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Abstract

The main objective of this paper is to build a macroeconomic model that takes into account a non-linearity in the interest rate rule. We assume that the monetary authority considers, to determine the rate of interest, the interaction between the actual inflation and the capacity utilization, so that the sensitivity of the interest rule to the inflation gap varies in accordance with the business cycle. The macroeconomic policy framework proposed in this work enables the monetary authority to give as much weight to inflation as to the product without losing sight of the expected anchor role of the inflation target, either in the assumption of closed economy as in the open-economy assumption.

Keywords: Macroeconomic dynamics, non-linear interest rate rule, expectational consistency. JEL Classification E10, E12, E52, E58

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1 Introduction

The inflation targets (IT) regime has become the monetary policy norm in a number of central banks around the world since 1990. There are currently about 35 countries which adopt this monetary regime (Schmidt-Hebbel and Carrasco, 2016) , and there are many empirical studies that evaluate the macroeconomic performance of countries who followed this policy framework (Ball and Sheridan, 2003; Goncalves and Salles, 2008; de Mendonça and de Guimarães e Souza, 2012).

Even with the advancements in conventional macroeconomic literature over the last two decades in terms of the construction of theoretical models that explicitly consider the IT regime (for example, Clarida et al. (1999)), the post-Keynesian literature only began to consider the IT framework in formal models recently, in particular through the seminal works of Setterfield (2006) and of Lima and Setterfield (2008). In fact, in its original form, the inflation target regime derives from a set of hypotheses that are apparently incompatible to post-Keynesian tradition, especially when it comes to the emphasis given to low-inflation in detriment of concerns related to the level of output. According to Setterfield (2006), this apparent mismatch, between the regime and the typical Keynesian concerns, could be overcome provided that: (i) the output is considered properly as primordial part of the goals of monetary policy; (ii) the distributive conflict component of inflation is not neglected and; (iii) the role of aggregate demand is considered to determine the real output.

Based on the works of Setterfield (2006) and Lima and Setterfield (2008), some other works began to incorporate the possibility of an inflation target regime under an alternative framework, with different modeling strategies. A common feature in this literature, both for closed and open economies, lies in the assumption of linearity in the monetary policy rule, as can be seen in the dynamic models developed by Porcile (2011), Santos (2011), Drumond and Porcile (2012), Bertella et al. (2015) and by Drumond and Jesus (2016). When a central bank uses a linear monetary policy rule it is implicitly assumed that its coefficients are constant. If this is the case, the monetary authority's concern with inflation, for example, is always the same, regardless of whether the economy is in expansion or contraction. Reflecting on this issue, Blinder (1997, p.06), has recognized that academic macroeconomists tend to use "quadratic loss functions for reasons of mathematical convenience" (which culminates in a linear monetary policy rule) "without full consideration as to their substantive implications". In addition, empirical studies with data from different countries have proposed that cases in which central banks have asymmetric preferences are not infrequent (Martin and Milas, 2004; Dolado et al.; Surico; Cukierman and Muscatelli, 2008). Considering the above, it seems unreasoned not to assume that the parameters of a monetary policy rule time-varying, so that, for example, they will depend on the phase of the economic cycle that the economy is in.

Additionally, another substantive issue arises when the IT regime is designed under an alternative framework which concomitant focus on inflation and on the output. In this case, the monetary police will be fully consistent only in the presence of certain policies that will complement the interest rate rule, as is the case of the income policy in the model developed by Lima and Setterfield (2008). It turns out that, as this article will try to demonstrate further ahead, in the absence of the natural output hypothesis, as well as in the absence of complementary policies to the monetary policy, a typical interest rule with dual mandate cannot guarantee the convergence of inflation to its target.

Taking the above observations into account, the main objective of this article is to build a macrodynamic model that takes non-linearity in the interest rule into account. We have considered that the closer (distant) the economy to its full level of capacity utilization, the more (less) sensitive the monetary authority will be to deviations of the inflation in relation to the targets. This dependence of the inflation parameter (in the interest rate rule) on the level of economic activity is a clear distinction between the model developed here and those available in the post-Keynesian literature.

In terms of the macroeconomic policy, the main contribution of the present work is to present a monetary policy rule, which allows the monetary authority to put as much weight on inflation as on the output without losing sight of the role of the expected anchor inflation target when conditioned by institutional constraints, especially regarding incomes policy. No less important is the fact that the model should be built without imposing the ad hoc convergence of inflation to its target, and without imposing some limiting form of expectational behavior, such as the hypothesis of rational expectations.

The paper is organized in five sections in addition to this introduction and the conclusions. In the second section we present a brief literature review, in the third section we develop a post-Keynesian standard model as the starting point for our analysis, in the fourth section we develop a model with a non-linear interest rate rule in context of closed economy and, finally, in the sixth section we analyze the open-economy case.

2 Literature Review and Motivation

The main ingredients of Post-Keynesians models that take the IT regime into account are as follows: (i) the role of aggregate demand is central to determine the macroeconomic equilibrium; (ii) the market for goods operates below full employment (there is excess capacity in the economy); (iii) the Phillips curve is derived from a wage-bargaining process; (iv) the monetary authority handles the interest rate to achieve specific macroeconomic objective(s) (money supply is endogenous); (v) there is no natural interest rate; and (vi) expectational inflation matters in determining the current level of inflation. In addition, considerations of political of income[1], open economy and fiscal policy have also been incorporated into post-Keynesians macro models without too much difficulty.

In the models built by Setterfield (2006), Lima and Setterfield (2008) and Santos (2011), the compatibility between IT and a post-Keynesian macroeconomics scheme is possible because there is an important role for the incomes policy, which makes it feasible for political authorities to place so much emphasis on the level of activity and inflation and, at the same time, ensure the stability of the economic system. According to Setterfield (2006, p. 665), one of the peculiarities of their extended post-Keynesian model lies in the fact that this structural model "involves policy making that explicitly recognizes the importance of aggregate demand conditions for real economic activity and the ' conflicting claims ' basis of the inflation process".

