

# Collusion and Competition

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## 1 Introduction

This paper examines the relationship between collusion and competition in a three-layer hierarchy model in which there are a principal, a supervisor, and an agent who has private information. Unlike the literature on collusion under asymmetric information, we examine optimal contracts in a setting in which the agent's cost is composed of not only a variable cost but also a fixed one, both of which depend on private information. We assume that the agent has two types. One type has a high marginal cost and a low fixed cost. The other has a low marginal cost and a high fixed cost. When the supervisor receives a signal about the agent's type, he can either transmit his information to the principal truthfully or conceal what was received. Then the possibility of collusion between the supervisor and the agent arises. We show that when a difference in the amount of fixed costs with respect to the agent's type is sufficiently large, countervailing incentives may result. This implies that the low marginal cost type produces a higher quality product than the first best quality level and that the high marginal cost type obtains an informational rent. We examine optimal contracts and the relationship between corruption and competition when countervailing incentives exist.

This paper is closely related to the literature on corruption and competition. Rose-Ackerman (1975) addresses the issue of how competition affects corruption. Laffont and N'Guessan (1999) explore the relationship between corruption and competition in a principal-agent model.

This paper is also related to the literature on contracts and collusion under asymmetric information. Since the pioneering work of Tirole (1986), much work has been done in examining collusion under asymmetric information. Laffont (1990) examines optimal contracts in a principal-supervisor-agent model (see also Laffont and Tirole (1993)). Laffont and Martimort (1997) consider collusion-proof contracts and characterize optimal contracts with collusion-proofness. However, these papers do not consider fixed costs that depend on the agents' types. We consider a more general cost function that includes fixed costs.

Furthermore, this paper is related to the literature on countervailing incentives under asymmetric information. Lewis and Sappington (1989) assume one-dimensional uncertainty regarding marginal costs and fixed costs when examining the possibility of countervailing incentives. Maggi and Rodriguez-Clare (1995) further examine the issue on countervailing incentives. Jullien (2000) explores the effects of type-dependent participation constraints on optimal contracts. However, these papers do not address the question of collusion under asymmetric information (see also Laffont and

Martimort (2002)).

In this paper, we study optimal contracts and collusion in a three-layer hierarchy model in which the cost function of the agent includes fixed costs that depend on its type. We show that when the difference in the magnitude of fixed costs with respect to productivity types is sufficiently large, countervailing incentives may arise because the set of binding incentive compatibility constraints and participation constraints depends on the value of the difference.

To address the question of how competition affects the degree of corruption, we assume that a better supervising technology is considered as greater competition and examine the effects of an improvement in supervision technologies on corruption. We show that whether or not greater competition decreases corruption depends on the difference in fixed costs with respect to the agent's types.

The remainder of the paper is organized as follows. In Section 2, we present a principal-supervisor-agent model and note basic assumptions. In Section 3, we examine optimal contracts when the supervisor and the agent can collude. In Section 4, we explore the relationship between competition and corruption. Section 5 concludes the paper.

## 2 The Model

We consider a principal-supervisor-agent hierarchy model. Suppose that a government (the principal) contracts with a firm (the agent) implementing a public project. The quantity or quality of the project is denoted as  $q$  (which henceforth will be used to refer to quality). The project yields social benefit  $S(q)$ . For all  $q > 0$ , we assume that  $S(q)$  is twice continuously differentiable, strictly increasing and concave.

The cost function is given by

$$C(q, \theta) = \theta q + F(\theta),$$

where  $\theta > 0$  is the constant marginal cost and  $F(\theta)$  is the fixed cost. The parameter  $\theta$  is the relevant private information of the firm. We assume that parameter  $\theta$  takes either  $\theta_1$  or  $\theta_2$  with  $\theta_1 < \theta_2$  and that  $F(\theta_1) > F(\theta_2)$ . Thus we consider the case in which a higher marginal cost is associated with a lower fixed cost and vice versa. This inverse relationship may arise because in general a higher fixed cost guarantees a lower marginal cost and vice versa. Let  $\pi = \Pr(\theta = \theta_1)$ ,  $0 < \pi < 1$ . Let  $t$  denote monetary transfers from the government to the firm,  $t \geq 0$ .

