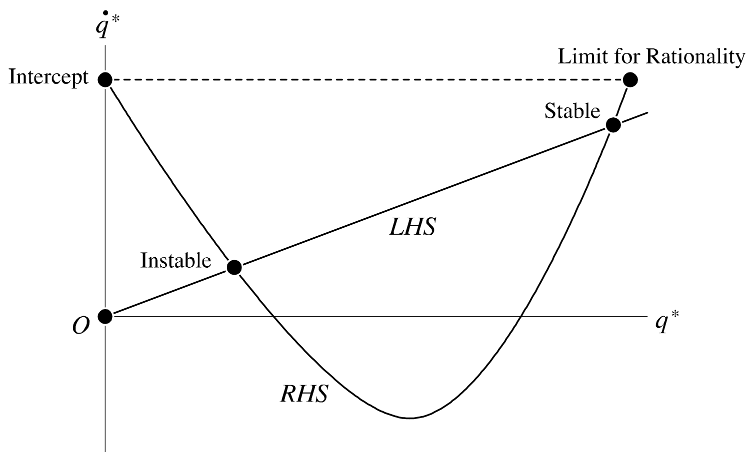

Figure 3: Determination of equilibria for $(1-\hat{\pi})(\theta-\mu)<0$

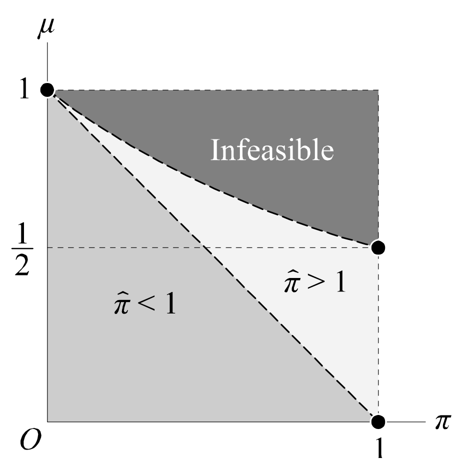

Figure 4: Feasibility of (π,μ) for $\hat{\pi} \gtrless 1$ with $\pi' < 1$

approach.[3] The overall effect of inflation is not monotone, as it is a combination of the expected value and the variance effects.

Let us focus on the variance effect. Since $\hat{\pi} \equiv \pi / (1-\mu)$, we can easily find that $\hat{\pi} \lesseqgtr 1$ implies $\pi \lesseqgtr 1-\mu$, or equivalently $\pi' \lesseqgtr \mu$ as (1). Feasibility sets of (π, μ) for $\hat{\pi} < 1$ and $\hat{\pi} > 1$ with $\pi' < 1$ are depicted in Figure 4, where the upper dashed curve and the lower one are $\mu = 1/(1+\pi)$ and $\mu = 1-\pi$, respectively. Proposition 2 then implies the followings. If $\pi < 1-\mu$, it indicates that the chance of losing money without meeting a seller is less than the chance of using money for a good for a buyer, or equivalently, the chance of obtaining money without meeting a buyer is less than the chance of obtaining money from a transaction for a seller. In such cases, where making business is better than waiting, a relative increase in buyer's risk devalues money at the stable equilibrium. If $\pi > 1-\mu$, it indicates that the chance of losing money without meeting a seller is higher than the chance of using money for a good for a buyer, or equivalently, the chance of obtaining money without meeting a buyer is higher than the chance of obtaining money from a transaction for a seller. In such cases, where waiting is better than making business, a relative increase in seller's risk devalues money at the stable equilibrium.

3 The Dual-Currency Model

3.1 Basic Framework

We extend the single-currency model of the previous section to a dual-currency environment. The currencies are indexed by $i = \{1, 2\}$. With this notation rule, we can represent no-money case as $i = 0$ as we have used in the previous section. Agents are still not allowed to carry money exceeding a unit. The life-cycle of the agent in this model is then shown as Figure **??**.

