Analysis of Asymmetry between Volatility and the Rate of Return on the Yen / US Dollar, Yen / Euro Exchange Rate

Hidetoshi Mitsui

1. Introduction

It is known that there is some kind of asymmetry and negative correlation between the fluctuation in the rate of return on asset prices and volatility. This is called the leverage effect[1)] in the stock market, and has been verified in many empirical studies. In this study, we discussed whether or not such asymmetry exists also in the foreign exchange market, because foreign currencies are recently used as operating assets of investors and so we speculated that there exists some relation between volatility and the rate of return also in foreign exchange rates. In this study, we conducted an empirical analysis about the relation between volatility and the rate of return on foreign exchange rates, using the asymmetric stochastic volatility model (hereinafter the "asymmetric SV model"), which was proposed by Harvey and Shephard [1996]. Since it is difficult to obtain likelihood in the SV model[2)] , Harvey and Shephard [1996] proposed that model parameters are estimated with the Quasi-Maximum Likelihood estimation (QML), which maximizes quasi-likelihood obtained through the Kalman Filter. This method was adopted in this study.

Engle [1982] proposed the Autoregressive Conditional Heteroskedasticity (ARCH) model, in which the volatility at each time is formulated as a linear function of the square of unpredicted shock in the past. In addition, Bollerslev [1986] extended this model into a more general model called the Generalized ARCH (GARCH) model, adding the past volatility values to the explanatory variable of volatility[3)] . The volatility in the stock market tends to respond to "bad news" rather than "good news." To cope with this empirical fact, Nelson [1991] proposed the exponential GARCH (EGARCH) model. In this study, we discussed whether or not the foreign exchange market also includes such asymmetry, based on the EGARCH model. It is known that the distribution of the rate of return on asset prices has thicker tails than the normal distribution. Accordingly, the model estimation was carried out, while assuming the Generalized Error Distribution (GED) for the EGARCH model.

The empirical analysis was conducted, using the daily data of the Yen / US dollar exchange rate and the Yen / Euro exchange rate in the period from January 4, 2000 to December 28, 2007. The main findings in this empirical analysis are as follows: (1) The shock of the volatility in the rate of return on the Yen / US dollar exchange rate and the Yen / Euro exchange rate is highly persistent

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(“volatility clustering”); (2) There is no asymmetry between volatility and the rate of return on the Yen / US dollar exchange rate or between volatility and the rate of return on the Yen / Euro exchange rate.

This paper’s contents are as follows: Chapter 2 explains the SV model and the asymmetric SV model briefly and mentions our investigation on the QML method for estimating the asymmetric SV model. Chapter 3 explains the EGARCH model. Chapter 4 mentions about the data of the Yen / US dollar exchange rate and the Yen / Euro exchange rate, and the results of the empirical analysis. The last Chapter 5 summarizes this study and gives the future themes.

2. Asymmetric SV Model

2.1 SV Model

A set of discrete-time data is used to estimate parameters describing the model. Then, the discrete-time SV model should be prepared as follows:

$$R_t = \sigma_t u_t, \tag{2.1}$$

$$\ln(\sigma_t^2) = \alpha + \beta \ln(\sigma_{t-1}^2) + \eta_t, \tag{2.2}$$

$$\begin{pmatrix} u_t \\ \eta_t \end{pmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix}\right), t = 1, \cdots, T.$$

where R_t represents the return rate; u_t is a disturbance term following a normal distribution with mean 0 and variance 1; η_t is a disturbance term following a normal distribution with mean 0 and variance σ_η^2. Equation (2.2) implies that the log-volatility follows the first-order autoregressive process[4)].

Defined $\phi = \{\alpha, \beta, \sigma_\eta^2\}$, the likelihood L becomes:

$$L = \int\int\cdots\int P(\{R_t\}_{t=1}^T | \{\sigma_t\}_{t=1}^T; \phi) P(\{\sigma_t\}_{t=1}^T) d\sigma_1^2 d\sigma_2^2 \cdots d\sigma_T^2 \tag{2.3}$$

$$\begin{aligned} &= \int\int\cdots\int P(R_1|\sigma_1^2;\phi)P(\sigma_1^2)P(R_1|\sigma_1^2) \\ &\times \prod_{t=2}^T \{P(R_t|\sigma_t^2;\phi)P(\sigma_t^2|\sigma_{t-1}^2;\phi)\} d\sigma_1^2 d\sigma_2^2 \cdots d\sigma_T^2. \end{aligned} \tag{2.4}$$

It is difficult to calculate likelihood in the SV model because σ is unknown at time $t-1$ in this model and the above function includes integrals as many as the number of sample data. It is impossible to calculate likelihood analytically with the above likelihood function. The SV model does not permit the exact ML estimation. Then, we need some technique to estimate the SV model. The technique we used in this study is the QML method in which Quasi-Likelihood is calculated through Kalman Filter and then we find the parameters that maximize the Quasi-Likelihood to estimate the SV model.