The government can contract with a supervisor (the regulatory agency) to bridge its information gap. The supervisor makes his report  $r$  to the government. Let  $m$  be a transfer from the government to the supervisor,  $m \geq 0$ . Following Laffont and Tirole (1993), we assume that there exists distortion or deadweight loss by funding the public project. Let  $\lambda > 0$  denote the cost of public funds.

Consumers have the following utility function:

$$V = S(q) - (1 + \lambda)(t + m).$$

The firm's payoff  $U$  is given by

$$U = t - \theta q - F(\theta).$$

The supervisor's utility function is given by

$$X = m - m_R \geq 0,$$

where  $m_R$  is his reservation utility. For simplicity, we assume that  $m_R = 0$ .

We assume that all parties are risk neutral.

The benevolent government maximizes social welfare  $W$ , which is given by

$$W = V + U + X = S(q) - (1 + \lambda) \{ \theta q + F(\theta) \} - \lambda U - \lambda X.$$

The government offers a contract  $(q_i, t_i)$  to the firm and a contract  $(r_i, m_i)$  to the supervisor,  $i = 1, 2$ . When designing optimal contracts, the government solves its payoff maximization problem subject to incentive compatibility constraints and participation constraints. An incentive compatibility constraint (ICC) guarantees that the firm prefers the contract that is designed for it. A participation constraint (PC) guarantees that the firm accepts the designated contract.

The sequence of events in the contracting game proceeds as follows.

At  $t = 1$ , nature determines a firm's productivity type  $\theta$ . Only the firm discovers it. The supervisor learns a signal.

At  $t = 2$ , the government offers contracts to the supervisor and the firm. Then the supervisor can sign a side contract with the firm.

At  $t = 3$ , the supervisor makes his report to the government and the firm undertakes the public project.

At  $t = 4$ , the government provides transfers to the firm and the supervisor.

### 3 Optimal contracts without collusion

In this section, as a benchmark, we derive an optimal contract when there is no collusion between the supervisor and the agent. First, under full information, the government maximizes the following expected social welfare:

$$\begin{aligned} W = & \pi [S(q_1) - (1 + \lambda) \{ \theta_1 q_1 + F(\theta_1) \} - \lambda U_1 - \lambda X_1] \\ & + (1 - \pi) [S(q_2) - (1 + \lambda) \{ \theta_2 q_2 + F(\theta_2) \} - \lambda U_2 - \lambda X_2], \end{aligned}$$

where  $U_i = t_i - \theta_i q_i - F(\theta_i)$  and  $X_i = m_i$ ,  $i = 1, 2$ .

Then, the optimal contract satisfies

$$S_q(q_1^{FB}) = (1 + \lambda)\theta_1$$

and

$$S_q(q_2^{FB}) = (1 + \lambda)\theta_2,$$

where  $S_q$  denotes  $\frac{dS(\cdot)}{dq}$  and  $q_i^{FB}$  the first best quality for  $\theta = \theta_i, i = 1, 2$ .

Next, we examine optimal contracts under asymmetric information. The government's problem in this case is to maximize the expected welfare subject to the following incentive compatibility constraints (ICCs):

$$U_1 = t_1 - \theta_1 q_1 - F(\theta_1) \geq t_2 - \theta_1 q_2 - F(\theta_1)$$

and

$$U_2 = t_2 - \theta_2 q_2 - F(\theta_2) \geq t_2 - \theta_2 q_1 - F(\theta_2)$$

and the participation constraints (PCs):

$$t_1 - \theta_1 q_1 - F(\theta_1) \geq 0$$

and

$$t_2 - \theta_1 q_2 - F(\theta_1) \geq 0.$$

These ICCs and PCs can be rewritten as follows:

$$U_1 \geq U_2 + (\theta_2 - \theta_1)q_2 + \{F(\theta_2) - F(\theta_1)\},$$

$$U_2 \geq U_1 - (\theta_2 - \theta_1)q_1 - \{F(\theta_2) - F(\theta_1)\},$$

$$U_1 \geq 0,$$

and

$$U_2 \geq 0.$$

The following result shows two regimes in the optimal contract with positive rents for the firm. The proof of this proposition is similar to that of Proposition 1 in Kobayashi (2018).