In order to compute value functions, we define μ and π'_i as

$$\mu \equiv m_1 + m_2 \in [0,1] \quad \text{and} \quad \pi'_i \equiv \frac{m_i \pi_i}{1-\mu}, \tag{19}$$

where m_i represents the share of money holders with Currency i and π_i the probability of confiscation when carrying Currency i. In addition, we also define

$$\pi' \equiv \pi'_1 + \pi'_2 \quad \text{and} \quad \hat{\pi}_i \equiv \pi_i + \pi'_i \equiv \frac{(1-m_j)\,\pi_i}{1-\mu} \quad \text{for} \quad i \neq j. \tag{20}$$

With these notations, the Bellman equations of a non money holder and a Currency i holder are given by

$$\begin{aligned}
(1+\tau r)\, V_0(t) \quad &= \sum_{j=\{1,2\}} \pi'_j \begin{pmatrix} \tau\lambda\sigma\,(1-\mu)\,\{u\,(q_j) + V_0\,(t+\tau)\} \\ +\tau\lambda\delta\,\{u\,(\bar{q}) - c\,(\bar{q}) + V_j\,(t+\tau)\} \\ +\{1-\tau\lambda\,\{\sigma\,(1-\mu)+\delta\}\}\, V_j\,(t+\tau) + \tau\gamma_j \end{pmatrix} \\
&+ (1-\pi') \begin{pmatrix} \tau\lambda\sigma \sum_{j=\{1,2\}} m_j\,\{V_j\,(t+\tau) - c\,(q)\} \\ +\tau\lambda\delta\,\{u\,(\bar{q}) - c\,(\bar{q}) + V_0\,(t+\tau)\} \\ +\{1-\tau\lambda\,(\sigma\mu+\delta)\}\, V_0\,(t+\tau) + \tau\gamma_0 \end{pmatrix} + o_0\,(\tau)\,,
\end{aligned} \tag{21}$$

and

$$(1+\tau r) V_i(t) = (1-\pi_i)\begin{pmatrix} \tau\lambda\sigma(1-\mu)\{u(q_i)+V_0(t+\tau)\} \\ +\tau\lambda\delta\{u(\bar{q})-c(\bar{q})+V_i(t+\tau)\} \\ +\{1-\tau\lambda\{\sigma(1-\mu)+\delta\}\}V_i(t+\tau)+\tau\gamma_i \end{pmatrix} + \pi_i\begin{pmatrix} \tau\lambda\sigma\sum_{j=\{1,2\}} m_j\{V_j(t+\tau)-c(q)\} \\ +\tau\lambda\delta\{u(\bar{q})-c(\bar{q})+V_0(t+\tau)\} \\ +\{1-\tau\lambda(\sigma\mu+\delta)\}V_0(t+\tau)+\tau\gamma_0 \end{pmatrix} + o_i(\tau). \tag{22}$$

Letting $k \in \{1,2\}$ and $k \neq i$, the above two equations provide

$$\begin{aligned} r\{V_i(t)-V_0(t)\} = {} & (1-\hat{\pi}_i)\lambda\sigma(1-\mu)\{u(q_i)+V_0(t+\tau)-V_i(t+\tau)\} \\ & -\pi_k'\lambda\sigma(1-\mu)\{u(q_k)+V_0(t+\tau)-V_k(t+\tau)\} \\ & -(1-\hat{\pi}_i-\pi_k')\lambda\sigma\sum_{j=\{1,2\}} m_j\{V_j(t+\tau)-V_0(t+\tau)-c(q)\} \\ & +(1-\hat{\pi}_i)\gamma_i-\pi_k'\gamma_k-(1-\hat{\pi}_i-\pi_k')\gamma_0 \\ & -\hat{\pi}_i\cdot\frac{V_i(t+\tau)-V_0(t+\tau)}{\tau}-\pi_k'\cdot\frac{V_k(t+\tau)-V_0(t+\tau)}{\tau} \\ & +\frac{V_i(t+\tau)-V_i(t)}{\tau}-\frac{V_0(t+\tau)-V_0(t)}{\tau}+\frac{o_i(\tau)}{\tau}-\frac{o_0(\tau)}{\tau}, \end{aligned} \tag{23}$$