In fact, one of the main results of the model by Lima and Setterfield (2008) is that when the more orthodox is the mix of policies, the more adverse are the consequences for macroeconomic stability and the viability of the IT regime in a post-Keynesian economy. The model developed by Santos (2011) in turn, which is an extension to the model of Lima and

[1] Setterfield (2007, p. 129) defines income policies as either "formal and/or informal institutions that frame and mediate aggregate wage and price setting behavior in such a way as to reduce conflict over income shares and better reconcile conflicting income claims ". It is in this sense that the idea of incomes policies has been considered in the present article.

Setterfield (2008), suggests that for the dynamic equilibrium in a post-Keynesian economy to be stable, "the incomes policy should at least react to the inflation gap, while the monetary policy should at least react to the output gap" (Santos, 2011, p. 316). As shown by Carvalho (2015, p. 127), although many followers of Keynes – including certain Keynesians of the neoclassical synthesis such as Tobin (1985) – have recognized that the incomes policy can be a supplementary stabilization policy, it is not clear whether Keynes would propose income policies permanently. In practice, most of the experiments with an incomes policy in the post-war period were based on control or on the monitoring of wages. However, it is known that a barrier to the permanent implementation of incomes policies in modern capitalist economies is the institutional framework of wage-bargaining in each country. Differences in the process of wage moderation and trade-union bargaining can derail the implementation of incomes policies to mediate the distributive conflict and fight inflation, from a political point of view. The greater the country (and its workforce) and the more fragmented its society (and unions), the less possible the incomes policy becomes.

In this sense, an alternative to the use of the incomes policy as a tool to combat inflation in a post-Keynesian environment is to consider a linear interest rule or IROP (Interest Rate Operating Procedure) that takes both inflation and the output/employment into account. The models of Porcile (2011), Drumond and Porcile (2012) and Drumond and Jesus (2016) have been constructed based on this perspective. In these works, when considering a set of fundamentally post-Keynesian hypotheses for a small open economy, the authors are able to show that depending on the arrangement of the macroeconomic policy under consideration, the dynamic equilibrium derived for the model is stable, even without consideration to any kind of incomes policies. On the other hand, as will be demonstrated further below, despite generating a stable dynamic equilibrium, a linear interest rule generates an important expectational inconsistency by failing to ensure the convergence of inflation to its target.

In the following section, we will seek to describe a standard model that will serve as a basis for the subsequent exercises proposed in this work.

3 A standard post-Keynesian model

3.1 The equilibrium in the goods market and the IS curve

In this section, we will attempt to construct a standard post-Keynesian model considering the existence of excess capacity in the economy. Consider the following aggregate demand for a closed economy:

$$Y = C + F + I \tag{1}$$

where Y represents the aggregate output/demand, C represents the aggregate consumption, F represents the government spending and I represents the aggregate investment. The aggregate consumption is written as a linear function of income, so that $C = cY$, with $0 < c < 1$. For purposes of simplicity and in view of the objectives of this work, differences between the marginal propensity to consume has been disregarded both for capitalists and workers.

The following is finally obtained after normalizing the aggregate demand in terms of capital stock:

$$uv = cuv + f + g \tag{2}$$

with $u = Y/\bar{Y}$ representing the rate of capacity utilization, $\bar{Y}$ representing the product of full employment, v stands for the inverse of the capital-product relationship, considered to be constant, c represents the marginal propensity to consume, f represent the government spending as a proportion of capital stock and g represents investment as a proportion of capital stock.

Investment as a proportion of capital stock, in turn, can be described as a positive function of the rate of capacity utilization and as a negative function of the real interest rate:

$$I/K = g = g_0 + \delta_1 u - \delta_2 r \tag{3}$$

with g_0 representing a positive parameter that seeks to represent the animal spirit of entrepreneurs, r standing for the real interest rate and δ_1 and δ_2 as positive parameters. The correct specification of the investment function leads to controversy in post-Keynesian/Kaleckian literature, considering that different closings for the models can be obtained from different investment functions, as can be seen in Setterfield (2017) and Hein et al. (2016). In this article, we disregard the possible influence that a rate of capacity considered normal by the capitalists could exert on their decision to invest. The use of the interest rate on the investment function, in turn, requires, in similar manner as for Setterfield (2009) and Rochon and Setterfield (2007), the indirect incorporation of the impact of interest on the net profit rate earned by the capitalists. When considering the equilibrium in the market for goods and replacing equation (2) in equation (3), it is possible to obtain the following IS curve normalized in relation to capital stock:

$$u = \frac{(g_0 + f - \delta_2 r)}{(v(1-c) - \delta_1)} \tag{4}$$

Based on the Keynesian condition for equilibrium $v(1-c) > \delta_1$, the IS curve is negatively inclined in the plan rate of capacity utilization versus the real interest rate: a positive (negative) variation in the real interest rate, everything else constant, will decrease (increase) the level of capacity utilization.

