Merger Incentives in a Mixed Duopoly with Asymmetric Market Structures*

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Abstract

This paper reconsiders the merger profitability problem between a public firm and a private firm in a mixed duopoly with substitutable goods. In this paper, we focus on the situation wherein the strategic contracts of the public firm and the private firm are different. We show that the area in the plane between the degree of product differentiation and the share of the owner of the pre-merger public firm in the merged firm when both firms want to merge is larger when the public firm chooses a quantity contract and the private firm chooses a price contract than when the public firm chooses a price contract and the private firm chooses a quantity contract. Moreover, in the four situations that are classified according to the strategic contracts of the public and private firms, a merger between firms is most likely when the public firm chooses a quantity contract and the private firm chooses a price contract. In contrast, in the market structure wherein the public firm chooses a price contract and the private firm chooses a quantity contract, the area wherein both firms want to merge with each other is smallest among the four games, implying that in this market structure, a merger between them occurs the least frequently.

Keywords: Mixed duopoly; Merger between public firm and private firm; Asymmetric competition with respect to strategic contracts

JEL classification: L13, L20, L32

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1 Introduction

This paper revisits likelihood in the public-private merger problem in a mixed duopoly with differentiated goods composed of one social welfare-maximizing public firm and one profit-maximizing private firm. By considering the two types of asymmetric market structures with respect to the strategic contracts of the firms, we depart from the existing works on such mixed duopoly merger problems. More concretely, we introduce the following two asymmetric market structures: (i) the market structure in which the public firm chooses a price contract and the private firm chooses a quantity contract and (ii) the market structure in which the public firm chooses a quantity contract and the private firm chooses a price contract.

Many works on horizontal mergers in the field of Industrial Organization have addressed the mergers of private firms owned by the private sector (private shareholders). Bárcena-Ruiz and Garzón (2003) considered a merger between one social welfare-maximizing public firm and one profit-maximizing private firm in which they formulated a merged multiproduct firm after the merger.[1)] Under this setting, Bárcena-Ruiz and Garzón (2003) found the condition for the degree of product differentiation and the shares of the public sector and the private sector in the merged firm such that the merger holds. Nakamura and Inoue (2007) explained the accomplishment of the merger between the public firm and a private firm in a mixed oligopolistic market with a homogeneous good composed of one public firm and multiple private firms in a pre-merger market structure, focusing on the efficiency-improving effect of such a merger. Subsequently, Méndez-Naya (2008) derived the condition for the shares of the owners of the pre-merger public and private firms in the merged firm such that the merger is satisfied by supposing a mixed oligopolistic market with a homogeneous good composed of one social welfare-maximizing public firm and a generalized number of profit-maximizing private firms before such a merger.[2)] However, in all the above-mentioned works on the

1) Recent real-world examples include the following. Volkswagen acquired the publicly owned Spanish firm SEAT in 1986 and Renault acquired Dacia. In the air line industry, the partially privatized SAS purchased Braathens, a rival firm. In China, the (local) government has pursued mergers among its public enterprises, including township and village enterprises and private firms.

2) Other studies on the merger between the public firm and the private firm include Kamaga and Nakamura (2007) and Kamijo and Nakamura (2009), who considered the achievement of the merger from the viewpoint of the cooperative game by using the core solution concept. In their 2007 paper, Kamaga and Nakamura considered the core before and after the possible merger in a mixed triopoly with one public firm and two symmetric private firms in the situation wherein the technology of each firm is represented as the quadratic cost function of its quantity level, whereas, in their 2009 paper, in a mixed triopoly, Kamijo and Nakamura conducted a similar analysis under the assumption that the technology of each firm is represented as the constant marginal cost function. Kamijo and Nakamura (2009) extended the model of Kamaga and Nakamura (2007) in a mixed oligopoly with one public firm and a generalized number of private firms analogous to that in Méndez-Naya (2011) and characterized the condition for (i) the degree of the difference in productivity between the public firm and the symmetric private

merger of a public firm and a private firm in a mixed duopoly or oligopoly, the situation in which their strategic contacts differed was not considered. In this paper, we characterize the condition of the degree of product differentiation and the share of the owner of the pre-merger public firm in the merged firm such that the merger between the public firm and the private firm occurs the the most frequently among the four combinations of strategic contracts.[3)]

In the context of a mixed duopoly, a growing number of works have addressed competition in an asymmetric market structure with respect to the strategic contracts of the public and private firms.[4)] Most recently, taking into account the influence of the choices of the strategic contracts of (the owners of) the firms on the market outcome, including their quantities, profits, and social welfare, Matsumura and Ogawa (2012) studied the endogenous selection problem for the strategic contracts of the public firm and the private firm in a mixed duopoly with differentiated goods using the approach adopted in Singh and Vives (1984). Scrimitore (2013) extended the simple model of Matsumura and Ogawa (2012) by introducing production subsidization by the government. Scrimitore (2013) showed that the two types of asymmetric market structures with respect to the firms' strategic contracts can become the equilibrium market structures depending on the value of the government's subsidy level. This result indicates that it is important to consider the asymmetric market structures with respect to the firms' strategic contracts under various economic situations in a mixed duopoly with differentiated goods. Choi (2012) investigated the endogenous selection of strategic contracts by a public firm and a private firm in a mixed duopoly with unionization with wage bargaining, and Chirco et al. (2014) explored the endogenous determination of strategic contracts of such firms in the context of the hiring problem. Most recently, Haraguchi and Matsumura (2014) considered the influence of foreign penetration on domestic market outcomes and the endogenous selection of contracts by both firms. The results indicate that it is important to consider the asymmetric market structures with respect to the firms' strategic contracts under several different economic situations in a mixed duopoly with

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firms and (ii) the number of symmetric private firms such that the market structure with a merger between one public firm and one private firm belongs to the core.

3) Since, in this paper, we suppose a mixed duopoly with differentiated goods in the fashion of Bárcena-Ruiz and Garzón (2003), Méndez-Naya (2008), and Andree (2013) as a pre-merger market structure, the pre-merger market structures that are classified on the basis of the strategic contracts of the public firm and the private firm are as follows: (i) the market structure in which both the public firm and the private firm choose price contracts (*p*-*p* game); (ii) the market structure in which the public firm chooses its price contract and the private firm chooses its quantity contract (*p*-*q* game); (iii) the market structure in which the public firm chooses a quantity contract and the private firm chooses a price contract (*q*-*p* game); and (iv) the market structure in which both the public firm and the private firm choose their quantity contracts (*q*-*q* game).