The optimal contract has the following features:

(i)

$$S_q(q_1^{SB}) = (1 + \lambda)\theta_1$$

and

$$S_q(q_2^{SB}) = (1 + \lambda)\theta_2 + \lambda \frac{\pi}{1 - \pi} (\theta_2 - \theta_1), \quad q_2^{SB} < q_2^{FB},$$

where superscript *SB* denotes the second best. Note that  $q_1^{SB}$  satisfies  $q_1^{SB} \leq \frac{F(\theta_1) - F(\theta_2)}{\theta_2 - \theta_1}$ .

(ii)

$$S_q(q_1^{CI}) = (1 + \lambda)\theta_1 - \lambda \frac{1 - \pi}{\pi} (\theta_2 - \theta_1), \quad q_1^{CI} < q_1^{FB},$$

and

$$S_q(q_2^{SB}) = (1 + \lambda)\theta_2,$$

where superscript *CI* denotes the countervailing incentives. Note that  $q_1^{CI}$  satisfies  $q_1^{CI} \leq \frac{F(\theta_1) - F(\theta_2)}{\theta_2 - \theta_1}$ .

This result says that in regime 1 (case (i) above),  $\theta_1$ -agent obtains a positive rent and there exists a downward distortion for  $\theta_2$ .

It also shows that in regime 2 (case (ii) above), countervailing incentives exist and thus we have an upward distortion for  $\theta_1$ . The intuition behind this result is as follows. When a difference in fixed costs with respect to the firm's productivity types,  $F(\theta_1) - F(\theta_2)$ , is sufficiently large, countervailing incentives may arise, and, thus, there is an upward distortion for the low marginal cost type  $\theta_1$ .

## 4 Optimal contracts and collusion

Now we examine collusion between the supervisor and the agent in the three-level hierarchy. Suppose that the supervisor observes a signal  $\sigma$  with probability  $\beta$  that the firm is of type  $\theta$  and otherwise  $\sigma = \phi$  with probability  $1 - \beta$ . The supervisor can hide his information and report that the signal is empty. If the supervisor reveals it to the government, the government can learn the signal. We assume that a signal is hard information.

Furthermore, suppose that there are two types of supervisors. With probability  $\mu$ , we have a type-one supervisor. The type-one supervisor will not collude with the agent if he obtains  $M_1$  when he reports  $r = \theta$  such that  $M_1 \geq [(\theta_2 - \theta_1)q_2 + \{F(\theta_2) - F(\theta_1)\}]$ . Recall that  $r$  denotes the supervisor's report to the government. With probability  $(1 - \mu)$ , we have a type-two supervisor. The type-two supervisor is less corruptible, and will not engage in collusion when he receives  $M_2 \geq (\theta_2 - \theta_1)q_2 + \{F(\theta_2) - F(\theta_1)\} - x$ ,  $x > 0$ .