3.2 Inflation, distributive conflict and Phillips Curve

Considering an environment of imperfect competition in which companies have monopoly power, the Phillips curve for this type of economy is built from the distributive conflict between workers and capitalists, in line with the tradition started by Rowthorn (1977). In this economy with imperfect competition, variations in the price level are equal to the sum of the nominal wage variation, variation of the mark-up of the companies and labor productivity. Whereas, for simplicity purposes, when considering mark-up and labor productivity as constants, inflation can be fully explained by variations in nominal wages, which are described as a function of the gap between the wage share in the workers ' desired income and the effective wage share on income determined by the market power of the companies. In addition, workers also take the expected inflation rate into account, in order to preserve his current income 's purchasing power. Based on these assumptions, we obtain the following Phillips curve:

$$\pi = \pi^e + \phi(w^d - w^f) \tag{5}$$

where π represent the effective inflation rate, π^e represent the expected inflation rate and w^d and w^f, respectively, represent the wage share on the income desired by workers and the wage share on income as determined by the capitalists, where ϕ is a strictly positive parameter. The bargaining power of workers is modeled as a function of the rate of capacity utilization, so that $w^d = \alpha u$, with α as a positive parameter. This suggests that, as the economy approaches the full use of its capacity utilization, workers acquire more bargaining power and can generate pressure for higher wages. Rewriting the equation (5):

$$\pi = \pi^e + \phi(\alpha u - w^f) \tag{6}$$

It is worth noting that the equilibrium in the wage-bargaining process – equality between the share on income desired by the workers and the effective wage share on income – implies that there is expectational consistency, with current inflation equal to the level of inflation expected by the workers. However, this is not a derived hypothesis, which imposes no predetermined expectational model, and is fully compatible with heuristic behaviors of agents in an attempt to anticipate inflation.

4 Dual mandate monetary policy and expectational consistency

In light of the existence of a negative relationship between the rate of unemployment and the capacity utilization, the interest rate rule is written as a function of the inflation and the rate of capacity utilization. In a first moment, the attempt is to build a typical linear interest rule that takes both inflation and the rate of capacity utilization into account. As explained further below, when embedded in a post-Keynesian model, this kind of rule generates expectational inconsistencies that must be circumvented.

4.1 Expectational consistency and interest rate operating procedure

Consider a dynamic interest rule in which the actual interest rate variation over time, $\dot{r} = \frac{dr}{dt}$, is a linear function of the gap between the current inflation and the inflation target, as well as the difference between the rate capacity utilization and its desired level by the political authorities.

$$\dot{r} = \gamma \left(\pi - \pi^T\right) + \beta \left(u - u^T\right) \tag{7}$$

with γ and β as positive parameters, to the extent that inflation is higher/lower than the current inflation target π^T, the real interest rates will rise/fall. On the other hand, the real interest also responds to the gap between the rate of capacity utilization and its target, u^T, that is pre-established by the political authorities. This interest rule can be modified considering monetary policy schemes that are exclusively focused on inflation (if $\beta = 0$) or exclusively concerned with the output (if $\gamma = 0$).

Once the interest rule has been defined, it is important to consider the expectational convergence process. In the present work, an expectational dynamic identical to that presented by Tobin and Buiter (1976), Turnovsky (1995) and Yoshida and Asada (2007) has been taken as reference, in which the forecast errors are continually reviewed by the agents in a learning process.

$$\dot{\pi}^e = k\left(\pi - \pi^e\right) \tag{8}$$

Equation (8) describes the expectational dynamics for inflation as a response to the deviation of the current inflation from the expected inflation, with $k > 0$ as a parameter that measures inflation memory. One of the advantages to using this equation is that it does not require the imposition of some *ex-ante* expectational anchor process such as, for example, imposing that inflation necessarily converges toward a predetermined goal.

Equations (7) and (8) form a two-dimensional linear dynamic model that can be analyzed when combining the Phillips curve, the *IS* curve, the interest rate rule and the expectational dynamics. After some algebraic manipulation, the interest rates dynamics and the expected inflation can be described as follows:

$$\dot{r} = \gamma \left[\pi^e + \phi \left(\alpha \left(\frac{g_0 + f - \delta_2 r}{v(1-c) - \delta_1} \right) - w^f \right) - \pi^T \right] + \beta \left[\left(\frac{g_0 + f - \delta_2 r}{v(1-c) - \delta_1} \right) - u^T \right] \tag{9a}$$

$$\dot{\pi}^e = k\phi \left(\alpha \left(\frac{g_0 + f - \delta_2 r}{v(1-c) - \delta_1} \right) - w^f \right) \tag{9b}$$

The expected inflation converges toward the effective inflation in the stationary state, provided that when $\dot{\pi}^e = 0$, then $\pi = \pi^e$. This necessarily implies that the wage share on income as desired by workers converges toward the effective wage share on income. On the other hand, the model generates an inconsistency of monetary policy, since there is no guarantee that the expected inflation and effective inflation converge toward the inflation target as determined by the monetary authority.

Proposition 1. *In the long run, the equality between the inflation target and the effective inflation, as the equality between the inflation target and the inflation expectation, are not ensured by the model.*

Proof Consider equation (9a) in the stationary state with $\pi = \pi^e$:

$$0 = \gamma \left[\pi + \phi \left(\alpha \left(\frac{g_0 + f - \delta_2 r}{v(1-c) - \delta_1} \right) - w^f \right) - \pi^T \right] + \beta \left[\left(\frac{g_0 + f - \delta_2 r}{v(1-c) - \delta_1} \right) - u^T \right] \tag{10}$$

Equation (9b) leads to $\left(\alpha \left(\frac{g_0+f-\delta_2 r}{v(1-c)-\delta_1} \right) - w^f \right) = 0$ in the stationary state. Therefore, equation (10) can be rewritten as follows:

$$\gamma \left[\pi - \pi^T \right] = \beta \left[u^T - \left(\frac{g_0 + f - \delta_2 r}{v(1-c) - \delta_1} \right) \right] \tag{11}$$

Based on equation (11), it is possible to realize that for given values of γ and β, and only in the case in which the government can achieve the target for the rate of capacity utilization, inflation will converge toward the inflation target (and vice versa). However, there is no endogenous mechanism in the model that guarantees the long-term equality between u and u^T or between π and π^T, so that achieving the two goals simultaneously is something completely random.[2]

□

[2] We can demonstrate a similar result from the minimization of the quadratic loss function $L = \beta_1 \left(\pi - \pi^T\right)^2 + \beta_2 (y - y^n)^2$ where $(y - y^n)$ is the output gap and β_1 and β_2 are positive parameters. For the Phillips-curve $\pi = \pi^e + \alpha (y - y^n)$ we have the following first order condition: $\alpha\beta_1 \left(\pi - \pi^T\right) = \beta_2 (y^n - y)$. When we consider the natural output as a attractor to the output, that necessarily works in absence of frictions, there is no expectation inconsistence. It is the new-Keynesian traditional view about the output trend. Otherwise, in the absence of this assumption, we have a similar kind of expectational inconsistency as suggest previously.