4) Many works on private oligopolies (and duopolies) have taken into account asymmetric market structures with respect to each firm's strategic contract, including Singh and Vives (1984), Häckner (2000), Zanchettin (2006), and Arya et al. (2008).

differentiated goods.

In this paper, in a mixed duopolistic market with substitutable goods, taking into account the two types of asymmetric market structures with respect to the firms' strategic contracts, we find that a merger between two firms is more likely to occur in the market structure in which the public firm chooses a quantity contract and the private firm chooses a price contract (the *q-p* game) than in the market structure in which the public firm chooses a price contract and the private firm chooses a quantity contract (the *p-q* game). Furthermore, we show that a merger occurs most and least frequently in the *q-p* game and the *p-q* game, respectively. These results are mainly explained by the ranking order of consumer surplus and social welfare, which is implied by the ranking orders of the quantity and price levels of the public firm and the private firm. Thus, on the basis of the approach of Bárcena-Ruiz and Garzón (2003), Méndez-Naya (2008), and Andree (2013), we find that the likelihood of the merger between the public firm and the private firm is highest and lowest in the two types of asymmetric market structures with respect to the firms' strategic contracts. This result implies that the corresponding authority must pay attention to both firms' strategic contracts, including the symmetric price competition and quantity competition as well as the two types of asymmetric market structures, that is, the *p-q* game and the *q-p* game, in determining an appropriate merger policy in a mixed oligopolistic market with differentiated goods.

The remainder of this paper is organized as follows. In Section 2, in order to consider the likelihood of the merger between the public firm and the private firm, we formulate a mixed duopolistic model with substitutable goods composed of a social welfare-maximizing public firm and a profit-maximizing private firm. In Section 3, using the model formulated in Section 2, we derive the market outcomes including the quantity and price levels, firms' profits, consumer surplus, producer surplus, social welfare, and payoffs of the (pre-merger) public and private firms in the *p-q* game and the *q-p* game. In Section 4, we derive the condition of the shares of both the public firm and the private firm such that the there are merger incentives on the basis of the consumer surplus, producer surplus, and the payoff of each firm for any degree of product differentiation, and we compare the difference in the areas in the plane between the degree of product differentiation and the share of the owner of the pre-merger public firm in the merged firm such that both firms want to merge in the *p-q* game and the *q-p* game. We also characterize the market structures in which the merger between the public firm and the private firm occurs most and least frequently among the four games (the *p-p* game, the *q-q* game, the *p-q* game, and the *q-p* game, the former two of which are considered in Andree (2013)). Section 5 concludes with several remarks.

2 Model

We consider a mixed duopolistic market with differentiated goods composed of one social welfare-maximizing public firm (firm 0) and one profit-maximizing private firm (firm 1) in a pre-merger market structure. The basic structure of the model follows a standard product differentiation model as in Dixit (1979) and Singh and Vives (1984). Firms 0 and 1 produce a differentiated good. A representative consumer's utility is given by $U(q_0, q_1) = a(q_0 + q_1) - (q_0^2 + 2bq_0q_1 + q_1^2)/2 - p_0q_0 - p_1q_1$, where $b \in (0, 1)$ represents the degree of product differentiation.[5] Note that q_0 and q_1 denote the output levels of firms 0 and 1, respectively. Moreover, similar to Bárcena-Ruiz and Garzón (2003) and Andree (2013), the above specification of the representative consumer's utility implies the following inverse demand functions for positive demand:

$$p_i(q_i, q_j; b) = a - q_i - bq_j,$$
$$q_i(p_i, p_j; b) = \frac{a}{1+b} - \frac{1}{1-b^2}p_i + \frac{a}{1-b^2}p_j, \quad i, j = 0, 1; i \neq j.$$

Furthermore, we assume that both firms 0 and 1 employ the same technology which is represented as the following cost function: $C(q_i) = q_i^2$, $(i = 0,1)$. This cost function is the same as that employed in Bárcena-Ruiz and Garzón (2003), Méndez-Naya (2008), Méndez-Naya (2011), and Andree (2013), all of which considered the merger problem between the public firm and the private firm (s). Then, the per-merger profits of firms 0 and 1 are given for the *p-q* and *q-p* games in a mixed duopolistic market as follows:

$$[p\text{-}q \text{ game}] \begin{cases} \pi_0(p_0, q_1; b) = [p_0 - q_0(p_0, q_1)]q_0(p, q_1) = (a - p_0 - bq_1)(-a + 2p_0 + bq_1), \\ \pi_1(p_0, q_1; b) = [p_1(p_0, q_1) - q_1]q_1 = [a(1-b) + bp_0 - 2q_1 + b^2q_1]q_1, \end{cases}$$
$$[q\text{-}p \text{ game}] \begin{cases} \pi_0(q_0, p_1; b) = [p_0(q_0, p_1) - q_0]q_0 = [a(1-b) + bp_1 - 2q_0 + b^2q_0]q_0, \\ \pi_1(q_0, p_1; b) = [p_1 - q_1(q_0, p_1)]q_1(q_0, p_1) = (a - p_1 - bq_0)(-a + 2p_1 + bq_0), \end{cases}$$

where p_i represents the pric elevel of firm i or j, $(i, j = 0, 1)$. In this paper, we focus on the market structure in which the strategic contracts of firm 0 and firm 1 are different from each other. Consumer surplus CS is represented as follows for the *p-q* and *q-p* games:

$$[p\text{-}q \text{ game}] \quad CS(p_0, q_1; b) = [a^2 - 2ap_0 + p_0^2 + (1-b^2)q_1^2]/2,$$
$$[q\text{-}p \text{ game}] \quad CS(q_0, p_1; b) = [a^2 - 2ap_1 + p_1^2 + (1-b^2)q_0^2]/2.$$