2.2 Asymmetric SV Model

The asymmetric stochastic volatility model is a model in which the fluctuation in volatility is advanced for one term, in order to grasp the effects of the rate of return on foreign exchange rates on the previous day on volatility. Assuming that R_t is the rate of return on foreign exchange rates, σ is volatility, u_t is the disturbance term following the normal distribution with a mean of 0 and a variance of 1, η_t is the disturbance term following the normal distribution with a mean of 0 and a variance of σ_η^2, and ρ is the correlation coefficient between u_t and η_t, the discrete-time SV model can be expressed by the following equations:

$$R_t = \sigma_t u_t, \tag{2.5}$$

$$\ln \sigma_{t+1}^2 = \alpha + \beta \ln \sigma_t^2 + \eta_t, \tag{2.6}$$

$$\begin{pmatrix} u_t \\ \eta_t \end{pmatrix} \sim NID\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho\sigma_\eta \\ \rho\sigma_\eta & \sigma_\eta^2 \end{bmatrix} \right), t = 1, \cdots, T.$$

Equation (2.6) indicates that the logarithm of volatility follows the AR(1) process (firstorder autoregressive process). *NID* means that the values are normally and independently distributed[5)]. Here, unknown parameters are $\{ \alpha, \beta, \sigma_\eta^2, \rho \}$. The parameter that indicates asymmetry is the correlation coefficient ρ. The parameters $\phi = \{ \alpha, \beta, \sigma_\eta^2, \rho \}$ are estimated with the QML[6)].

2.3 Estimation Method for the Asymmetric SV Model

The following is a brief explanation of the QML method targeted at the asymmetric SV model consisting of Equations (2.5) and (2.6). This method was pioneered by Harvey and Shephard [1996]. First, Equation (2.5) is squared and taken its logarithm to obtain[7)] :

$$Y_t = x_t + \xi_t, \qquad \text{(measurement equation)} \tag{2.7}$$

$$x_{t+1} = \alpha + \beta x_t + s_t \psi + \eta_t^*, \qquad \text{(transition equation)} \tag{2.8}$$

$$\begin{pmatrix} \xi_t \\ \eta_t^* \end{pmatrix} | s_t \sim ID\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\xi^2 & \gamma s_t \\ \gamma s_t & \sigma_\eta^2 - \psi^2 \end{bmatrix} \right), t = 1, \cdots, T.$$

This expression can be regarded as a linear state space model[8)] with Y_t being the observed variable and x_t the unobserved state variable. *ID* means that the values are independently distributed. Although this distribution is not a normal one, we assumed it is a normal distribution and applied Kalman Filter[9)] with the purpose of calculating quasi-likelihood. Kalman Filter consists of a prediction equation and an updating equation. These equations for Equations (2.7) and (2.8) are expressed as follows[10)] :

prediction equation:

$$x_{t|t-1} = \alpha + \beta x_{t-1|t-1}, \tag{2.9}$$

$$P_{t|t-1} = \beta^2 P_{t-1|t-1} + \sigma_\eta^2. \tag{2.10}$$

updating equation:

$$x_{t|t} = x_{t|t-1} + \frac{P_{t|t-1}}{f_t}\epsilon_t, \tag{2.11}$$

$$P_{t|t} = P_{t|t-1} - \frac{P_{t|t-1}^2}{f_t}, \tag{2.12}$$

$$\epsilon_t = Y_t - x_{t|t-1} - M. \tag{2.13}$$

$x_{t|t-1}$: estimate of x_t given information at $t-1$,

$x_{t|t}$: estimate of x_t given information at t,

$P_{t|t-1}$: variance of x_t given information at $t-1$,

$P_{t|t}$: variance of x_t given information at t,

ϵ_t : prediction error,

f_t : variance of prediction error $(f_t = P_{t|t-1} + V)$.

