Regulating Illegal Items Under Variety Effect

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Abstract

This paper considers regulatory policies against illegal items, such as drugs, under variety effect using numerical simulations. In the model, there are legal and illegal items as differentiated products and the numeraire. We then find that fines levied on consumers may increase consumptions of illegal items while those on suppliers unambiguously reduce them. This finding is not surprising, as previous studies indicate such outcomes. The new finding of this paper is the impacts of policies in illegal varieties. Fines levied on consumers may expand illegal varieties and those on suppliers unambiguously expand them. The principal objective of regulatory policies is to reduce illegal consumptions, but expansions of varieties may mislead policy evaluations and may also raise investigation costs. The results of this paper thus provide important cases for further studies in the relationship among regulatory policies and patterns of illegal consumptions.

Keywords: Variety effect; regulatory policies; expansions of illegal varieties; expansions of illegal consumptions

JEL Classifications: D40; L51; K14; K42
1 Introduction

Some policies may not work as expected, but it is not surprising. For example, Lott and Mustard (1997), Duggan (2001), and Ehrlich and Saito (2010) provide discussions if controlling guns reduce crimes. A recent survey by Pollack et. al. (2011) discusses how drugs are not wiped out of the United States. Another survey by Donohue et. al. (2011) shows several empirical results to doubt policies against drugs. In the virtual space, there are several black markets, such as Agora, Blackmarket Reloaded, and Silk Road 2.0 (recently seized). In Japan, we currently face increasing number of varieties in quasi-legal drugs despite stringent regulations.

We still cannot wipe out illegal items despite several efforts. This paper considers a taste for variety model with legal and illegal items to see the existence of variety effect against regulations by extending a part of Saito (2015).\textsuperscript{1} The varieties in this paper are primarily drugs and the likes, but they are not restricted to such items. For example, varieties of arms for offenses (guns, knives, and other items) are also considered within the same framework.

One result of this study suggests that taxes on consumers may not work to reduce consumptions of illegal items while taxes on suppliers will work. As Ehrlich (1981, 1996) summarizes, an increase in fines unambiguously decreases the crime rate if there is only one type of crime. However, fines in our model may not reduce the consumption of illegal items, because there are alternative illegal items. If there are alternative illegitimate activities, as Ehrlich (1973, 1974) analyzes, an increase in fines reduces some types of offenses while other offenses may increase. Smith (1976) and Becker and Murphy (2006) also suggest analogous results when there are several alternatives.

Another result of this study suggests that the illegal variety may expand when regulations gets stringent even if illegal consumptions decline. The principal objective of policies is to reduce consumptions of illegal items, but a larger illegal varieties may lead a larger investigation cost. In such a case, by comparing the social cost and benefit, the regulatory authority needs to focus on reducing the illegal varieties. In addition, the regulatory authority may consider their policy is inadequate if the illegal varieties expand while the illegal consumptions decline behind them.

The discussions are developed as follows. The base model is introduced in Section 2 and solved in Section 3 (technical). The equilibrium is numerically explored in Section 4 using two policies on consumers and suppliers to derive main results. Concluding remarks are stated in Section 5.

\textsuperscript{1}In Saito (2015), the base model is designed to be the workforce of Bitcoin exchange rate when there are legal and illegal items. Thus, the analysis is not focused on the base model.
2 The Baseline Model

We consider a “new trade theory” framework (NTT) à la Krugman (1979, 1980). In the NTT, consumers are distributed in more than two countries. In our model, however, consumers are distributed in two groups, compliant and noncompliant, that are active in a digital economy. There are \( n \geq 0 \) compliant consumers and \( m \geq 0 \) noncompliant consumers; hence, the total population \( N \) is provided by

\[
N = n + m.
\]  

(1)

For simplicity, these consumers are supposed to be risk-neutral. Each consumer is endowed with \( h > 0 \) labor hours to be active in the digital economy; hence, the labor endowment is \( Nh \). Compliant consumers consume only legal items and engage only in legal productions. Noncompliant consumers consume both legal and illegal items and engage in both legal and illegal productions. For simplicity, suppliers are supposed to have long-run perspectives, so that, they earn no economic profit.