Taking into account the fact that the producer surplus is equal to the sum of the profits of firms 0 and 1, social welfare is defined as follows:

$$\begin{aligned}[p\text{-}q \text{ game}] \quad W(p_0, q_1; b) &= CS(p_0, q_1; b) + PS(p_0, q_1; b) \\ &= [-a^2 - 3p_0^2 - 4bp_0q_1 - (3+b^2)q_1^2 + 2a(2p_0 + q_1 + bq_1)]/2,\end{aligned}$$

5) More precisely, the fact that the value of b is restricted to the open interval $(0, 1)$ means that the goods produced by public firm 0 and private firm 1 are substitutes.

$$[q\text{-}p \text{ game}] \quad W(q_0, p_1; b) = CS(q_0, p_1; b) + PS(q_0, p_1; b)$$
$$= [-a^2 - 3p_1^2 - 4bp_1q_0 - (3+b^2)q_0^2 + 2a(2p_1 + q_0 + bq_0)]/2,$$

where $PS(i_0, j_1) = \pi_0(i_0, j_1) + \pi_1(i_0, j_1)$, $(i, j = p, q)$.[6)]

Following Bárcena-Ruiz and Garzón (2003), Méndez-Naya (2008), Méndez-Naya (2011), and Andree (2013), we consider that when firms 0 and 1 determine whether the merger is completed or not, such a merged multiproduct firm is supposed to be owned by both the public sector (the government) and the private sector (the owner of the pre-merger private firm). Similar to Bárcena-Ruiz and Garzón (2003), Méndez-Naya (2008), Méndez-Naya (2011), and Andree (2013), we suppose that the owner of public firm 0, the government, owns s percent of the shares of the public-private merged firm 01 whereas the owner of the pre-merger private firm owns $1-s$ percent of the shares of merged firm 01. Under the above assumption, merged multiproduct firm 01 maximizes the weighted sum of social welfare, W, and the sum of the profits of firms 0 and 1, which is equal to the producer surplus, $PS_{01}^m(i_0, j_1) \equiv \pi_{01}^m(i_0, j_1) = \pi_0^m(i_0, j_1) + \pi_1^m(i_0, j_1)$, $V_{01}^m(i_0, j_1) = sW^m(i_0, j_1) + (1-s)\pi_{01}^m(i_0, j_1)$, where the superscript m denotes the objective function of merged firm 01, social welfare, and the profit of firm 01, $(i, j = p, q)$.

Following Andree (2013), it is supposed that the game runs in the following two stages. In the first stage, the owners of firms 0 and 1 decide whether or not to merge with each other, and in the second stage, either or both of the firms compete in terms of their quantity or price levels in the differentiated goods mixed market. We adopt the subgame perfect Nash equilibrium as the solution concept of the game, and thus, our approach is based on backward induction.

3 Analysis in the *p-q* and *q-p* games

In this section, we derive the two types of asymmetric market structures with respect to the strategic contracts of firms 0 and 1, that is, the *p-q* game and the *q-p* game before and after the merger between the firms.

3.1 Market outcomes after the merger of firms 0 and 1 in the *p-q* and *q-p* games

After the merger between public firm 0 and private firm 1 in the *p-q* game and the *q-p* game, the objective function of merged firm 01 is $V_{01}(i_0, j_1; b) = sW(i_0, j_1; b) + (1-s)\pi_{01}(i_0, j; b)$, $(i, j = p, q)$. Consequently, we obtain the following market outcomes:

6) More precisely, the producer surplus is given as follows for the *p-q* game and the *q-p* game:

$$[p\text{-}q \text{ game}] \quad PS(p_0, q_1; b) = -a^2 + a(3p_0 + q_1 + bq_1) - 2(p_0^2 + bp_0q_1 + q_1^2),$$
$$[q\text{-}p \text{ game}] \quad PS(q_0, p_1; b) = -a^2 + a(3p_1 + q_0 + bq_0) - 2(p_1^2 + bp_1q_0 + q_0^2).$$

$$p^{pq}_{0am} = p^{pq}_{1am} = a[3 + b(1 - s) - s]/[4 + b(2 - s) - s] = p^{qp}_{0am} = p^{qp}_{1am},$$
$$q^{pq}_{0am} = q^{pq}_{1am} = a/[4 + b(2 - s) - s] = q^{qp}_{0am} = q^{qp}_{1am},$$
$$\pi^{pq}_{am} = \pi^{pq}_{0bm} + \pi^{pq}_{1bm} = 2a^2[2 + b(1 - s) - s]/[4 + b(2 - s) - s]^2 = \pi^{qp}_{0bm} + \pi^{qp}_{1bm} = \pi^{qp}_{am},$$
$$CS^{pq}_{am} = CS^{qp}_{am} = a^2(1 + b)/[4 + b(2 - s) - s]^2, \quad W^{pq}_{am} = W^{qp}_{am} = a^2[5 + b(3 - 2s) - 2s]/[4 + b(2 - s) - s]^2.$$

Therefore, we obtain the payoffs of pre-merger firm 0 and pre-merger private firm 1, which are equal to the equilibrium social welfare and the share of per-merger firm 1 in the equilibrium profit of merged-firm 01 as follows:

$$U^{pq}_{0am} = W^{pq}_{am} = a^2[5 + b(3 - 2s) - 2s]/[4 + b(2 - s) - s]^2 = W^{qp}_{am} = U^{qp}_{0am},$$
$$U^{pq}_{1am} = (1 - s)\pi^{pq}_{am} = 2a^2(1 - s)[2 + b(1 - s) - s]/[4 + b(2 - s) - s]^2 = (1 - s)\pi^{qp}_{am} = U^{qp}_{1am}.$$

Note that the subscript *am* denotes the payoffs of pre-merger public firm 0 and pre-merger private firm 1 on the basis of the market outcomes after their merger.[7)]

3.2 Market outcomes before the merger of firms 0 and 1 in the *p-q* game

Before the merger between public firm 0 and private firm 1 in the *p-q* game, their objective functions are $V_0(p_0, q_1; b) = W(p_0, q_1; b)$ and $V_1(p_0, q_1; b) = \pi_1(p_0, q_1; b)$, respectively. Consequently, we obtain the following market outcomes before the merger in the *p-q* game:

$$p^{pq}_{0bm} = a(4 - b - b^2)/2(3 - b^2), \quad p^{pq}_{1bm} = a(3 - b)/4, \quad q^{pq}_{0bm} = a(4 - b - b^2)/4(3 - b^2),$$
$$q^{pq}_{1bm} = a(3 - b)/4(3 - b^2), \quad \pi^{pq}_{0bm} = a^2(4 - b - b^2)^2/16(3 - b^2)^2, \quad \pi^{pq}_{1bm} = a^2(3 - b)^2(2 - b^2)/16(3 - b^2)^2,$$
$$CS^{pq}_{bm} = a^2(25 + 10b - 20b^2 - 2b^3 + 3b^4)/32(3 - b^2)^2, \quad W^{pq}_{bm} = a^2(93 - 30b - 48b^2 + 14b^3 + 3b^4)/32(3 - b^2)^2.$$