Proposition 2. *The steady state of the dynamic system formed by (9a) and (9b) is locally asymptotically stable.*

Proof From now on, it is possible to evaluate the dynamic properties of the model taking the Jacobian matrix of the system formed by equations (9a) and (9b) into account.

$$J = \begin{bmatrix} \frac{\partial \dot{r}}{\partial r} & \frac{\partial \dot{r}}{\partial \pi^e} \\ \frac{\partial \dot{\pi}^e}{\partial r} & \frac{\partial \dot{\pi}^e}{\partial \pi^e} \end{bmatrix} \tag{12}$$

It is easy to notice that matrix (12) has the following elements:

$$\frac{\partial \dot{r}}{\partial r} = \frac{-\delta_2(\phi\gamma\alpha + \beta)}{v\,(1-c) - \delta_1} < 0 \tag{13a}$$

$$\frac{\partial \dot{r}}{\partial \pi^e} = \gamma > 0 \tag{13b}$$

$$\frac{\partial \dot{\pi}^e}{\partial r} = \frac{-k\phi\alpha\delta_2}{v\,(1-c) - \delta_1} < 0 \tag{13c}$$

$$\frac{\partial \dot{\pi}^e}{\partial \pi^e} = 0 \tag{13d}$$

Since J has negative trace and positive determinant, it can be concluded that the dynamical system formed by equations (9a) and (9b) converges to a stationary state that is a stable equilibrium point.

$$Det\,(J) = \frac{k\phi\alpha\delta_2\gamma}{v\,(1-c) - \delta_1} > 0 \tag{14a}$$

$$Tr\,(J) = \frac{-\delta_2(\phi\gamma\alpha + \beta)}{v\,(1-c) - \delta_1} < 0 \tag{14b}$$

□

In case of open economies, similar results are found in the models developed by Drumond and Porcile (2012) and Drumond and Jesus (2016).

The inconsistency between inflation target and current inflation is certainly a problem for the operation of monetary policies over time. Given this inconsistency, it is possible to consider two opposing alternatives in terms of monetary policy, namely a monetary regime exclusively focused on inflation (by setting $\beta = 0$), and a monetary regime exclusively focused on the use of capacity utilization (by setting $\gamma = 0$).

In fact, in the case of a monetary regime exclusively focused on inflation, the system is stable and the economy converges to a steady state wherein not only the current inflation equals the expected inflation, it also equals the inflation target. The determinant of the Jacobian matrix remains as shown in equation (14a) while the line equals to $\frac{-\delta_2(\emptyset\gamma\alpha)}{v(1-c)-\delta_1} < 0$. In the case of a monetary regime exclusively focused on the use of installed capacity ($\gamma = 0$), on the other hand, the model generates a dynamic indetermination that cannot be considered as stability, so that the determinant of matrix J becomes null. Jesus and Correia (2016) have develop a macroeconomic model with an active fiscal policy and arrived at the same conclusion when assuming that the movements of the interest rate within a closed economy depend solely on the output gap.

Proposition 3. *The trajectory of convergence to the steady state in the model with $\beta = 0$ depends on the value of k, that reflects the inflationary expectations adjustment speed .*

Proof By analyzing the matrix 's J discriminant, we can investigate what kind of convergence is present in the model. The characteristic equation generates by the dynamic system is represented as:

$$x^2 - Tr(J)x + Det(J) = 0 \tag{15}$$

By investigating the sign of the discriminant

$$D = Tr(J)^2 - 4Det(J) \tag{16}$$

We can determine completely whether the roots are real or imaginary. The characteristic equation has two real roots if $D > 0$, and two complex conjugate roots if $D < 0$. First, it is important to notice from (15) and (16) that there is the following relationship in this case:

$$Det(J) = -kTr(J) \tag{17}$$

Thus, the discriminant is expressed as:

$$D = Tr(J)[Tr(J) + 4k].. \tag{18}$$

From the above discussions, it is clear that the discriminant is positive or zero if

$$0 < k \leq \frac{\gamma\phi\alpha\delta_2}{4[v(1-c) - \delta_1]} \tag{19}$$

In this situation, the roots are real. Thus, the solution trajectories on the phase diagram converge to the steady state without the spiral. On the other hand, the discriminant is negative if:

$$k > \frac{\gamma\phi\alpha\delta_2}{4[v(1-c) - \delta_1]} \tag{20}$$

In this case, the roots are imaginary. Thus, the solution trajectories converge to the steady state with the spiral. These results suggest that increasing the speed of adjustment of inflationary expectations changes the dynamic properties of the steady state. If inflationary expectations adjust faster, the economy will experience the oscillatory convergence to the steady state.