We assume that each individual has one’s own variety. Thus, there are \( n \) legal varieties and \( m \) illegal varieties. However, there is no demand and supply of legal variety if \( n = 0 \) and similarly no demand and supply of illegal variety if \( m = 0 \); hence, \( n \) and \( m \) as indices of varieties start from \( n = 1 \) and \( m = 1 \) instead of \( n = 0 \) and \( m = 0 \), respectively. Each item is monopolistically competitive, but free entry is allowed. In this case, \( N \) remains open while \( n/N \) and \( m/N \), or \( m/n \), are going to be endogenously determined.

In addition to these \( n + m \) varieties, there is a numéraire good (legal). The numéraire may be omitted from the model, but it simplifies the determination of equilibrium wage rate in our general equilibrium framework. Consumers then derive utility from consuming these goods in accordance with a Dixit-Stiglitz utility function (Dixit and Stiglitz, 1977), as discussed in the next two subsections, 2.1 and 2.2.

2.1 Compliant Consumers

Compliant consumers consume only legal items indexed by \( i = 1, \ldots, n \). Letting \( z \) and \( u \) be the quantity of consumption of the numéraire good and the utility from legal varieties, respectively, the utility function of a representative compliant consumer is provided in a Cobb-Douglas form:

\[
U(z, u) = z^\alpha u^{1-\alpha},
\]

(2)

where \( \alpha \in [0, 1) \) is the expenditure share of the numéraire; hence, \( 1 - \alpha \in (0, 1] \) is the expenditure share of the differentiated goods. The utility from varieties of legal items is
provided in a CES (constant-elasticity-of-substitution) form:
\[ u(x) = \left( \sum_{i=1}^{n} x_i^\theta \right)^{1/\theta}, \]

where \( x_i \) is the consumption of \( i \)th legal variety, \( x = (x_i)_{i=1}^{n} \) the vector representing a consumption bundle of legal items, and \( \theta \) the preference parameter associated with elasticity of substitution \( \sigma \) as \( \theta = (\sigma - 1) / \sigma \). In this model, all items are substitutes of each other and \( \sigma > 1 \) is bounded; hence, \( \theta \in (0, 1) \). This compliant consumer maximizes \( U(z, u) \) subject to the budget constraint:
\[ z + \sum_{i=1}^{n} p_i x_i = w h, \]

where \( w > 0 \) is the average wage rate and \( p_i > 0 \) the price of \( i \)th legal variety. The average wage rate is computed as a convex combination of wage rates engaging in productions of the numéraire good and a legal variety (two wage rates coincide with each other at the equilibrium).

### 2.2 Noncompliant Consumers

Noncompliant consumers consume both legal and illegal items indexed by \( i = 1, \ldots, n \) and \( j = 1, \ldots, m \), respectively. Similarly to the legal consumers, letting \( v \) be the utility from legal and illegal varieties, the utility function of a representative noncompliant consumer is represented by
\[ U(z, v) = z^\alpha v^{1-\alpha}. \]

The utility from varieties of legal and illegal items is represented by a CES form:
\[ v(x, y) = \left( \sum_{i=1}^{n} x_i^\theta + \sum_{j=1}^{m} y_j^\theta \right)^{1/\theta}, \]

where \( y_j \) is the consumption of \( j \)th illegal variety and \( y = (y_j)_{j=1}^{m} \) the vector representing a consumption bundle of illegal items.

The noncompliant consumer receives \( w \) by engaging in legal item production and \( w' > 0 \) in illegal item production. Letting \( \varepsilon \in (0, 1) \) be the share of working hours engaging in the legal item production, the average wage income is provided as
\[ \omega h = \varepsilon w h + (1 - \varepsilon) w' h. \]
At the labor market equilibrium, however, no worker has an incentive to work for the illegal item production if the expected wage rate is less than the market wage rate $w$. Thus, $w' = w$ in (7), and wage income of noncompliant consumer is provided by $\omega = w$. This noncompliant consumer maximizes the utility function subject to the budget constraint provided by

$$z + \sum_{i=1}^{n} p_i x_i + \sum_{j=1}^{m} \pi_j y_j = wh - \gamma,$$  

(8)

where $\omega > 0$ is the average wage rate, $\gamma \geq 0$ the fine against illegal consumption, and $\pi_j > 0$ the price of illegal variety $j$. The collected fines compensate for the cost of investigations.