Note that the subscript *bm* denotes the market outcomes and the payoffs of public firm 0 and private firm 1 on the basis of their pre-merger market outcomes. Therefore, in the *p-q* game, we obtain the payoffs of pre-merger public firm 0 and pre-merger private firm 1, which are equal to the equilibrium social welfare and the equilibrium profit of private firm 1, respectively, as follows:

$$U^{pq}_{0bm} = W^{pq}_{m} = a^2(93 - 30b - 48b^2 + 14b^3 + 3b^4)/32(3 - b^2)^2,$$
$$U^{pq}_{1bm} = \pi^{pq}_{1bm} = a^2(3 - b)^2(2 - b^2)/16(3 - b^2)^2.$$

3.3 Market outcomes before the merger between firms 0 and 1 in the *q-p* game

Before the merger between public firm 0 and private firm 1 in the *q-p* game, their objective functions are $V_0(q_0, p_1; b) = W(q_0, p_1; b)$ and $V_1(q_0, p_1; b) = \pi_1(q_0, p_1; b)$, respectively. Consequently, we obtain the following market outcomes before the merger in the *q-p* game:

7) In the model of this paper, the market outcomes and the payoffs of firms 0 and 1 after their merger are the same among the four games since the merger between firms 0 and 1 results in a complete monopoly.

$$q^{qp}_{0bm} = a(2-b)/(6-b^2),\quad q^{qp}_{1bm} = a(3-b)/2(6-b^2),\quad p^{qp}_{0bm} = a(8-b-b^2)/2(6-b^2),$$
$$p^{qp}_{1bm} = 3a(3-b)/2(6-b^2),\quad \pi^{qp}_{0bm} = a^2(2-b)(4+b-b^2)/2(6-b^2)^2,\quad \pi^{qp}_{1bm} = a^2(3-b)^2/2(6-b^2)^2,$$
$$CS^{qp}_{bm} = a^2(25+2b-15b^2+4b^3)/8(6-b^2)^2,\quad W^{qp}_{bm} = a^2(93-30b-23b^2+8b^3)/8(6-b^2)^2.$$

Therefore, in the q-p game, we obtain the payoffs of pre-merger public firm 0 and pre-merger private firm 1, which are equal to the equilibrium social welfare and the equilibrium profit of pre-merger firm 1,

respectively, as follows:

$$U^{qp}_{0bm} = W^{qp}_{bm} = a^2(93-30b-23b^2+8b^3)/8(6-b^2)^2,\quad U^{qp}_{1bm} = \pi^{qp}_{1bm} = a^2(3-b)^2/2(6-b^2)^2.$$

4 Comparisons of several market outcomes before and after the merger between firms 0 and 1 in the four market structures

In this section, we determine the merger incentives for the owners of firm 0 and 1 for several market outcomes in the four games.

4.1 Consumer surplus before and after the merger between firms 0 and 1 in the four market structures

In this subsection, we compare the consumer surplus before and after the merger between firms 0 and 1 in all four games.

1. In the p-p game, we have

$$CS^{pp}_{am} \gtreqless CS^{pp}_{bm} \Longleftrightarrow s \gtreqless s^{pp}_{CS} := \frac{100-42b+46b^2+40b^3-6b^5+2b^6-\sqrt{2}\ (12-5b^2+b^4)\sqrt{25-23b+10b^3-5b^4+b^5}}{25+2b-23b^2+10b^3+5b^4-4b^5+b^6}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the p-p game when $s > s^{pp}_{CS}$.

2. In the q-q game, we have

$$CS^{qq}_{am} \gtreqless CS^{qq}_{bm} \Longleftrightarrow s \gtreqless s^{qq}_{CS} := \frac{100+190b+62b^2-44b^3-12b^4+4b^5-(1+b)(12-b^2)\sqrt{50+70b-4b^2-20b^3+4b^4}}{(1+b)^2(25+10b-12b^2+2b^3)}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the q-q game when $s > s^{qq}_{CS}$.

3. In the p-q game, we have

$$CS^{pq}_{bm} \gtreqless CS^{pq}_{bm} \Longleftrightarrow s \gtreqless s^{pq}_{CS} := \frac{2\left[50-5b-25b^2-b^3+3b^4-2(3-b^2)\sqrt{50-30b-10b^2+6b^3}\right]}{25+10b-20b^2-2b^3+3b^4}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the p-q game when $s > s^{pq}_{CS}$.

4. In the q-p game, we have

$$CS_{am}^{qp} \gtreqless CS_{bm}^{qp} \Longleftrightarrow s \gtreqless s_{CS}^{qp} := \frac{2\left[50+79b+b^2-35b^3-3b^4+4b^5-(1+b)(6-b^2)\sqrt{50+54b-26b^2-22b^3+8b^4}\right]}{(1+b)^2(25+2b-15b^2+4b^2)}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the *q-p* game when $s > s_{CS}^{qp}$.

Furthermore, form easy calculations, we obtain the following result: $s_{CS}^{pq} > \max\{s_{CS}^{pp}, s_{CS}^{qq}\}$ and $\min\{s_{CS}^{pp}, s_{CS}^{qq}\} > s_{CS}^{qp}$.[8)]

Therefore, from a viewpoint of consumer surplus, we realize that the merger between firms 0 and 1 is most likely to occur in the *q-p* game while it is least likely to occur in the *p-q* game.[9)]

4.2 Producer surplus before and after the merger between firms 0 and 1 in the four market structures

In this section, we compare the producer surplus before and after the merger between firms 0 and 1 in all four games.