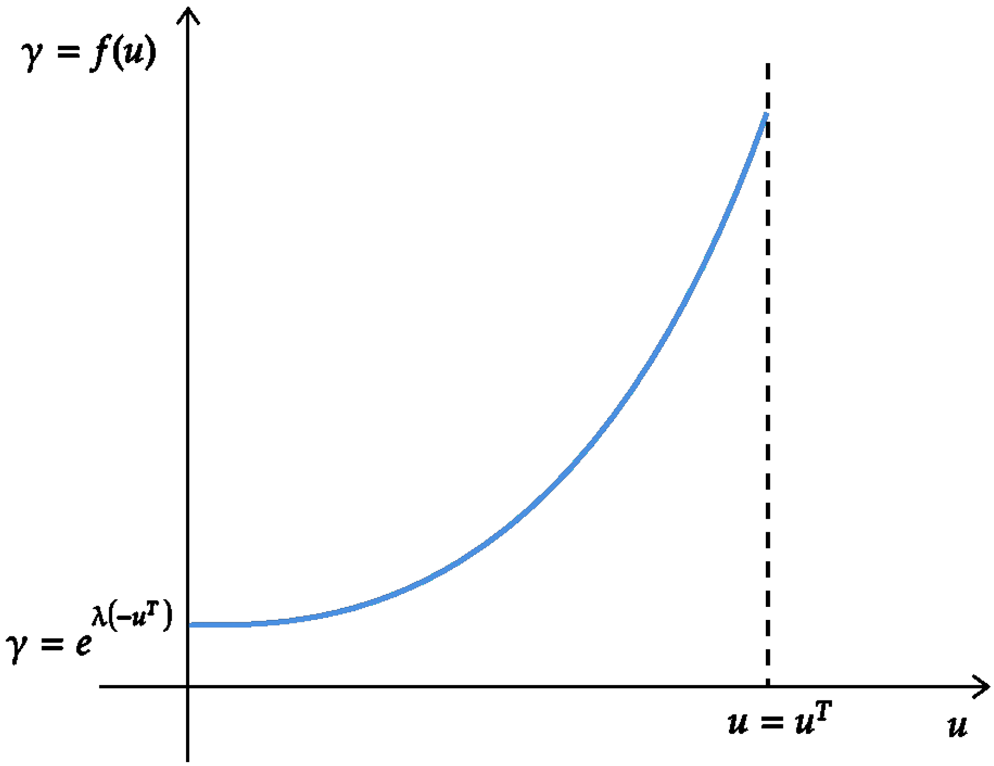
□

5 Non-linear interest rate operating procedure

The main objective of this paper is to propose an alternative monetary policy rule, which on the one hand becomes possible a dual mandate monetary regime and, on the other hand, circumvents the aforementioned inconsistency problem in the monetary police. In this work we propose an interest rule in which the monetary authority's sensitivity to deviations of inflation regards the inflation target is endogenous and responds to changes in the rate of capacity utilization.

This non-linear interest rule is described below:

Figure 1: Real interest rate speed of adjustment



$$\dot{r} = \gamma(\pi - \pi^T) \tag{21a}$$

$$\gamma = \exp[\lambda(u - u^T)] \tag{21b}$$

$$\dot{r} = \exp[\lambda(u - u^T)](\pi - \pi^T), \quad \lambda > 0 \tag{21c}$$

where u^T is an exogenous target for the rate of capacity utilization, setting by the central bank. An important aspect of the equation (21a) is that the real interest rate speed of adjustment in response to a change in the current inflation is an endogenous variable. Note that the function $\gamma = f(u)$, that is convex in u, has a local maximum in the interval $0 < u < u^T$, when $u = u^T$.

Solving the model with this new interest rule results in the following two-dimensional dynamical system:

—labelkey

$$\dot{r} = \exp[\lambda\left(\frac{g_0 + f - \delta_2 r}{v(1-c) - \delta_1 - u^T}\right)]\left[\pi^e + \phi\left(\alpha\left(\frac{g_0 + f - \delta_2 r}{v(1-c) - \delta_1}\right) - w^f\right) - \pi^T\right] \tag{22a}$$

$$\dot{\pi^e} = k\phi\left(\alpha(\frac{g_0 + f - \delta_2 r}{v(1-c) - \delta_1}) - w^f\right) \tag{22b}$$

Proposition 4. *The model has a steady state equilibrium where* $\pi^e = \pi = \pi^T$ *and the trajectory of convergence to the steady state in the model with* $\beta = 0$ *depends on the value of* k*, that reflects the inflationary expectations adjustment speed .*

Proof The pair of equations (22a) and (22b) form a nonlinear dynamical system whose stability should be evaluated by taking the partial derivatives of the equations in their respective stationary points:

$$J = \begin{bmatrix} \left(\frac{\partial \dot{r}}{\partial r}\right)_{(r,\pi^e)} & \left(\frac{\partial \dot{r}}{\partial \pi^e}\right)_{(r,\pi^e)} \\ \left(\frac{\partial \dot{\pi}^e}{\partial r}\right)_{(r,\pi^e)} & \left(\frac{\partial \dot{\pi}^e}{\partial \pi^e}\right)_{(r,\pi^e)} \end{bmatrix} \tag{23}$$

$$\left(\frac{\partial \dot{r}}{\partial r}\right)_{(r,\pi^e)} = -\frac{\phi\alpha\delta_2}{v(1-c)-\delta_1}\exp[\lambda\left(\frac{g_0+f-\delta_2 r}{v(1-c)-\delta_1} - u^T\right)] \tag{24a}$$

$$\left(\frac{\partial \dot{r}}{\partial \pi^e}\right)_{(r,\pi^e)} = \exp[\lambda\left(\frac{g_0+f-\delta_2 r}{v(1-c)-\delta_1} - u^T\right)] \tag{24b}$$

$$\left(\frac{\partial \dot{\pi}^e}{\partial r}\right)_{(r,\pi^e)} = \frac{-k\phi\alpha\delta_2}{v(1-c)-\delta_1} \tag{24c}$$

$$\left(\frac{\partial \dot{\pi}^e}{\partial \pi^e}\right)_{(r,\pi^e)} = 0 \tag{24d}$$

where the steady state values are given by the following equations:

$$\pi^e = \pi^T = \pi \tag{25a}$$

$$\alpha(\frac{g_0+f-\delta_2 r}{v(1-c)-\delta_1}) = w^f \tag{25b}$$

The Jacobian matrix in (23) has the following trace and determinant.