### 2.3 Production Technologies

A unit of numéraire good uses a constant returns to scale technology with unit marginal cost. Letting $L_0$ be the labor input and $Z^S$ be the quantity of supply for production of the numéraire good, the production function is represented by

$$L_0 = Z^S.$$  

(9)

Letting $L_i$ be the labor input and $X^S_i$ be the quantity of supply for production of a legal item $i$, the production technology of this item is represented by

$$L_i = f + aX^S_i,$$  

(10)

where $f > 0$ and $a > 0$ are a common fixed input and a common input coefficient across varieties, respectively.

The production technologies of illegal items are identical to legal items, but additional fixed costs represented by $\eta$ are imposed, as fines. Letting $L'_j$ be the labor input, $\eta$ be fines levied on noncompliant suppliers, and $Y^S_j$ be the quantity of supply for production of illegal item $j$, the production technology of this item is represented by

$$L'_j = (f + \eta) + aY^S_j.$$  

(11)

The collected fines again compensate for the cost of investigations.

In our framework, legal and illegal items are distinguished by fines, so that, there is no difference between them if $\gamma = 0$. In addition, there is no noncompliant consumer if $\gamma = h$. If noncompliant consumers are fined, they expand varieties of illegal items in consumption to compensate for the loss in their income. The volume of consumption of illegal items is
then either increase or decrease depending on the scale effect in production. Hence, it is expected that fines suppress the production of illegal items if fines are sufficiently large, but an increase in fines increases the production of illegal items if fines are insufficient.\footnote{Such a mechanism seems similar to the argument about the existence of the worst configuration of trading blocs, as shown in Krugman (1991) and Bond and Syropoulos (1996): complete separation and unification are identical to each other (free trade).}

## 3 Market Equilibrium

In the labor market, the wage rate $w^*$ is determined competitively so as to equalize wage rates in all productions including the numéraire good. Since the production technology of the numéraire is classical, the zero-profit condition is applied to obtain

$$wL_0 = Z^S \Rightarrow w^* = 1.$$ \hspace{1cm} (12)

Preference and production structures are symmetric. At the equilibrium, consumptions of numéraire good and legal items by compliant consumers are provided by $z = z^*$ and $x_i = x^*$ for all legal items for all compliant consumers, respectively. Similarly, consumptions of the numéraire and legal and illegal items by noncompliant consumers are provided as $z = z^*$, $x_i = x^*$, and $y_j = y^*$ for all legal and illegal items for all noncompliant consumers. Let $Z^D$, $X^D$, and $Y^D$ be aggregated demands for the numéraire, a legal item, and an illegal item. At the symmetric equilibrium, respective aggregate demands are provided by

$$Z^D = nz^* + mz^*,$$ \hspace{1cm} (13)
$$X^D = nx^* + mx^*,$$ \hspace{1cm} (14)
$$Y^D = my^*.$$ \hspace{1cm} (15)

At the equilibrium, the budget constraint of the compliant consumer is provided by

$$z^* + pn x^* = w^* h \Rightarrow \left\{ \begin{array}{l}
z^* = \alpha w^* h \\
pn x^* = (1 - \alpha) w^* h
\end{array} \right.,$$\hspace{1cm} (16)

where the last equation follows from the two-stage budgeting procedure based on the Cobb-Douglas utility function (2) whose expenditure share of the numéraire is $\alpha$. Similarly, the budget constraint of the noncompliant consumer is provided by