1. In the *p-p* game, we have

$$PS^{pp} \gtreqless PS^{pp} \Longleftrightarrow s \gtreqless s_{PS}^{pp} := \frac{-8+12b-8b^2-14b^3+15b^4-3b^5-3b^6+b^7+(12-5b^2+b^4)\sqrt{8-20b+28b^2-14b^3-b^4+4b^5-b^6}}{34-16b-21b^2+5b^3+10b^4-2b^5-3b^6+b^7}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the *p-p* game when $s < s_{PS}^{pp}$.

2. In the *q-q* game, we have

$$PS_{am}^{qq} \gtreqless PS_{bm}^{qq} \Longleftrightarrow s \gtreqless s_{PS}^{qq} := -\frac{8+20b+16b^2-2b^3-5b^4+b^5-(12+12b-b^2-b^3)\sqrt{8+12b+4b^2-6b^3+b^4}}{(1+b)^2(34-20b+3b^2)}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the *q-q* game when $s < s_{PS}^{qq}$.

3. In the *p-q* game, we have

$$PS_{am}^{pq} \gtreqless PS_{bm}^{pq} \Longleftrightarrow s \gtreqless s_{PS}^{pq} := \frac{2\left[-2-b+b^2+2\sqrt{2-b}(3-b^2)\right]}{17-10b-7b^2+4b^3}.$$

Thus, we obtain the result that the owners of both firms have a strict incentive to merge in the *p-q*

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8) More concretely, we have

$$\begin{cases} CS_{bm}^{pq} > CS_{bm}^{qq} \geq CS_{bm}^{pp} > CS_{bm}^{qp}, & \text{if } b \leq 0.7223065, \\ CS_{bm}^{pq} > CS_{bm}^{pp} > CS_{bm}^{qq} > CS_{bm}^{qp}, & \text{otherwise} \end{cases}$$

Thus, we find that the market competition is most intense in the *p-q* game, while the market competition is the least intense in the *q-p* game.

9) More precisely, we obtain the following result: $s_{CS}^{pp} \gtreqless s_{CS}^{qq} \Leftrightarrow b \gtreqless 0.723065$. Thus, from a viewpoint of consumer surplus, we find that the merger between firms 0 and 1 is more likely to be achieved in the *q-q* game than in the *p-p* game when *s* is sufficiently high (*b* > 0.723065), whereas the merger between firms 0 and 1 is more likely to be achieved in the *p-p* game than in the *q-q* game otherwise (*b* < 0.723065).

game when $s < s_{PS}^{pq}$.

4. In the q-p game, we have

$$PS_{am}^{qp} \gtreqless PS_{bm}^{qp} \Longleftrightarrow s \gtreqless s_{PS}^{qp} := \frac{2\left[-2+6+b-b^2\sqrt{2-b}\right]}{(1+b)(17-8b-2b^2+b^3)}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the q-p game when $s < s_{PS}^{qp}$.

Furthermore, from easy calculations, we obtain the following result: $s_{PS}^{pq} > \max\{s_{PS}^{pp}, s_{PS}^{qq}\}$ and min $\{s_{PS}^{pp}, s_{PS}^{qq}\} > s_{PS}^{qq}$.[10] Therefore, from the viewpoint of producer surplus, the merger between firms 0 and 1 is most likely to occur in the p-q game while it is least likely to occur in the q-p game.[11]

4.3 Social welfare before and after the merger between firms 0 and 1 in the four market structures

In this section, we compare the social welfare, which is equal to the payoff of firm 0 before and after the merger between firms 0 and 1 in all four games.

1. In the p-p game, we have

$$U_{0am}^{pp} \gtreqless U_{0bm}^{pp} \Longleftrightarrow s \gtreqless s_W^{pp} := \frac{84-18b-62b^2+12b^3+30b^4-12b^5-4b^6+2b^7-\sqrt{2(9-3b+3b^2-b^3)(12-12b-5b^2+5b^3+b^4-b^5)}}{93-30b-65b^2+20b^3+25b^4-8b^5-5b^6+2b^7}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the p-p game when $s > s_W^{pp}$.

2. In the q-q game, we have

$$U_{0am}^{qq} \gtreqless U_{0bm}^{qq} \Longleftrightarrow s \gtreqless s_W^{qq} := \frac{84+66b-36b^2-4b^3+2b^4-\sqrt{6(3-b)(12-b^2)}}{(1+b)(93-30b-6b^2+2b^3)}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the q-q game when $s > s_W^{qq}$.

3. In the p-q game, we have

$$U_{0am}^{pq} \gtreqless U_{0bm}^{pq} \Longleftrightarrow s \gtreqless s_W^{pq} := \frac{2\left[42-9b-21b^2+b^3+3b^4-2\sqrt{6(3-b)}(3-3b-b^2+b^3)\right]}{93-30b-48b^2+14b^3+3b^4}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the p-q game when $s > s_W^{pq}$.

---

10) More precisely, we have

$$\begin{cases} PS_{bm}^{qp} > PS_{bm}^{pp} \geq PS_{bm}^{qq} > PS_{bm}^{pq}, & \text{if } b < 0.820132 \text{ and} \\ PS_{bm}^{qp} > PS_{bm}^{qq} > PS_{bm}^{pp} > PS_{bm}^{pq}, & \text{otherwise.} \end{cases}$$

11) More precisely, we obtain the following result: $s_{PS}^{pp} \gtreqless s_{PS}^{pp} \Longleftrightarrow b \gtreqless 0.820132$. Thus, from the viewpoint of producer surplus, we find that the merger is more likely in the q-q game than in the p-p game when b is sufficiently high ($b > 0.820132$), whereas the merger is more likely in the q-q game otherwise ($b < 0.820132$).

4. In the q-p game, we have

$$U_{0am}^{qp} \gtreqless U_{0bm}^{qp} \Longleftrightarrow s \gtreqless s_W^{qp} := \frac{2\left[42+75b+5b^2-35b^3-3b^4+4b^5-\sqrt{2(9-3b+3b^2-b^3)(6+6b-b^2-b^3)}\right]}{(1+b)^2(93-30b-23b^2+8b^3)}.$$

Thus, we obtain the result that the owners of both firms have a strict incentive to merge their firms in the q-p game when $s > s_W^{qp}$.