$$Tr(J) = -\frac{\phi\alpha\delta_2}{v(1-c)-\delta_1}\exp[\lambda(\frac{g_0+f-\delta_2 r}{v(1-c)-\delta_1} - u^T)] < 0 \tag{26a}$$

$$Det(J) = \frac{k\phi\alpha\delta_2}{v(1-c)-\delta_1}\exp[\lambda(\frac{g_0+f-\delta_2 r}{v(1-c)-\delta_1} - u^T)] > 0 \tag{26b}$$

We can see that the steady state is stable by applying the Routh-Hurwitz stability criterion. The characteristic equation is represented as:

$$x^2 - Tr(J)x + Det(J) = 0 \tag{27}$$

By investigating the sign of the discriminant:

$$D = Tr(J)^2 - 4Det(J) \tag{28}$$

We can determine completely whether the roots are real or imaginary. The characteristic equation has two real roots if $D > 0$, and two complex conjugate roots if $D < 0$. First, it is important to notice that there is the following relationship in this case $Det(J) = -kTr(J)$. Thus, the discriminant is expressed as:

$$D = Tr(J)[Tr(J) + 4k] \tag{29}$$

From the above discussions, it is clear that the discriminant is positive or zero if k $<=$ $-(1/4)Tr(J)$. In this situation the roots are real. Thus, the solution trajectories on the phase diagram converge to the steady state without the spiral. On the other hand, the discriminant is negative if k $> -(1/4)Tr(J)$. In this case, the roots are imaginary. Thus, the solution trajectories converge to the steady state with the spiral.

□

In summary, these results suggest that increasing the speed of adjustment of inflationary expectations changes the dynamic properties of the steady state. If inflationary expectations adjust faster, the economy will experience the oscillatory convergence to the steady state.

The solution of the model indicates that the economy converges to a stable stationary state in which the inflation target is achieved in a way that there are no inter-temporal inconsistencies in the monetary policy. There is also expectational coherence, in a way that the effective inflation converges toward the expected inflation, which in turn converges toward the target set by the monetary authority.

In the next section we will extend the analysis to the open-economy case. In particular, we are interested in the small-open economy case, which is, from our point of view, a good first step to understand most of developing economies.

6 The small-open Economy case

6.1 The equilibrium in the goods market and the IS curve in open economy

To make the IS relation an open-economy IS curve we have to put the net exports into the the aggregate demand. Once again, we will write the IS relation in terms of the rate of capacity utilization, like in (2). The net exports, normalizing in terms of the capital stock K can be written as follows:

$$X/K = h = h_0 - b_1 u + b_2 \rho \tag{30}$$

Where h_0 is the autonomous part of the net exports and b_1 and b_2 are positive parameters. Following the Marshall-Lerner conditions, the real exchange rate $\rho = EP^f/P$ has positive effects on the net exports. As usual, the real exchange rate is a function of the nominal exchange rate E, the foreign level of prices P^f and the the local level of domestic prices P. Rewriting the IS relation, we have:

$$u = u(r, \rho) = \frac{f + g_0 + h_0 - \delta_2 r + b_2 \rho}{v(1-c) - \delta_1 + b_1} \tag{31}$$

Following the Keynesian equilibrium condition ($v(1-c) + b_2 > \delta_1$), the IS curve has a negative slope in the (r, u) space.

6.2 Phillips curve and exchange rate dynamics

Like in Drumond and Porcile (2012) we consider a Phillips Curve based on the conflict claims between capitalists and workers. As workers consume imported goods, the level of the real exchange rate will affect real consumption. The real exchange rate has impact on the total consumption, trough the workers purchase power. Thus, a real depreciation cause a reduction in the workers purchase power and, all more constant, increase the wage bargain.

$$\pi = \pi^e + \phi(\alpha u - w^f) + (1-\phi)\rho, \quad 0 < \phi < 1 \tag{32}$$

The exchange rate dynamics can be written from the typical relation $\rho = EP^f/P$, where E is the nominal exchange rate, P^f the foreign price level and P the domestic price level. Taking the logs and differentiating in relation to the time, the real exchange rates dynamics can be written as a function of the nominal exchange rate dynamics, the foreign inflation π^f and the domestic inflation π:

$$\frac{\dot{\rho}}{\rho} = \frac{\dot{E}}{E} + \pi^f - \pi \tag{33}$$

Departing from the UIP (Uncovered Interest Rate Parity) condition, we can write the nominal exchange rate dynamics $\frac{\dot{E}}{E}$ as a function of the difference between the domestic nominal interest rate i and the foreign nominal interest rate i^f, take as constant in the small-open economy assumption.

$$\frac{\dot{E}}{E} = \mu(i^f - i), \quad \mu > 0 \tag{34}$$

The nominal exchange rate response to the nominal interest rate differential is modeled following the adaptive expectation regards to the exchange rate market assumption. For more details see Drumond and Porcile (2012) and Libman (2018).

From the Fisher equation, we write the nominal interest rate as a function of the real interest rate and the expected inflation. For simplicity purposes we consider the foreign inflation equal to the expected foreign inflation.The foreign inflation as the foreign real interest rate (r^f) are both taken as constants.

$$i = r + \pi^e \tag{35a}$$

$$i^f = r^f + \pi^f \tag{35b}$$

Putting (35) into (34) and then into (33):

$$\dot{\rho} = [(1+\mu)\pi^f - \mu(r - r^f) - \mu\pi^e - \pi]\rho \tag{36}$$

Combining the open-economy IS curve, the open-economy Phillips curve, the real exchange dynamics, the expectation dynamic in (8) and the non-linear interest rate operating procedure in (21c), we have a three-dimensional dynamic system.