To begin with, we consider the case of regime 1 (case (i) above). Let us examine optimal contracts when the government has full information and then analyze the case of asymmetric information. Suppose that  $\sigma = r = \theta$ . Recall that  $r$  denotes the supervisor's report to the government. Then, the government is informed. Under full information, social welfare  $W^F$  is

$$W^F = \pi[S(q_1^{FB}) - (1 + \lambda) \{ \theta_1 q_1^{FB} + F(\theta_1) \}] \\ + (1 - \pi) [S(q_2^{FB}) - (1 + \lambda) \{ \theta_2 q_2^{FB} + F(\theta_2) \}].$$

Next, suppose that  $\sigma = \phi$ . Then, the government is uninformed. Under asymmetric information, the expected social welfare  $\Phi^{NC}$  is

$$\Phi^{NC} = \beta W^F + (1 - \beta) [W(q_1^{SB}) - \lambda \pi \beta \delta \{ F(\theta_1) - F(\theta_2) + (\theta_2 - \theta_1) q_2^{SB} \}].$$

Then, the first order conditions for the maximization of the expected social welfare  $\Phi^{NC}$  are

$$S_q(q_1^{NC}) = (1 + \lambda) \theta_1$$

and

$$S_q(q_2^{NC}) = (1 + \lambda) \theta_2 - \lambda \cdot \frac{\pi}{(1 - \pi)(1 - \beta)} (\theta_2 - \theta_1) [1 - \beta + \beta \delta],$$

where the superscript *NC* denotes the case of no collusion.

Next, we consider the case in which the supervisor and the firm can collude. Suppose that  $\sigma = \theta$ . Then, the firm obtains an informational rent,  $[(\theta_2 - \theta_1) q_2^S + F(\theta_1) - F(\theta_2)]$ , if the supervisor conceals the information of  $\sigma = \theta$ . We assume that there exists a transaction cost between the supervisor and the firm. Let  $\rho$  represent the transaction cost. We assume  $\rho > \lambda$ . Let  $\delta \equiv \frac{1}{1 + \rho}$ .

The expected social cost is  $\lambda \cdot \pi \cdot \beta [(\theta_2 - \theta_1) q_2^S + F(\theta_1) - F(\theta_2)] \cdot \delta$  with probability  $\pi \cdot \beta$ . Thus, the expected social welfare  $\Phi^C$  is

$$\Phi^C = \pi \beta (1 - \mu) [S(q_1^{FB}) - (1 + \lambda) \{ \theta_1 q_1^{FB} + F(\theta_1) \}] \\ + \pi (1 - \beta + \beta \mu) [S(q_1^S) - (1 + \lambda) \{ \theta_1 q_1^S + F(\theta_1) \} - \lambda \{ (\theta_2 - \theta_1) q_2^S + F(\theta_1) - F(\theta_2) \}] \\ + (1 - \pi) \beta [S(q_2^{SB}) - (1 + \lambda) \theta_2 q_2^{SB} + F(\theta_2)] \\ + (1 - \pi) (1 - \beta) [S(q_2^{SB}) - (1 + \lambda) \theta_2 q_2^{SB} + F(\theta_2)] \\ - \lambda \cdot \pi \cdot \beta \cdot (1 - \mu) [ \{ (\theta_2 - \theta_1) q_2^S + F(\theta_1) - F(\theta_2) \} \cdot \delta - x].$$

The maximization of  $\Phi^C$  yields the following:

$$S_q(q_1^C) = (1 + \lambda) \theta_1$$

and

$$S_q(q_2^C) = (1 + \lambda) \theta_2 - \lambda \cdot \frac{\pi}{(1 - \pi)(1 - \beta)} (\theta_2 - \theta_1) [1 - \beta(1 - \mu)(1 - \delta)],$$

where the superscript *C* denotes the case of collusion.

Thus far we have considered the case of regime 1 (case (i) above). Next, we analyze optimal contracts and the possibility of collusion for the case in which countervailing incentives can emerge at

equilibrium. Thus we consider regime 2 (case (ii) above). We assume that the supervisor observes a signal  $\sigma$  with probability  $\beta$  that the firm is of type  $\theta$  and otherwise  $\sigma = \phi$  with probability  $1 - \beta$ .