Furthermore, form easy calculations, we obtain the following result: $s_W^{pq} > s_W^{pp} > s_W^{qq} > s_W^{qp}$. Therefore, from the viewpoint of social welfare, the merger between firms 0 and 1 is most likely to occur in the q-p game, while it is least likely to occur in the p-q game.[12)]

4.4 Profit of firm 1 before and after the merger between firms 0 and 1 in the four market structures

In this subsection, we compare the profit of firm 1, which is equal to the payoff of firm 1 before and after the merger between firms 0 and 1 in all four games.

1. In the p-p game, we have

$$U_{1am}^{pp} \gtreqless U_{1bm}^{pp} \Longleftrightarrow s \gtreqless s_\pi^{pp} := \frac{\left[\begin{array}{c}360+228b-296b^2-198b^3+129b^4+92b^5-28b^6-20b^7+3b^8+2b^9-(12-5b^2+b^4)\\ \times\sqrt{360+252b-252b^2-190b^3+61b^4+36b^5-10b^6-2b^7+b^8}\end{array}\right]}{270+264b-227b^2-220b^3+94b^4+94b^5-19b^6-20b^7+2b^8+2b^9}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the p-p game when $s > s_\pi^{qp}$.

2. In the q-q game, we have

$$U_{1am}^{qq} \gtreqless U_{1bm}^{qq} \Longleftrightarrow s \gtreqless s_\pi^{qq} := \frac{360+228b-44b^2-36b^3-b^4+2b^5-(12-b^2)\sqrt{360+252b-48b^2-64b^3+b^4+4b^5}}{2(135+132b-22b^2-23b^3+b^5)}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the p-q game when $s > s_\pi^{qq}$.

3. In the p-q game, we have

$$U_{1am}^{pq} \gtreqless U_{1bm}^{pq} \Longleftrightarrow s \gtreqless s_\pi^{pq} := \frac{2\left[180+114b-112b^2-75b^3+15b^4+13b^5+b^6-2(3-b^2)\sqrt{360+252b-228b^2-190b^3+28b^4+36b^5-2b^7}\right]}{270+264b-179b^2-172b^3+28b^4+28b^5+b^6}.$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the p-q game when $s < s_\pi^{pq}$.

4. In the q-p game, we have

$$\pi_{1am}^{qp} \gtreqless \pi_{1bm}^{qp} \Longleftrightarrow s \gtreqless s_\pi^{qp} := \frac{2\left[90+57b-29b^2-21b^3+2b^4+2b^5-(6-b^2)\sqrt{90+63b-18b^2-16b^3+b^4+b^5}\right]}{135+132b-46b^2-44b^3+3b^4+4b^5}.$$

12) More concretely, we have

$$U_{0bm}^{pq} > U_{0bm}^{pp} > U_{0bm}^{qq} > U_{0bm}^{qp}, \quad \forall b \in (0,1)$$

Thus, we obtain the result that there is a strict incentive for firms 0 and 1 to merge in the *q-p* game when $s < s_\pi^{qp}$.

Furthermore, from easy calculations, we obtain the following result: $s_\pi^{pq} > \max\{s_\pi^{pp}, s_\pi^{qq}\}$ and $\min\{s_\pi^{pp}, s_\pi^{qq}\} > s_\pi^{qp}$.[13] Therefore, from the viewpoint of the profit of firm 1, the merger between firms 0 and 1 is most likely to occur in the *p-q* game, while it is least likely to occur in the *q-p* game.[14]

Summing the comparisons of the consumer surplus, producer surplus, social welfare (which is equal to the payoff of the owner of firm 0), and the profit of firm 1, (which is equal to the payoff of the owner of firm 1) before and after the merger between firms 0 and 1, we find that the merger incentives from the viewpoints of consumer surplus and social welfare are sharply opposed to those from viewpoints of producer surplus and the profit of firm 1 with respect to the value of s which depends on b.

Our next concern is the area in the (b, s) plane such that the merger incentives with respect to the payoffs of the owners of firms 0 and 1 are compatible, particularly in the two types of market structures with their asymmetric strategic contracts. Specifically, the owner of public firm 0 wants to merge his firm with firm 1 most strongly in the *p-q* game for any $b \in (0,1)$, whereas the owner of firm 1 wants to merge his firm with firm 0 most strongly in the *q-p* game. We determine which of the areas among the four games in the (b, s) plane such that the merger between firms 0 and 1 is achieved is largest and smallest given opposing merger incentives between the owners of firms 0 and 1.

4.5 Compatibility of the merger incentive between firms 0 and 1

From the above analyses, we find that the merger incentive of firm 0 contrasts with that of firm 1, particularly between the *p-q* game and the *q-p* game. Our next concern is to determine in which of the market structures the merger between the public firm and the private firm is most and least likely to occur. We derive the area in which the merger incentives of both firms in the four games are compatible. For this purpose, first, we define the degree of product differentiation on the intersection point of s_π^{pq} and s_W^{pq} as b^{pq} in the *p-q* game and that on the intersection point of s_π^{qp} and s_W^{qp} as b^{qp} in the *q-p* game. Then, from easy calculations, we obtain $b^{pq} \approx 0.274145$. Here, we obtain the following results from rigorous calculation on the concrete area in the (b, s) plane in which that both firms want to merge with each other in the *p-q* game. Area (i) in Figure 1 shows the area in the (b, s) plane in

13) More concretely, we have

$$\begin{cases} U_{1bm}^{qp} > U_{1bm}^{pp} \geq U_{1bm}^{qq} > U_{1bm}^{pq}, & \text{if } \; b \leq 0.913327, \\ U_{1bm}^{qp} > U_{1bm}^{qq} > U_{1bm}^{pp} > U_{1bm}^{pq}, & \text{otherwise.} \end{cases}$$

14) More precisely, we obtain the following result: $s_\pi^{pp} \gtreqless s_\pi^{qq} \Longleftrightarrow b \gtreqless 0.913327$. Thus, from the viewpoint of firm 1's profit, we find that the merger between firms 0 and 1 is more likely to occur in the *q-q* game than in the *p-p* game when s is sufficiently high ($b > 0.913327$), whereas it is more likely to occur in the *q-q* game otherwise ($b < 0.913327$).

which both firms want to merge with each other in the p-q game.[15)]