$$z^* + pn x^* + \pi my^* = w^* h - \gamma \Rightarrow \left\{ \begin{array}{l}
z^* = \alpha (w^* h - \gamma) \\
pn x^* + \pi my^* = (1 - \alpha) (w^* h - \gamma)
\end{array} \right..$$ \hspace{1cm} (17)
From (16) and (12), \( x^* \) is computed as
\[
z^* = \alpha h \quad \text{and} \quad x^* = \frac{(1 - \alpha) h}{np},
\]
and the corresponding indirect utility function as
\[
u^* = \frac{(1 - \alpha) hn^{1/(\sigma-1)}}{p}.
\]
In the equilibrium, the first order conditions of the noncompliant consumer provide
\[
\frac{p}{\pi} = \left( \frac{x^*}{y^*} \right)^{-1/\sigma} \Rightarrow y^* = \left( \frac{p}{\pi} \right)^{\sigma} x^*,
\]
which is substituted into (17) with (12) to obtain
\[
z^* = \alpha (h - \gamma) \quad \text{and} \quad x^* = \frac{(1 - \alpha) (h - \gamma)}{np + m \pi (p/\pi)^{\sigma}}.
\]
From (20) and (21), the indirect utility function is computed as
\[
u^* = \frac{(1 - \alpha) (h - \gamma)}{p} \left( n + m \left( \frac{p}{\pi} \right)^{\sigma-1} \right)^{1/(\sigma-1)}.
\]
Comparing \( U(z^*, u^*) \) and \( U(z^*, v^*) \), noncompliant consumers can dispose illegal items if \( u^* > v^* \), where \( \zeta \) is defined as
\[
\zeta \equiv \left( \frac{z^*}{z^*} \right)^{\alpha/(1-\alpha)} = \left( \frac{h}{h - \gamma} \right)^{\alpha/(1-\alpha)}.
\]
Similarly, potentially noncompliant consumers purchase illegal items if \( u^* < v^* \). Thus, equilibrium utility levels of compliant and noncompliant consumers must be indifferent, à la open-city model that chooses locations:
\[
z^{*\alpha} u^{*1-\alpha} = z^{*\alpha} v^{*1-\alpha} \Rightarrow \zeta u^* = v^*.
\]
Substituting (18) and (21) into (24), varieties of illegal items relative to legal ones, \( \mu = m/n \), are computed as
\[
\mu = \frac{1}{(p/\pi)^{\sigma-1}} \left[ \left( \frac{\zeta h}{h - \gamma} \right)^{\sigma-1} - 1 \right].
\]
At the equilibrium, by symmetry, \( L_i = L^* \) and \( L'_j = L'^* \) are held for all \( i \) and \( j \). From
the zero-profit condition for each item, \( pX^S = w^*L^* \) and \( \pi Y^S = w^*L^* \), the value of supply of a legal item and that of an illegal item are provided as

\[
pX^S = \frac{pf}{p - a} \quad \text{and} \quad \pi Y^S = \frac{\pi (f + \eta)}{\pi - a}.
\]

(26)

From (14), (15), (18), (20), and (21), the value of aggregate demand for a legal item and that of an illegal item are computed as

\[
pX^D = (1 - \alpha) \left( h + \frac{h - \gamma}{\mu^{-1} + (p/\pi)^{\sigma-1}} \right),
\]

(27)

and

\[
\pi Y^D = \frac{(1 - \alpha) (h - \gamma) (p/\pi)^{\sigma-1}}{\mu^{-1} + (p/\pi)^{\sigma-1}},
\]

(28)

respectively. Using (25), expression \( \mu^{-1} + (p/\pi)^{\sigma-1} \) in (27) and (28) is arranged into two alternative forms, such as

\[
\frac{1}{\mu} + \left( \frac{p}{\pi} \right)^{\sigma-1} = \frac{1}{\mu} \left( \frac{\zeta h}{h - \gamma} \right)^{\sigma-1},
\]

(29)

\[
\frac{1}{\mu} + \left( \frac{p}{\pi} \right)^{\sigma-1} = \left( \frac{p}{\pi} \right)^{\sigma-1} \left[ 1 - \left( \frac{h - \gamma}{\zeta h} \right)^{\sigma-1} \right]^{-1}.
\]

(30)

Substituting (29) into (27) provides

\[
pX^D = F(\mu) \equiv (1 - \alpha) h \left[ 1 + \zeta \mu \left( \frac{h - \gamma}{\zeta h} \right)^{\sigma} \right].
\]