Suppose that  $\sigma = r = \theta$ . Then, the government is informed. Thus, we have, with probability  $\beta$ ,

$$W^{CI} = \pi [S(q_1^{FB}) - (1 + \lambda) \{ \theta_1 q_1^{FB} + F(\theta_1) \}] + (1 - \pi) [S(q_2^{FB}) - (1 + \lambda) \{ \theta_2 q_2^{FB} + F(\theta_2) \}].$$

Suppose that  $\sigma = \phi$ . Then, the government is uninformed. Thus, we have, with probability  $1 - \beta$ ,

$$W^U = \pi [S(q_1^{CI}) - (1 + \lambda) \{ \theta_1 q_1^{CI} + F(\theta_1) \}] \\ + (1 - \pi) [S(q_2^{FB}) - (1 + \lambda) \theta_2 q_2^{FB} - \lambda \{ F(\theta_1) - F(\theta_2) - (\theta_2 - \theta_1) q_1^{CI} \}].$$

Hence, the expected social welfare is

$$\beta W^{CI} + (1 - \beta) W^U.$$

Suppose that  $r = \phi$ . Then, the government is uninformed.

Now, we analyze the case in which the supervisor and the firm can collude when there are countervailing incentives.

The expected social cost is  $\lambda \cdot (1 - \pi) \cdot \beta \cdot [F(\theta_1) - F(\theta_2) - (\theta_2 - \theta_1) q_1^{CI}] \cdot \delta$  with probability  $(1 - \pi) \cdot \beta$ .

For the regime with no collusion, the expected social welfare  $\Psi^{NC}$  is

$$\Psi^{NC} = \beta W^{CI} + (1 - \beta) [W(q_1^{SB}) - \lambda \pi \beta \delta \{ F(\theta_1) - F(\theta_2) + (\theta_2 - \theta_1) q_2^{SB} \}].$$

The following proposition shows the optimal contract for the regime without collusion.

**Proposition 1** *The optimal contract is characterized as follows.*

$$S_q(q_2^{NC}) = (1 + \lambda) \theta_1 - \lambda \frac{(1 - \pi)}{\pi} (\theta_2 - \theta_1) [1 + \frac{\beta \delta}{1 + \rho}]$$

and

$$S_q(q_2^{NC}) = (1 + \lambda) \theta_2.$$

This proposition says that countervailing incentives exist and thus, too, upward distortion for the lower marginal cost type. It also shows that if  $\beta$  increases,  $q_1^{NC}$  will increase.

For the regime with collusion, the expected social welfare  $\Psi^C$  is

$$\Psi^C = \pi \beta [S(q_1^{FB}) - (1 + \lambda) \{ \theta_1 q_1^{FB} + F(\theta_1) \}] \\ + \pi (1 - \beta) [S(q_1^{CI}) - (1 + \lambda) \{ \theta_1 q_1^{CI} + F(\theta_1) \}] \\ + \pi (1 - \beta) (1 - \mu) [S(q_2^{FB}) - (1 + \lambda) \{ \theta_2 q_2^{FB} + F(\theta_2) \}]$$

$$\begin{aligned}
& + (1 - \pi) (1 - \beta + \beta\mu) [S(q_2^s) - (1 + \lambda) \{ \theta_2 q_2^s + F(\theta_2) \}] \\
& - \lambda \{ F(\theta_1) - F(\theta_2) - (\theta_2 - \theta_1) q_1^{CI} \} \\
& - \lambda \cdot (1 - \pi) \beta \cdot \mu [ \{ F(\theta_1) - F(\theta_2) - (\theta_2 - \theta_1) q_1^{CI} \} \cdot \delta - x ].
\end{aligned}$$

The expected social cost is  $\lambda \cdot (1 - \pi) \cdot \beta \cdot (1 - \mu) [ (F(\theta_1) - F(\theta_2) - (\theta_2 - \theta_1) q_1^{CI}) \cdot \delta ]$  with probability  $(1 - \pi) \cdot \beta$ .

The following proposition shows the optimal contract for the regime with collusion.