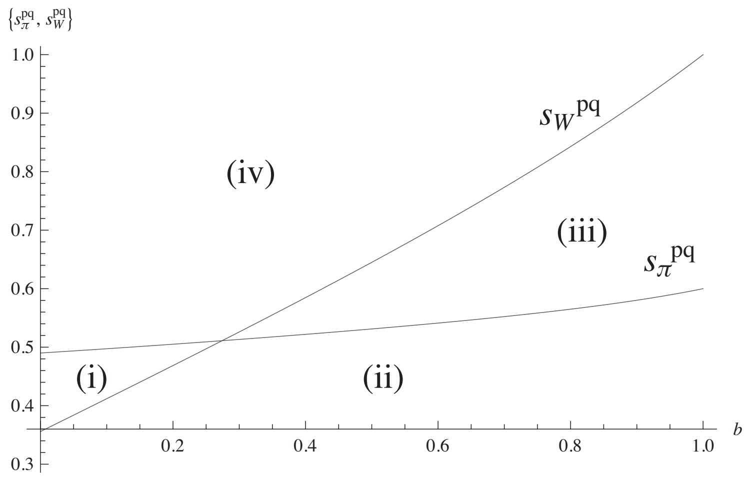

Figure 1: Incentives for firms 0 and 1 to merge with each other in the *p-q* game

Then, defining the concrete area in the (b, s) plane such that both firms wish to merge with each other as ϕ^{pq}, we obtain the following result:

$$\phi^{pq} := \int_0^{b^{pq}} s_\pi^{pq} db - \int_0^{b^{pq}} s_W^{pq} db = 0.137151 - 0.11872 = 0.018431.$$

Similar to the p-q game, we define the degree of product differentiation on the intersection point of s_π^{qp} and s_W^{qp} as b^{qp} in the q-p game and that on the intersection point of s_π^{qp} and s_W^{qp} as b^{qp} in the q-p game. Then, from easy calculations, we obtain $b^{qp} \approx 0.590175$. Here, we obtain the following results from rigorous calculation on the concrete area in the (b, s) plane in which both firms want to merge with each other in the q-p game. Area (i) in Figure 2 describes the area in the (b, s) plane in which both firms want to merge with each other in the q-p game.[16)]

15) Figure 1 has areas (ii) to (iv) other than area (i) in the p-q game. We realize

$$\text{area}(ii)\begin{cases} U_{0bm}^{pq} > U_{0am}^{pq} \Longleftrightarrow s_W^{pq} > s, \\ U_{1am}^{pq} > U_{1bm}^{pq} \Longleftrightarrow s_\pi^{pq} > s, \end{cases} \quad \text{area}(iii)\begin{cases} U_{0bm}^{pq} > U_{0am}^{pq} \Longleftrightarrow s_W^{pq} > s, \\ U_{1bm}^{pq} > U_{1am}^{pq} \Longleftrightarrow s > s_\pi^{pq}, \end{cases} \quad \text{area}(iv)\begin{cases} U_{0am}^{pq} > U_{0bm}^{pq} \Longleftrightarrow s > s_W^{pq}, \\ U_{1bm}^{pq} > U_{1am}^{pq} \Longleftrightarrow s > s_\pi^{pq}. \end{cases}$$

Therefore, we recognize that in area (ii), firms 0 doers not want to merge, while firm 1 does; that in (iii) neither firm wants to merge; and that in (iv), firm 0 wants to merge and firm 1 does not.

16) Figure 2 has areas (ii) to (iv) other than area (i) in the q-p game. We realize

$$\text{area}(ii)\begin{cases} U_{0bm}^{qp} > U_{0am}^{qp} \Longleftrightarrow s_W^{qp} > s, \\ U_{1am}^{qp} > U_{1bm}^{qp} \Longleftrightarrow s_\pi^{qp} > s, \end{cases} \quad \text{area}(iii)\begin{cases} U_{0bm}^{qp} > U_{0am}^{qp} \Longleftrightarrow s_W^{qp} > s, \\ U_{1bm}^{qp} > U_{1am}^{qp} \Longleftrightarrow s > s_\pi^{qp}, \end{cases} \quad \text{area}(iv)\begin{cases} U_{0am}^{qp} > U_{0bm}^{qp} \Longleftrightarrow s > s_W^{qp}, \\ U_{1bm}^{qp} > U_{1am}^{qp} \Longleftrightarrow s > s_\pi^{qp}. \end{cases}$$

Therefore, we recognize that in area (ii) firms 0 does not want to merge, while firm 1 does; that in (iii), neither firm wants to merge; and that in (iv) firm 0 wants to merge and firm 1 does not.

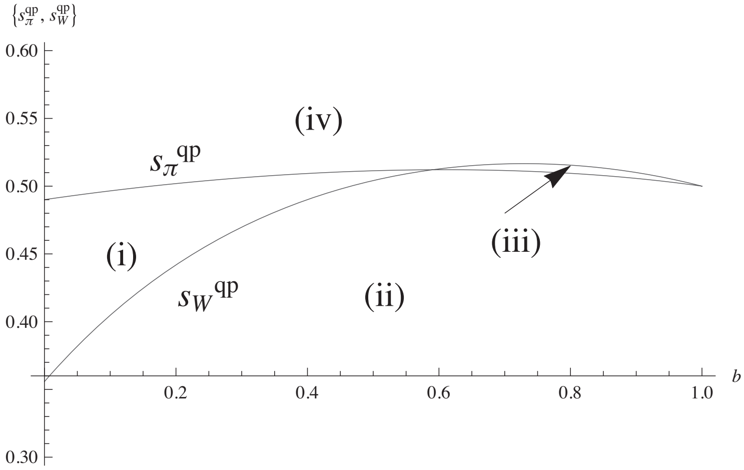

Figure 2: Incentives for firms 0 and 1 to merge with each other in the *q-p* game

Then, by defining the concrete area in which both firms want to merge with each other as ϕ^{qp}, we obtain the following result:

$$\phi^{qp} := \int_0^{b^{pq}} s_\pi^{pq}\, db - \int_0^{b^{pq}} s_W^{qp}\, db = 0.297671 - 0.269867 = 0.0278047.$$

Comparing the areas in the (b, s) plane in which firms 0 and 1 want to merge with each other in the *p-q* game and the *q-p* game, we obtain the following proposition.[17)]

Proposition 1. *The merger area in the (b, s) plane when the strategic contract of public firm 0 is a quantity contract and the strategic contract of private firm 1 is a price contract (that is, the q-p game) is larger than when the strategic contract of public firm 0 is a price contract and the strategic contract of private firm1 is a quantity contract (that is, the p-q game).*[18)]

Following Bárcena-Ruiz and Garzón (2003) and Andree (2013), we define the areas in the (b, s) plane in which both firms 0 and 1 want to merge with each other in the *p-p* game and the *q-q* game as ϕ^{pp} and ϕ^{qq}, respectively. Then, on the basis of the analysis presented in Bárcena-Ruiz and Garzón (2003) and Andree (2013), $\phi^{pp} \approx 0.02002$ and $\phi^{qq} \approx 0.02468$. By comparing the values of ϕ^{ij} among the four games, we obtain the following corollary $(i, j = p, q)$.