where ϕ_i represents the marginal benefit of money for Currency i. For $\tau \to 0$, this equation further provides

$$\begin{aligned} r(V_i-V_0) = {} & (1-\hat{\pi}_i)\lambda\sigma(1-\mu)\{u(q_i)+V_0-V_i\}-\pi_k'\lambda\sigma(1-\mu)\{u(q_k)+V_0-V_k\} \\ & -(1-\hat{\pi}_i-\pi_k')\lambda\sigma\sum_{j=\{1,2\}} m_j\{V_j-V_0-c(q)\} \\ & +(1-\hat{\pi}_i)(\gamma_i-\gamma_0)-\pi_k'(\gamma_k-\gamma_0)-\hat{\pi}_i\phi_i-\pi_k'\phi_k+\dot{V}_i-\dot{V}_0. \end{aligned} \tag{24}$$

Similarly to the single-currency model, the bargaining solution conforms to the splitting rule, such as

$$V_i - V_0 = \phi_i = (1-\theta)u(q_i^*)+\theta c(q_i^*), \tag{25}$$

which is inserted into (24) to get

$$\begin{aligned} \dot{V}_i-\dot{V}_0 = {} & r\{(1-\theta)u(q_i^*)+\theta c(q_i^*)\}-(1-2\pi_i)\lambda\sigma(1-\mu)\theta\{u(q_i^*)-c(q_i^*)\} \\ & +\pi_k\lambda\sigma(1-\mu)\theta\{u(q_k^*)-c(q_k^*)\} \\ & +(1-2\pi_i-\pi_k)\lambda\sigma(1-\theta)\sum_{j=\{1,2\}} m_j\{u(q_j^*)-c(q_j^*)\} \\ & -(1-2\pi_i)(\gamma_i-\gamma_0)+\pi_k(\gamma_k-\gamma_0)+2\pi_i\phi_i+\pi_k\phi_k. \end{aligned} \tag{26}$$

The bargaining rule also provides

$$\dot{V}_i-\dot{V}_0 = \dot{q}_i^*\{(1-\theta)u'(q_i^*)+\theta c'(q_i^*)\}. \tag{27}$$

Therefore, $\dot{q}_i \gtreqless 0$ implies

$$\begin{aligned}
& \lambda\sigma \left\{\pi_k' (1-\mu)\theta + (1-\hat{\pi}_i - \pi_k')(1-\theta) m_k\right\} \{u(q_k^*) - c(q_k^*)\} \\
& + \pi_k' \left\{(1-\theta) u(q_k^*) + \theta c(q_k^*)\right\} + \pi_k' (\gamma_k - \gamma_0) \\
\gtreqless\ & \lambda\sigma \left\{(1-\hat{\pi}_i)(1-\mu)\theta - (1-\hat{\pi}_i - \pi_k')(1-\theta) m_i\right\} \{u(q_i^*) - c(q_i^*)\} \\
& - (r+\hat{\pi}_i) \left\{(1-\theta) u(q_i^*) + \theta c(q_i^*)\right\} + (1-\hat{\pi}_i)(\gamma_i - \gamma_0). \qquad (28)
\end{aligned}$$

3.2 A Special Case: Fiat Money Vs. Commodity Money

Let us provide $\theta = 1$, $\pi_1 > 0$ and $\pi_2 = 0$. This parameter set implies that buyers are dictatorial and Currencies 1 and 2 are a fiat money and a commodity money, respectively. Then, $\dot{q}_1 \gtreqless 0$ implies

$$c(q_1^*) \gtreqless \frac{(1-\hat{\pi}_1)\left[\lambda\sigma(1-\mu)\{u(q_1^*) - c(q_1^*)\} - \eta_1\right]}{r + \hat{\pi}_1}. \qquad (29)$$