**Proposition 2** *The optimal contract is characterized as follows.*

$$S_q(q_1^C) = (1 + \lambda) \theta_1 - \lambda \frac{(1 - \pi)}{\pi} (\theta_2 - \theta_1) \left[ 1 + \frac{\beta}{1 + \beta} (\rho + (1 - \rho) \delta) \right]$$

and

$$S_q(q_2^C) = (1 + \lambda) \theta_2.$$

This proposition says that countervailing incentives exist and thus, too, upward distortion for the lower marginal cost type. It also shows that if  $\beta$  increases,  $q_1^C$  will increase.

## 5 Competition and Corruption

In this section, we examine the effects of competition on corruption. Following Laffont and N'Guessan (1999), we consider a setting in which an improvement on the supervising technology is considered as an increase in competition. Thus, we analyze the effects of changes in probability  $\beta$  on corruption for both a standard case ( regime (i) ) and a case where countervailing incentives occur ( regime (ii) ).

From the results in the previous section, we examine whether an increase in probability  $\beta$  raises the possibility of collusion between the supervisor and the agent.

Suppose that the difference in fixed costs with respect to productivity types is sufficiently small. Thus, we consider regime (i).

Let us define  $x^*$  the value of  $x$  that satisfies  $\Phi^{NC} = \Phi^C$ . Then we obtain the following result regarding the relationship between competition and corruption.

**Proposition 3** *An increase of competition increases corruption between the supervisor and the firm, that is,*

$$\frac{dx^*}{d\beta} > 0.$$

Proof:

Let us define  $J(\beta, x) = \Phi^{NC} = \Phi^C$ .



Then, by the definition of  $x^*$ , we have  $J(\beta, x^*) = 0$ .

Because  $\frac{\partial J}{\partial x}(\beta, x^*) \neq 0$ , by the implicit function theorem, we have

$$\frac{dx^*}{d\beta} = -\frac{\frac{\partial J}{\partial \beta}}{\frac{\partial J}{\partial x^*}}.$$

Now we have

$$\begin{aligned} \frac{\partial J}{\partial \beta} &= -W(q_2^{NC}) - \lambda\pi\beta\delta[F(\theta_1) - F(\theta_2) + (\theta_2 - \theta_1)q_2^{NC}] \\ &\quad - \lambda\pi(1-\mu)[\{(\theta_2 - \theta_1)q_2^{NC} + F(\theta_1) - F(\theta_2)\}\delta] \\ &\quad + \lambda\pi\mu[(\theta_2 - \theta_1)q_2^C + F(\theta_1) - F(\theta_2)] \\ &\quad + W(q_2^C) + \lambda\pi(1-\mu)[\{(\theta_2 - \theta_1)q_2^C + F(\theta_1) - F(\theta_2)\} \cdot \delta - x^*] \end{aligned}$$

and

$$\frac{\partial J}{\partial x^*} = -\lambda\pi\beta(1-\mu).$$

By the definition of  $J$ , we have

$$\begin{aligned} \frac{1-\mu}{\mu} [W(q_2^C) - W(q_2^{NC})] &= [\lambda\pi\mu\{(\theta_2 - \theta_1)q_2^C + F(\theta_1) - F(\theta_2)\} \\ &\quad - \lambda\pi\delta\{F(\theta_1) - F(\theta_2) + (\theta_2 - \theta_1)q_2^{NC}\} \\ &\quad + \lambda\pi(1-\mu)[\{(\theta_2 - \theta_1)q_2^C + F(\theta_1) - F(\theta_2)\} \cdot \delta - x^*]]. \end{aligned}$$

Let  $\tilde{q}_2$  denote a maximizer of  $W(q_2)$ .

Then we have  $\tilde{q}_2 < q_2^C < q_2^{NC}$ .

Because  $W(q_2)$  is concave,  $W(q_2^C) > W(q_2^{NC})$ .

Hence, we obtain

$$\frac{dx^*}{d\beta} > 0.$$

This complete the proof.