6.3 *Non-linear interest rate operating procedure in a small open-economy context*

The new three-dimensional dynamic system can be written as follows:

$$\dot{r} = \exp[\lambda(u(r,\rho) - u^T)][\pi^e + \phi(\alpha u(r,\rho) - w^f) + (1-\phi)\rho - \pi^T] \tag{37a}$$

$$\dot{\pi}^e = k[\phi(\alpha u(r,\rho) - w^f) + (1-\phi)\rho] \tag{37b}$$

$$\dot{\rho} = [(1+\mu)\pi^f - \mu(r - r^f) - (1+\mu)\pi^e - \phi(\alpha u(r,\rho) - w^f) - (1-\phi)\rho]\rho \tag{37c}$$

where we use the IS relation in (31) and the utilization rate u depends on r and ρ in the following way:

$$u_r = \frac{\partial u}{\partial r} = \frac{-\delta_2}{v(1-c)-\delta_1+b_1} < 0 \tag{38a}$$

$$u_\rho = \frac{\partial u}{\partial \rho} = \frac{b_2}{v(1-c)-\delta_1+b_1} > 0 \tag{38b}$$

By using $\dot{r} = \dot{\pi}^e = \dot{\rho} = 0$, we obtain the values of the steady state:

$$(1+\mu)(\pi^f - \pi^T) = \mu(r_* - r^f) \tag{39}$$

$$\pi^e_* = \pi^T \tag{40}$$

$$\phi(\alpha u(r_*, \rho_*) - w^f) + (1-\phi)\rho_* = 0 \tag{41}$$

We can find an interesting policy implication about (39). If the central bank sets the target rate of inflation above the rate of foreign inflation, the equilibrium rate of domestic interest is lower than the rate of foreign interest. The opposite holds for the opposite case.

Let us now turn to the topic of the dynamic stability properties of the model. We can obtain the following Jacobian matrix evaluated at the steady-state point (r_*, π^T, ρ_*). We henceforth suppress an asterisk that indicates steady state values when no confusion can arise.

$$J = \begin{bmatrix} \exp[\lambda(u-u^T)]\phi\alpha u_r & \exp[\lambda(u-u^T)] & \exp[\lambda(u-u^T)][\phi\alpha u_\rho + (1-\phi)] \\ k\phi\alpha u_r & 0 & k[\phi\alpha u_\rho + (1-\phi)] \\ -(\mu+\phi\alpha u_r)\rho & -(1+\mu)\rho & -[\phi\alpha u_\rho + (1-\phi)]\rho \end{bmatrix} \tag{42}$$

For this system, we have the following corresponding characteristic equation.

$$x^3 + a_1 x^2 + a_2 x + a_3 = 0 \tag{43}$$

where the coefficients a_i are determined as follows:

$$a_1 = -\mathrm{tr}(J) = -\exp[\lambda(u-u^T)]\phi\alpha u_r + [\phi\alpha u_\rho + (1-\phi)]\rho > 0 \tag{44a}$$

$$a_2 = \{(1+\mu)\rho[\phi\alpha u_\rho + (1-\phi)] - \exp[\lambda(u-u^T)]\phi\alpha u_r\}k \tag{44b}$$

$$+ \exp[\lambda(u-u^T)][\phi\alpha u_\rho + (1-\phi)]\rho\mu > \tag{44c}$$

$$a_3 = -\det(J) = \mu\rho\exp[\lambda(u-u^T)]k[\phi\alpha u_\rho + (1-\phi)] > 0 \tag{44d}$$

Note that $a_1 > 0, a_2 > 0$, and $a_3 > 0$ with no additional requirements. By using (44a), (44b), and (44d), we obtain:

$$L = a_1 a_2 - a_3 = D_1 k + D_2 \tag{45}$$

where

$$\begin{aligned} D_1 = \{&-\exp[\lambda(u-u^T)]\phi\alpha u_r + [\phi\alpha u_\rho + (1-\phi)]\rho\} \\ &\times \{(1+\mu)\rho[\phi\alpha u_\rho + (1-\phi)] - \exp[\lambda(u-u^T)]\phi\alpha u_r\} \\ &- \mu\rho\exp[\lambda(u-u^T)][\phi\alpha u_\rho + (1-\phi)] \end{aligned} \tag{46}$$

$$D_2 = \{-\exp[\lambda(u-u^T)]\phi\alpha u_r + [\phi\alpha u_\rho + (1-\phi)]\rho\} \times \exp[\lambda(u-u^T)][\phi\alpha u_\rho + (1-\phi)]\rho\mu > 0 \tag{47}$$

We shall investigate two cases by using the Routh-Hurwitz conditions, which are necessary and sufficient conditions for local asymptotic stability.

Case 1: $D_1 \geq 0$. In this case, we can obtain the following proposition.

Proposition 5. *Suppose that $D_1 \geq 0$. The steady state is locally asymptotically stable for all $k > 0$.*

Proof It is clear from (44a), (44d), and (45) that the Routh-Hurwitz conditions ($a_1 > 0, a_3 > 0, a_1a_2 - a_3 > 0$) are satisfied since $D_1 \geq 0$ and $D_2 > 0$.