In this case, the stable equilibrium is determined as in the single-currency model (Figure 2 or 3) and Propositions 1 and 2 are directly applicable for Currency 1; whence, $\hat{\pi}_1 < 1$ and $\eta_1 > 0$ or $\hat{\pi}_1 > 1$ and $\eta_1 < 0$ are required to find a stable equilibrium. Similarly, $\dot{q}_2 \gtreqless 0$ implies

$$\pi_1' \left[\lambda\sigma(1-\mu)\{u(q_1^*) - c(q_1^*)\} + c(q_1^*) - \eta_1\right] \gtreqless \lambda\sigma(1-\mu)\{u(q_2^*) - c(q_2^*)\} - rc(q_2^*) - \eta_2, \qquad (30)$$

where $\eta_2 \equiv \gamma_0 - \gamma_2$.

At the steady state, (29) holds with equality and that provides

$$\lambda\sigma(1-\mu)\{u(q_1^*) - c(q_1^*)\} + c(q_1^*) - \eta_1 = \frac{1+r}{1-\hat{\pi}_1} c(q_1^*). \qquad (31)$$

Thus, $\dot{q}_2 \gtreqless 0$ implies

$$rc(q_2^*) + \Gamma \gtreqless \lambda\sigma(1-\mu)\{u(q_2^*) - c(q_2^*)\}, \qquad (32)$$

where Γ is the intercept of the left-hand-side of (32) given by

$$\Gamma = \eta_2 + \frac{(1+r)\pi_1'}{1-\hat{\pi}_1} c(q_1^*). \qquad (33)$$

The stable equilibrium is then determined as shown in Figure 5, where LHS and RHS in this figure indicate the left-hand-side and the right-hand-side of (32).

From (32), however, we cannot specify the directions of changes of the value of two currencies. For instance, as $1-\hat{\pi}$ affects the equilibrium, an increase in one currency's value may or may not improve the value of the other. However, we can find that risks of one currency influence the value of the other, as recent models for counterfeiting money have suggested (Nosal and Wallace [11], Li and Rocheteau [10], and Shao [13]). Furthermore, since Proposition 2 directly applies to Currency 1, the purchasing power of Currency 2 decreases as that of Currency 1 decreases. In addition, a decrease in η_2 reduces the purchasing power of Currency 2, but it does not alter the purchasing power of Currency 1; hence, the fiat money looks as if it is independent of the commodity money. Yet, there may be implicit correlations between the risks of the two currencies, η_1 and η_2 via several channels, though correlation patters are too complex to study analytically. The study for this point is delegated to our experimental project.

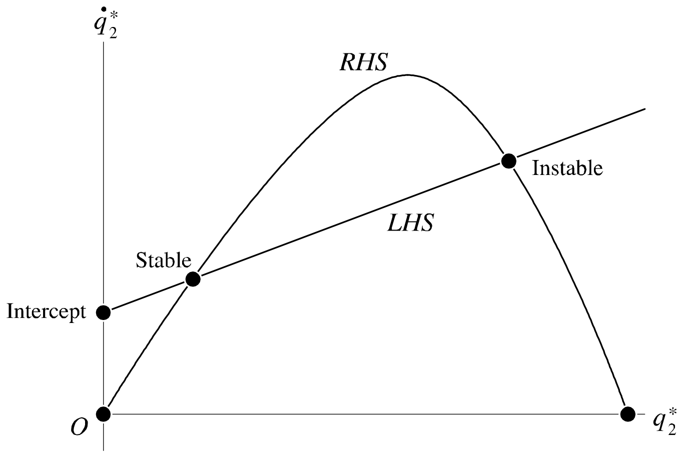

Figure 5: Determination of equilibria for Currency 2

3.3 Gresham's Law

The simplified model considered in Section 3.2 is applied to consider the Gresham's Law that is knows for a phrase "bad money drives out good money". In the model, Currency 1 is the bad money and Currency 2 is the good money, as $\pi_1 > \pi_2 \equiv 0$. In addition, we assume m_2 is constant. Since $\theta = 1 > \mu$, we can avoid a possibility of violating individual rationality (*cf.* Saito [12]). In addition, we assume Γ is positive to keep a stable equilibrium to exist.