Next we examine the case of countervailing incentives. Suppose that the difference in fixed costs with respect to productivity types is sufficiently large. Thus, we consider regime (ii).

Let us define  $x^{**}$  the value of  $x$  that satisfies  $\Psi^{NC} = \Psi^C$ .

Then the following result holds regarding the relationship between competition and corruption.

**Proposition 4** *An increase of competition decreases corruption between the supervisor and the firm, that is,*

$$\frac{dx^{**}}{d\beta} < 0$$

Proof:

Let us define  $L(\beta, x) = \Phi^{NC} - \Phi^C$ .

Then, by the definition of  $x^{**}$ , we have  $L(\beta, x^{**}) = 0$ .

Because  $\frac{\partial L}{\partial x}(\beta, x^{**}) \neq 0$ , by the implicit function theorem,

we have

$$\frac{dx^{**}}{d\beta} = - \frac{\frac{\partial L}{\partial \beta}}{\frac{\partial L}{\partial x^{**}}}.$$

Now we have

$$\begin{aligned} \frac{\partial L}{\partial \beta} &= -W(q_2^{NC}) - \lambda\pi\beta\delta[F(\theta_1) - F(\theta_2) + (\theta_2 - \theta_1)q_1^{NC}] \\ &\quad + \lambda\pi(1-\mu)[\{(\theta_2 - \theta_1)q_2^{NC} + F(\theta_1) - F(\theta_2)\}\delta] \\ &\quad + \lambda\pi\mu[(\theta_2 - \theta_1)q_2^C + F(\theta_1) - F(\theta_2)] \\ &\quad + W(q_2^C) + \lambda\pi(1-\mu)[\{(\theta_2 - \theta_1)q_1^C + F(\theta_1) - F(\theta_2)\} \cdot \delta - x^{**}] \end{aligned}$$

and

$$\frac{\partial L}{\partial x^{**}} = -\lambda(1-\pi)\beta(1-\mu).$$

By the definition of  $L$ , we have

$$\begin{aligned} \frac{1-\mu}{\mu}[W(q_2^C) - W(q_2^{NC})] &= [\lambda(1-\pi)\mu\{(\theta_2 - \theta_1)q_1^C + F(\theta_1) - F(\theta_2)\} \\ &\quad - \lambda(1-\pi)\delta\{F(\theta_1) - F(\theta_2) + (\theta_2 - \theta_1)q_1^{NC}\} \\ &\quad + \lambda(1-\pi)(1-\mu)[\{(\theta_2 - \theta_1)q_1^C + F(\theta_1) - F(\theta_2)\} \cdot \delta - x^{**}]]. \end{aligned}$$

Let  $\tilde{q}_1$  denote a maximizer of  $W(q_1)$ .

Then we have  $\tilde{q}_1 < q_1^{NC} < q_1^C$ .

Because  $W(q_1)$  is concave,  $W(q_1^{NC}) > W(q_1^C)$ .

Hence, we obtain

$$\frac{dx^{**}}{d\beta} < 0.$$

This complete the proof.

Thus, if an increase of competition prevails, then corruption between the supervisor and the firm decreases when the difference in fixed costs with respect to productivity types is sufficiently large or increases when the difference in fixed costs with respect to productivity types is sufficiently small.

## 6 Conclusion

In this paper, we have analyzed optimal contracts and collusion in a three level hierarchy model with adverse selection. We have characterized optimal contracts when the costs of production is composed of a variable cost and a fixed cost, both of which depend on the asymmetric information parameter. The optimal contract exhibits different regimes and there can be countervailing incentives. We have also characterized optimal contracts when the supervisor has an information technology and can collude with the firm. To examine how competition affects the degree of corruption, we have examined the effects of an improvement in supervision technologies on corruption. We have shown that in the regime with countervailing incentives, greater competition decreases corruption. On the other hand, we have proved that in the regime without countervailing incentives, greater competition increases corruption.

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