(31)

Similarly, substituting (30) into (28) provides

\[
\pi Y^D = W \equiv (1 - \alpha) (h - \gamma) \left[ 1 - \left( \frac{h - \gamma}{\zeta h} \right)^{\sigma-1} \right].
\]

(32)

It is straightforward to see that \( F(\mu) > W \). The equilibrium price of each variety is determined by equalizing respective values of demand and supply:

\[
pX^D = pX^S \quad \Rightarrow \quad p^* = \frac{aF(\mu)}{F(\mu) - f};
\]

(33)

\[
\pi Y^D = \pi Y^S \quad \Rightarrow \quad \pi^* = \frac{aW}{W - (f + \eta)}.
\]

(34)
From (33) and (34), conditions to obtain feasible \( p^* \) and \( \pi^* \) are provided by

\[
F(\mu) > f \quad \text{and} \quad W > f + \eta, \tag{35}
\]

respectively. In addition, (31) and (32) provide \( F(\mu) > W \); hence, illegal varieties vanish before legal ones do.

From (33) and (34), the equilibrium relative price of legal item in terms of illegal one is provided by

\[
\frac{p^*}{\pi^*} = \frac{F(\mu)}{F(\mu) - f} \cdot \frac{W - (f + \eta)}{W}. \tag{36}
\]

The obtained relative price (36) is substituted into (24) to obtain the fixed point problem to determine the ratio of varieties of illegal and legal items:

\[
\mu = G(\mu) \equiv \left[ \left( \frac{\zeta h}{h - \gamma} \right)^{\sigma - 1} - 1 \right] \left( \frac{F(\mu) - f}{F(\mu)} \cdot \frac{W}{W - (f + \eta)} \right)^{\sigma - 1}. \tag{37}
\]

Using the next remark, we obtain Figure 1 to graphically show the existence of the solution for the above fixed point problem.

![Diagram](image)

Figure 1: Determination of relative size of illegal varieties \( (\mu = m/n) \)

**Remark 1** For a constant \( \gamma \), if legal and illegal varieties are produced, \( G(\mu) \) satisfies \( G'(\mu) > 0 \) and \( G''(\mu) < 0 \) for all \( \mu > 0 \), and \( G(0) > 0 \) and \( \lim_{\mu \to +\infty} G(\mu) < +\infty \).

**Proof.** \( F(\mu) \) is a linear function and \( W \) is independent of \( \mu \). In addition, \( \{F(\mu) - f\} / F(\mu) \) is increasing and concave in \( \mu \); hence, \( G'(\mu) > 0 \) and \( G''(\mu) < 0 \). In addition, for \( F(\mu) > f \)
and $W > 0$, $G(0)$ and $\lim_{\mu \to +\infty} G(\mu)$ are directly computed to complete the proof:

$$G(0) = \left[ \left( \frac{\zeta h}{h - \gamma} \right)^{\sigma - 1} - 1 \right] \left( \frac{(1 - \alpha) h - f}{(1 - \alpha) h} \right) \left( \frac{W}{W - (f + \eta)} \right)^{\sigma - 1} > 0, \quad (38)$$

$$\lim_{\mu \to +\infty} G(\mu) = \left[ \left( \frac{\zeta h}{h - \gamma} \right)^{\sigma - 1} - 1 \right] \left( \frac{W}{W - (f + \eta)} \right)^{\sigma - 1} < +\infty. \quad (39)$$

4 Numerical Simulations

Impacts of fines, $\gamma$ and $\eta$, are examined numerically by firstly focusing on

$$\text{Illegal Varieties} = \frac{m}{N} \Rightarrow \frac{\mu^*}{1 + \mu^*}, \quad (40)$$

$$\text{Relative Demand} = \frac{mY^D}{NX^D} \Rightarrow \frac{\mu^*}{1 + \mu^*} \cdot \frac{W - (f + \eta)}{F(\mu^*) - f}, \quad (41)$$

where (41) follows from (31), (32), and (36). The parameters for simulations are provided in Table 1.