□

Case 2: $D_1 < 0$. In this case, we shall focus our attention on the forecasting ability of private agents, k. We can obtain the following proposition:

Proposition 6. *Suppose that $D_1 < 0$. The steady state is locally asymptotically stable for all $k \in (0, k_c)$. Otherwise, it is unstable.*

Proof First of all, it is clear that $a_1 > 0$ and $a_3 > 0$. In addition, It is easy to verify that the function $L(k)$ has a negative slope and a positive vertical intercept: $D_1 < 0$ and $D_2 > 0$. Thus, the function $L(k)$ has a positive horizontal intercept at k_c such that $L(k) = 0$. This implies that $L(k) > 0$ if $k < k_c$ and $L(k) \leq 0$ if $k \geq k_c$. It is concluded, from what has been said above, that the Routh-Hurwitz conditions are satisfied if and only if $0 < k < k_c$. This completes the proof.

□

The above results suggests that the steady state is locally unstable if the adjustment speed of inflation expectations is sufficiently high. This means that the assumption of perfect foresight does not guarantee the stability of the steady-state point. Furthermore, **Proposition 6** suggests that the value $k = k_c$ is a stability switching point. We can show that the emergence of endogenous and persistent business cycles in our model by using the Hopf bifurcation theorem, which guarantees the existence of limit cycles in the local sense[3].

Proposition 7. *Suppose that $D_1 < 0$. The dynamic system (37a)–(37c) generates persistent fluctuations at $k = k_c$ via a Hopf bifurcation.*

Proof To apply the Hopf bifurcation theorem, we have to check the following two conditions: (1) The characteristic equation (43) has a pair of pure imaginary roots and a non-zero real root, and (2) the real part of the pure imaginary roots is not stationary with respect to the bifurcation parameter, k. The above conditions are equivalent to $a_1 > 0, a_3(k) > 0, L(k) = 0, L'(k) \neq 0$ at a bifurcation point[4] In our model, we can confirm that $a_1 > 0, a_3(k_c) > 0, L(k_c) = 0, L'(k_c) = D_1 < 0$ at $k = k_c$.

□

The intuitive explanation for the **Proposition 7** as follows. Suppose that a temporary shock causes an increase in the domestic rate of real interest at the steady state. This has two effects on the domestic goods market. The first effect is related to domestic investment. The increase in the interest rate leads to a decrease in domestic investment, which induces

[3]For a technical discussion of the Hopf bifurcation theorem, see Guckenheimer and Holmes (1997)

[4]On this point, see Asada and Semmler (1995)

$$r\uparrow \;\rightarrow\; u\downarrow \;\rightarrow\; g\downarrow \quad \text{(first effect)}$$

$$\downarrow$$

$$\pi\downarrow \;\rightarrow\; \pi^e\downarrow \;\rightarrow\; \rho\uparrow \;\rightarrow\; h\uparrow \quad \text{(second effect)}$$

a reduction in the utilization rate. That is, the economy experiences the recessionary phase. On the other hand, the second effect is connected to net exports. The rate of inflation starts to decline in response to the recessionary situation in the domestic goods market by the Phillips curve effect. This decreasing tendency of the inflation rate accompanies a decline in inflationary expectations via the adaptive expectations mechanism. As a result, a depreciation of the domestic currency prevails and hence induces foreigners to purchase the domestic goods.

If k is sufficiently large, the adjustment speed of expected inflations is fast. In this case, the second effect has a prevalent power in the domestic goods market. That is, an increase in the real rate of interest yields an active result in the domestic goods market by enhancing a depreciation of the domestic currency. This would lead to a further increase in the real rate of interest through the Taylor rule. In this way, the dynamic process of the economy is unstable. On the other hand, if k is sufficiently small, the adjustment speed of expected inflations is slow. In this case, the first effect has a dominant position in the domestic goods market. This means that an increase in the real rate of interest leads to a negative effect on the domestic goods market. The central bank would reduce the real rate of interest through the interest rate operating procedure. In this way, the dynamic process of the economy is stable.

7 Conclusion

Based on the works of Setterfield (2006) and Lima and Setterfield (2008), a series of studies went on to consider the possibility of making the regime of inflation targets with the post-Keynesian assumptions about the functioning of modern economies. One of the solutions in this attempt to reconcile is the use of incomes policies in addition to monetary policy.

In the present work, we argue that in the absence of institutional feasibility of using incomes policies, it is important to assess an alternative form of policy. One option is the use of monetary policies with dual mandate interest rule, which consider both the output gap and the inflation gap in comparison to a target. We analyze the characteristics of the model in the context of this dual mandate interest rule and show that even though it converges to a stable stationary state, this rule implies an expectational inconsistency in the long run. As demonstrated in this work, in the absence of the natural output, as well as in the absence of complementary policies to the monetary policy, a typical double mandate interest rule cannot guarantee the convergence of inflation to its target.

Having affirmed that, we propose a non-linear interest rule in which the sensitivity of the monetary policy to the inflation gap is sensitive to the rate of capacity utilization. In this new formulation, the closer (further) the economy to the full use of capacity utilization, the more (less) sensitive the monetary authority will be to deviations of inflation to the target. We show that a monetary policy rule of this type is not only difficult to implement in the real world, it meets the conditions for stability in a macrodynamic post-Keynesian model, so that inflation converges to the long-term inflation target even when the monetary authority is concerned about smoothing out the business cycle. In terms of macroeconomic policies, the main contribution of this paper is to offer a monetary policy rule conditioned by institutional constraints, especially regarding incomes policies, allowing the monetary authority to consider both inflation and the unemployment/output without losing sight of the role of the expectational inflation target anchor. In particular, the monetary police framework proposed here works either in the assumption of closed economy as the open-economy assumption.

New researches can be carried out considering other non-linearities in the monetary policy rule, or even in the Phillips curve. Notwithstanding, a natural extension of this work is to verify that the dynamic properties of the model remain valid in the case of an open economy or when considering the active role of fiscal policy with a budget constraint to the public sector.

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