If Currency 1 is driving out Currency 2, we are going to observe an increase in the risk of Currency 1 increases the value of itself while it reduces the value of Currency 2. If $\hat{\pi}_1 < 1$, we use Figures 2 and 5 to find the equilibrium for Currency 1. In the stable equilibrium, an increase in π_1 associated with an increase in the relative risk of being a seller increases the value of Currency 1. Similarly, if $\hat{\pi}_1 > 1$, we use Figures 3 and 5 to find the stable equilibrium. In the stable equilibrium, an increase in π_1 associated with an increase in the relative risk of being a buyer with Currency 1 increases the value of Currency 1. In order for each case to reduce the value of Currency 2, Γ must decrease. For $\hat{\pi}_1 < 1$, η_2 is required to decrease sufficiently; hence, $|\gamma_0|$ must sufficiently increase relative to $|\gamma_2|$. For $\hat{\pi}_1 > 1$, in contrast, η_2 is required not to increase too much; hence, an increase in $|\gamma_2|$ must be modest relative to $|\gamma_0|$. These arguments are valid even if there is a positive correlation between π_1 and m_1, which is usually observed in the process of inflation.

Remark 1 (Gresham's Law) *Let $\pi_1 > \pi_2 \equiv 0$, $\theta = 1$, and m_2 to be constant. Suppose there is a stable equilibrium. The Gresham's Law is then observed at the stable equilibrium in the following two cases (in the statements, a possibility of a positive correlation between π_1 and m_1 in inflation is not excluded).*

1. *For $\hat{\pi}_1 < 1$, inflation of bad money sufficiently increases the risk of being a seller relative to the risks of being a buyer with any currency.*

2. *For $\hat{\pi}_1 > 1$, inflation of bad money increases the risk of being a buyer with bad money (Currency 1) relative to the risk of being a seller and does not too much affect the risk of being a buyer with good money relative to the risk of being a seller.*

According to Remark 1, we find that the requirement for the emergence of Gresham's Law seems much easier for $\hat{\pi}_1 > 1$ than for $\hat{\pi}_1 < 1$, where $\hat{\pi}_1 \gtrless 1$ implies $(1 - m_2)\pi_1 \gtrless 1 - \mu$. In addition, we require

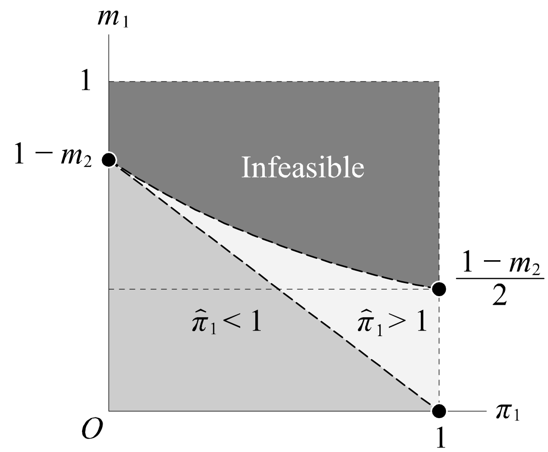

Figure 6: Feasibility of (π_1, m_1) for $\hat{\pi}_1 \gtrless 1$ with $\pi_1' < 1$

$m_1\pi_1 < 1-\mu$ for $\pi_1' < 1$. Thus, the feasibility set of (π_1, m_1) conditional on m_2, which is an analogue of Figure 4 in the single-currency model, is depicted in Figure 6. In this figure, the upper dashed curve and the lower one are $m_1 = (1-m_2)/(1+\pi_1)$ and $m_1 = (1-m_2)(1-\pi_1)$, respectively. The obtained result for the emergence of Gresham's Law is not so surprising. What is more important here is to obtain a testable hypothesis in experimental studies.