Certain values were assigned to parameters $\phi = \{\alpha, \beta, \sigma_\eta^2, \rho\}$, $x_{1|0}$, and $P_{1|0}$ (In general, $x_{1|0}$ and $P_{1|0}$ are unconditional expected value and variance respectively, that is, it is set that $x_{1|0} = \alpha/(1-\beta)$ and $P_{1|0} = \sigma_\eta^2/(1-\beta^2)$), and then Kalman Filter was applied stepwise to obtain the following Quasi-Log-Likelihood function:

$$\ln L = -\frac{T}{2}\ln(2\pi) - \sum_{t=1}^{T}\ln(f_t) - \frac{1}{2}\sum_{t=1}^{T}\frac{\epsilon_t^2}{f_t}. \tag{2.14}$$

where T is the number of observations. Because a normal distribution approximation was used, the above log-likelihood is referred to as Quasi-Log-Likelihood. Through this QML method we can estimate parameters $\phi = \{\alpha, \beta, \sigma_\eta^2, \rho\}$ by finding the conditions that maximize this Quasi-Log-Likelihood function. This QML is somewhat deficient in its efficiency due to the difference between the true likelihood and quasi-one. However, it is known that the estimates derived from QML have consistency, and its asymptotic distribution has been unveiled by Dunsmuir [1979], Ljung / Caines [1979][11).

3. EGARCH Model

In this study, the fluctuations in foreign exchange rates were analyzed, based on the EGARCH model developed by Nelson [1991]. In addition, since many empirical studies state that performance is not improved so much even if the order of the volatility fluctuation process is increased, the analysis was carried out, using the EGARCH(1,1) models in this study[12)]. These models are briefly described below. Letting R_t represent the rate of return (rate of change) at time t in discrete-time economy, the process of the rate of return is expressed as follows:

$$R_t = \mu + \epsilon_t, \tag{3.1}$$

$$\epsilon_t = \sigma_t z_t, \quad \sigma_t > 0, \tag{3.2}$$

$$z_t \sim i.i.d., E[z_t] = 0, Var[z_t] = 1. \tag{3.3}$$

Here, it is assumed that the constant term μ in Equation (3.1) represents expected rate of return, ϵ_t denotes the error term, and the rate of return has no autocorrelation. *i.i.d.* means "independent and identically distributed." $E[\cdot]$ and $Var[\cdot]$ represent an expected value and a variance, respectively.

EGARCH(1,1): Volatility is formulated, removing the condition that parameters must be positive by defining the logarithm of volatility as the explained variable[13)].

$$\ln(\sigma_t^2) = \omega + \alpha_1\{\theta_1 z_{t-1} + \theta_2(|z_{t-1}| - E|z_{t-1}|)\} + \beta_1 \ln(\sigma_{t-1}^2). \tag{3.4}$$

Here, if $\theta_1 < 0$, volatility is higher on the following day of the date on which asset prices decreased than the following day of the date on which asset prices increased. In this model, the logarithm of volatility is defined as the explained variable, and so it is unnecessary to assume that ω, β, θ_1, and θ_2 are not negative. $0 < \beta < 1$ is assumed, in order to ensure the steady property; plus, ω, $\theta_2 > 0$, and $\theta_1 < 0$ are assumed, considering the results of many previous empirical studies. In addition, when z follows the standard normal distribution, $E|z_{t-1}| = \sqrt{\pi/2}$ [14)].

If the error term follows the normal distribution, z_t in Equation (3.3) becomes as follows:

$$z_t \sim i.i.d.N(0,1). \tag{3.5}$$

If the error term follows the Generalized Error Distribution, z_t in Equation (3.3) becomes as follows:

$$z_t \sim GED(\eta). \tag{3.6}$$

Here, η is the degree of freedom of the distribution. The density function of the GED is given by:

$$f(z_t) = \frac{\eta \exp\left(-\frac{1}{2}\left|\frac{z_t}{\lambda}\right|^{\eta}\right)}{\lambda 2^{(1+\frac{1}{\eta})}\Gamma(1/\eta)}, \quad 0 < \eta \leq \infty, \tag{3.7}$$

$$\lambda = \frac{1}{2^{\frac{1}{\eta}}}\sqrt{\frac{\Gamma(1/\eta)}{\Gamma(3/\eta)}}.$$

When $\eta = 2$, z_t follows the standard normal distribution. When $\eta < 2$, z_t has thicker tails than the normal distribution. When $\eta > 2$, z_t has thinner tails than the normal distribution. In this study, the following two kinds of EGARCH models in which volatility changes. "-*n*" implies that the error term follows the normal distribution, and "-*GED*" means that the error term follows the Generalized Error Distribution.