Corollary 1. *Among the four games, the area in the (b, s) plane in which both public firm 0 and private firm 1 want to merge is largest in the q-p game, implying that the merger occurs the the most frequently in the q-p game. On the other hand, the area in the (b, s) plane in which both firms want to*

17) More precisely, we obtain $\phi^{qp} - \phi^{pq} = 0.00937366 > 0$.

18) Since $s_W^{pq} > s_W^{qp}$ and $s_\pi^{pq} > s_\pi^{qp}$ for any $b \in (0, 1)$, the area of (i) in Figure 1 is not a subset of the area of (i) in Figure 2, and vice versa. Thus, the result stated in Proposition 1 depends on the assumption that two variables, b and n, are uniformly distributed in area $(0,1) \times (0,1]$, similar to Andree (2013). We are grateful to the anonymous referee who indicated this fact.

merge is narrowest in the p-q game, implying that the merger between firms 0 and 1 occurs the least frequently in the p-q game.

Proposition 1 states that the merger between public firm 0 and private firm 1 occurs more frequently in the *q-p* game than in the *p-q* game, and Corollary 1 emphasizes these statements. More precisely, the merger between firms 0 and 1 occurs most and least frequently in the two asymmetric games. Here, we present the intuition behind the statements in Proposition 1 and Corollary 1. We obtain the following ranking order of the quantities and prices of firms 0 and 1 as follows:

1. $q_{0bm}^{qq} > q_{0bm}^{pq} > q_{0bm}^{pp} > q_{0bm}^{qp}$ for any $b \in (0, 1)$;
2. $q_{1bm}^{pp} > q_{1bm}^{pq} > q_{1bm}^{qp} > q_{1bm}^{qq}$ for any $b \in (0, 1)$;
3. $p_{0bm}^{qp} > p_{0bm}^{pp} \geq p_{0bm}^{qq} > p_{0bm}^{pq}$ for any $b \leq 0.792287$, while $p_{0bm}^{qp} > p_{0bm}^{qq} > p_{0bm}^{pp} > p_{0bm}^{pq}$ for any $b > 0.792287$;
4. $p_{1bm}^{qp} > p_{1bm}^{qq} > p_{1bm}^{pp} > p_{1bm}^{pq}$ for any $b \in (0, 1)$.

We find that the ranking order of the prices of firm 0 is the reverse of that of consumer surplus, while the ranking order of the prices of firm 1 is the reverse of that of social welfare.[19] In particular, in the two types of asymmetric market structures in pre-merger situations, that is, the *p-q* game and the *q-p* game, we note that the consumer surplus and social welfare are higher in the *p-q* game than in the *q-p* game since the pre-merger quantity levels of public firm 0 and private firm 1 are larger in the *p-q* game than in the *q-p* game, implying that the market competition is more intense in the *p-q* game. This property that pre-merger social welfare is higher in the *p-q* game than in the *q-p* game yields the result stated in Proposition 1 that the merger between public firm 0 and private firm 1 teoccur more frequently in the *q-p* game than in the *p-q* game. Therefore, the more competitive market structure in the pre-merger situation becomes the market structure in which the merger between firms 0 and 1 occurs less frequently. In addition, we find that the ranking order for the area in the plane between the degree of product differentiation and the share of the owner of the public firm in the merged firm in which both firm want to merge is the reverse of the ranking order for the price level of private firm 1.[20] Thus, Corollary 1 is explained by the ranking order of the intensity of market competition.

Finally, summing the statements of Proposition 1 and Corollary 1, we find that to prescribe an appropriate merger policy, the merger authority (*e. g.*, the government) must take into account not only the symmetric market structures with respect to the strategic contracts of the public firm and the private firm–that is, the *p-p* game and the *q-q* game–but also the asymmetric market structures in which the strategic contracts of the public firm and the private firm differ. One of the contributions of this paper is that it shows that the merger between the public firm and the private firm tends not to

19) In addition, the ranking order of the prices of firm 0 is the same as that of the producer surplus.

20) We note that the concrete ranking order of the area in the (b, s) plane in which public firm 0 and private firm 1 want to merge is given as $\phi^{pq} > \phi^{qq} > \phi^{pp} > \phi^{qp}$, where ϕ^{pp} and ϕ^{qq} denote the areas in the (b, s) plane in which public firm 0 and private firm 1 want to merge in the *p-p* game and the *q-q* game, respectively.

occur most or least frequently in pre-merger symmetric market structures. In addition, although the merger incentives of the public firm and the private firm on the basis of consumer surplus and social welfare are strikingly different from those on the basis of the profit of the private firm and producer surplus for any degree of product differentiation, the other contribution of this paper is that in order to appropriately regulate a possible merger between the public firm and the private firm, the government should prescribe a merger policy on the basis of consumer surplus and social welfare rather than on the profit of the private firm and producer surplus.

5 Concluding remarks

This paper explored the likelihood of the merger between a public firm and a private firm in a mixed duopoly with differentiated goods in which the firms' strategies in the market are differ. We showed that the area in the plane between the degree of product differentiation and the share of the public firm in the merged firm in which the merger incentives of both firms are compatible is largest (smallest) in the *q-p* (*p-q*) game. Future research focusing on the asymmetric market structures with respect to the firms' strategic contracts should analyze their merger problem under the assumption of separation between their ownership and management as in Barros (1995) and White (2001).

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