4 Concluding Remarks

This paper has extended the Trejos-Wright model in order to study intertemporal risks in a search-theoretic environment. The discussion for the single-currency model suggests that inflation reduces purchasing power of money when it increases relative risks instead when it increases the probability of confiscation. This finding will ease our annoyance about the behavior of money-search model in inflation, which seems caused by ignorance of intertemporal risks, as it suggests a channel to devalue money.

In the discussion for the dual-currency model, we find that the Gresham's Law is conditional on the expected value and the variance effects of the good and the bad monies. The conditionality of the Gresham's Law is not surprising. The result is more important as a hypothesis in the experiment than as a theory.

Notes

[1] There are several approaches to this problem. A recent approach is, for example, given by Ingersoll [5].

[2] It is known that $\bar{q}$ coincides with the social optimum: $\bar{q} = \arg\max\{u(q) - c(q)\}$.

[3] In the standard Trejos-Wright model, we also face another annoyance, as an increase in the storage cost increases the purchasing power of money at the stable equilibrium.

References

[1] Brown, Paul M. (1996). Experimental evidence on money as a medium of exchange. *Journal of Economic Dynamics and Control*, 20(4), 583-600.

[2] Duffy, John and Jack Ochs (1999). Emergence of money as a medium of exchange: an experimental study. *American Economic Review*, 89(4), 847-77.

[3] Duffy, John and Jack Ochs (2002). Intrinsically worthless objects as media of exchange: experimental evidence. *International Economic Review*, 43(3), 637-74.

[4] Epstein, Larry G. and Stanley E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption growth and asset returns: a theoretical framework. *Econometrica*, 57(4), 937–69.

[5] Ingersoll, Jonathan E., Jr. (2011). Time-additive consumption-wealth utility. *ICF Working Paper Series*, Yale School of Management.

[6] Kiyotaki, Nobuhiro and Randall Wright (1989). On money as a medium of exchange. *Journal of Political Economy*, 97(4), 927-54.

[7] Kiyotaki, Nobuhiro and Randall Wright (1991). A contribution to the pure theory of money. *Journal of Economic Theory*, 53(2), 215-35.

[8] Lagos, Ricardo and Randall Wright (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, 113(3), 463-84.

[9] Li, Victor (1995). The optimal taxation of fiat money in search equilibrium. *International Economic Review* 36(4), 927-42.

[10] Li, Yiting and Guillaume Rocheteau (2011). On the threat of counterfeiting. *Macroeconomic Dynamics*, 15(S1), 10-41.

[11] Nosal, Ed and Neil Wallace (2007). A model of (the threat of) counterfeiting. *Journal of Monetary Economics*, 54(4), 994ñ1001.

[12] Saito, Tetsuya (2012). Rationality and stability of equilibrium in a search-theoretic model of money. *Theoretical Economics Letters*, 2(3) 283-86.

[13] Shao, Enchuan (2013). The threat of counterfeiting in competitive search equilibrium. *Bank of Canada Working Paper Series*, No.2013-22.

[14] Trejos, Alberto and Randall Wright (1995). Search, bargaining, money, and prices. *Journal of Political Economy*, 103(1), 118-41.

[15] Wakai, Katsutoshi (2008). A model of utility smoothing. *Econometrica*, 76(1), 137-53.

[16] Wakai, Katsutoshi (2013). Intertemporal utility smoothing under uncertainty. *Theory and Decision*, 74(2), 285-310.