1. EGARCH(1,1)-*n*・・・Equations (3.1)－(3.5).
2. EGARCH(1,1)-*GED*・・・Equations (3.1)－(3.4), (3.6).

4. Data and Empirical Application

4.1 Data Description

In this study, daily data[15)] of the inter-bank spot Yen / US dollar exchange rate and Yen / Euro exchange rate at 17:00 in the Tokyo foreign exchange market were used to estimate the asymmetric SV model, which is described in Chapter 2 and "EGARCH(1,1)- *n*, " and " EGARCH(1,1)-*GED* ", which is described in Chapter 3. The sample data was taken from those in the period from Jan. 4, 2000 to Dec. 28, 2007 (see Fig. 1). The return was set to be $R_t = (\ln S_t - \ln S_{t-1}) \times 100$ (%) based on closing exchange rate values (refer to Figs. 2). The number of samples is 812 and the period from Jan. 5, 2000 to Dec. 28, 2007. The summary statistics are tabulated in table 2.

Figure 1. Exchange rate (Jan. 4, 2000 – Dec. 28, 2007)

Figure 2. Rate of return (Jan. 5, 2000 – Dec. 28, 2007)

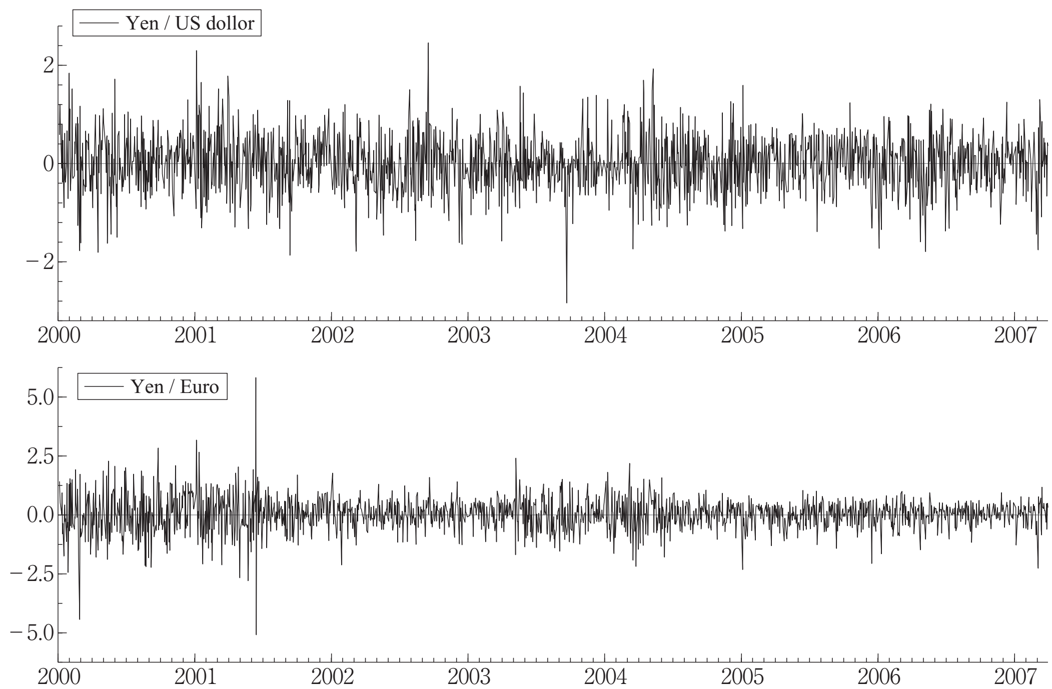


Table 1. Summary Statistics for the Rate of Returns R_t

Jan. 4, 2000 − Dec. 28, 2007

Sample Size: 1968

Yen / US dollar	Mean	Std Dev.	Skewness	Kurtosis	Max.	Min.	$LB^2(12)$
	0.005	0.573	−0.221	4.338*	2.458	−3.153	7.680
	(0.013)		(0.055)	(0.110)			
Yen / Euro	0.023	0.697	−0.173	8.808*	5.820	−5.086	7.680
	(0.016)		(0.055)	(0.110)			

( ) denotes standard error. The standard error of the mean, skewness, and kurtosis estimates calculate $\hat{\sigma}/\sqrt{T}$, $\sqrt{6/T}$, and $\sqrt{24/T}$ respectively, where T=sample size and $\hat{\sigma}$ = standard deviation. $LB^2(12)$ is the heteroskedasticity-corrected Ljung = Box statistic following Diebold [1988] and Diebold / Lopez [1995].
$*$ denotes statistical significance at the 5% level.