Figure 2 presents the result when fines are levied on noncompliant consumers, $0 \leq \gamma \leq h = 10,000$ for $\eta = 0$. In this case, if 1211.14... $\leq \gamma \leq 7908.50...$, illegal items are consumed. The results suggest that the illegal varieties start increasing if fines are above a certain level and the illegal consumptions do not decline if fines are insufficient. Smaller $\gamma$ than 1211.14... implies that the difference between legal and illegal items are too close for consumers to exploit variety effect (scale effect in the legal sector is outweighed). Larger $\gamma$ than 7908.50... implies that the risk of conviction is so high that variety effect cannot compensate it.

Figure 3 presents the result when fines are levied on noncompliant suppliers, $0 \leq \eta \leq 1,000$ for $\gamma = 0.3h$. In this case, larger $\eta$ than 785 implies that prices of illegal items are not affordable for any consumers. The results suggest that the illegal varieties keep increasing
Table 2: Critical values

<table>
<thead>
<tr>
<th>Fines</th>
<th>Relative Demand</th>
<th>Relative Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>4747.0</td>
<td>0.0992611</td>
<td>0.537044</td>
</tr>
<tr>
<td>4226.5</td>
<td>0.0964289</td>
<td>0.544123</td>
</tr>
</tbody>
</table>

while the illegal consumptions keep declining.\(^3\)

For noncompliant consumers, larger fines induce lower consumptions that must be compensated by variety effect if they cannot compensate the losses by scale effect. The variety effect is exploited by consuming illegal goods while the scale effect by concentrate on legal goods that have potentially larger demands than illegal goods. In this case, regulations do not work monotonically. For noncompliant suppliers, larger fines induce higher prices that must be compensated by either variety effect or scale effect, but higher prices do not induce larger demands; hence, noncompliant suppliers only can rely on the variety effect. In this case, regulations work monotonically.

For reference, Figure 4 depicts the relative price of legal items in terms of illegal ones. When fines are levied on noncompliant consumers, fines increase the relative price of legal items when fines are below the threshold level. Around the same domain of fines, consumptions of illegal items also increase. When fines are larger than the threshold level, the relative price of legal items starts declining. Around the same domain of fines, consumptions of illegal items also start declining. Yet, the critical value of \(\gamma\) does not coincide with the threshold values for relative price and relative demand, as shown in Table 2: the threshold value for the relative demand is \(\gamma = 4747\) and that for the relative price is \(\gamma = 4226.5\).

When fines are levied on noncompliant suppliers, as Figure 4 shows, fines reduce the relative price of legal items to reduce consumptions of illegal items. Becker and Murphy (1988) suggest that addiction is caused by complementarity effect among inter-temporal consumptions. In their analysis, higher prices in the future reduce current consumptions of addictive goods. The variety effect may generate a similar effect, as inter-temporal consumptions are considered as varieties, as considered by a version of Epstein and Zin (1989). In this sense, the result of this paper is consistent with Becker and Murphy’s: regulations on noncompliant suppliers induce higher illegal item prices to reduce illegal consumptions.

\(^3\)A similar theoretical result in the impact of policy in varieties is also seen in a recent international trade paper by Qui and Yu (2014): tariffs increase scope of varieties. In their study, an empirical evidence is also provided.
Figure 2: Impacts of fines on noncompliant consumers ($\gamma$)

Figure 3: Impacts of fines on noncompliant suppliers ($\eta$)

Figure 4: Relative price of legal items vis-à-vis illegal ones ($p^*/\pi^*$)
5 Concluding Remarks

This paper numerically suggests there are cases in which regulations increase illegal consumptions and varieties while increases in illegal varieties are not necessarily accompanied by increases in illegal consumptions. These results are caused by variety effect that compensates for losses from regulations. The cases in which regulations increase illegal consumptions are suggested by Smith (1976) and Becker and Murphy (2006). However, the cases in which regulations increases illegal varieties are not considered in previous studies.

For further studies, in addition to empirical attempts, the relationship between the complementarity effect of addiction as discussed in Becker and Murphy (1988) and the variety effect should be examined. In addition, the relationships among expansions of varieties, investigation costs, and social costs must be explored to find optimum policies.

References