The sample mean of the rate of return on the Yen / US dollar exchange rate is 0.005, but this is not statistically significant. The skewness is −0.221, but this is also not statistically significant. The kurtosis is 4.338, which is statistically significant. Since this value exceeds 3, it can be understood that the distribution of the rate of return on the Yen / US dollar exchange rate has thicker tails than the normal distribution. The histogram of the rate of return on the Yen / US dollar exchange rate is shown in Fig. 3. In this figure, density and normal approximation are superimposed. N(s = 0.573) means that the normal approximation follows the normal distribution N(0.005, 0.573) with a mean of 0.005 and a variance of 0.573. With regard to the Ljung-Box test statistic, the null hypothesis that the first- to twelfth-order autocorrelation coefficients of the Yen / US dollar exchange rate are all zero is not nullified. This indicates that there is no significant autocorrelation in the rate of return on the Yen / US dollar exchange rate. Fig. 4, Fig. 5 and Fig 6. show the autocorrelation function (ACF), spectral density and periodogram, respectively.

The sample mean and skewness of the rate of return on the Yen / Euro exchange rate are 0.023 and −0.173, respectively, but these values are not statistically significant. The kurtosis is 8.808, which is statistically significant, and since this value exceeds 3, it can be understood that the distribution of the rate of return on the Yen / Euro exchange rate has thicker tails than the normal distribution like the distribution of the rate of return on the Yen / US dollar exchange rate. The histogram of the rate of return on the Yen / Euro exchange rate is shown in Fig. 3. The normal approximation of the rate of return on the Yen / Euro exchange rate follows N(0.023, 0.697). With regard to the Ljung-Box test statistic, the null hypothesis that the first- to twelfth-order autocorrelation coefficients of the Yen / Euro exchange rate are all zero is not nullified. This indicates that there is no significant autocorrelation in the rate of return on the Yen / Euro exchange rate. Fig. 4, Fig. 5 and Fig 6. show the autocorrelation function (ACF), spectral density and periodogram, respectively.

Figure 3. Density (Jan. 5, 2000 − Dec. 28, 2007)

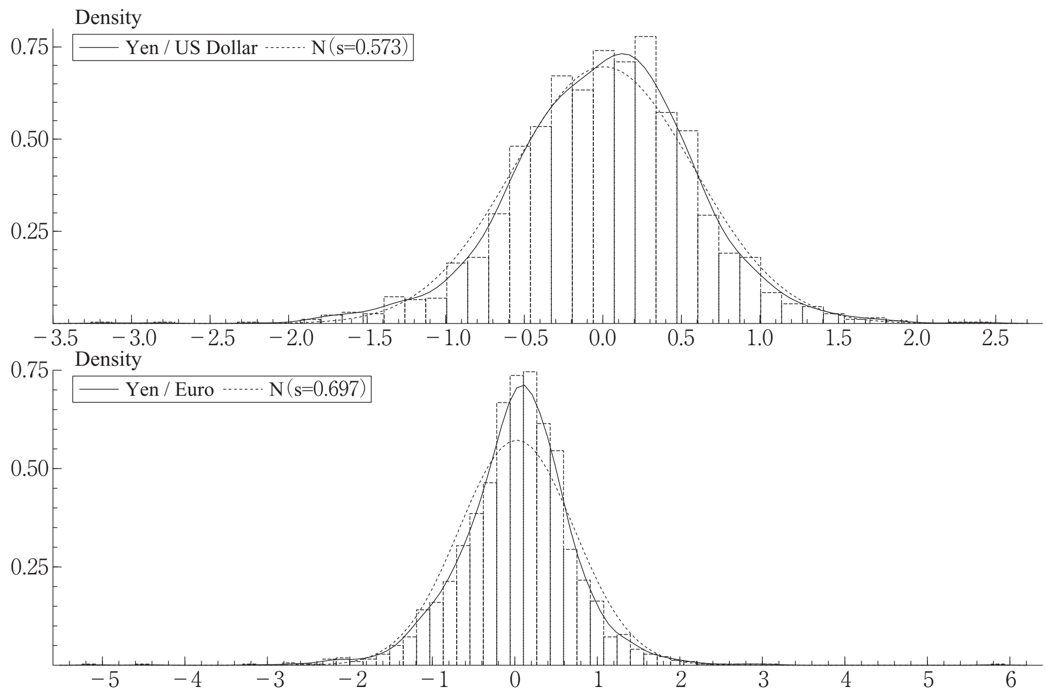


Figure 4. ACF

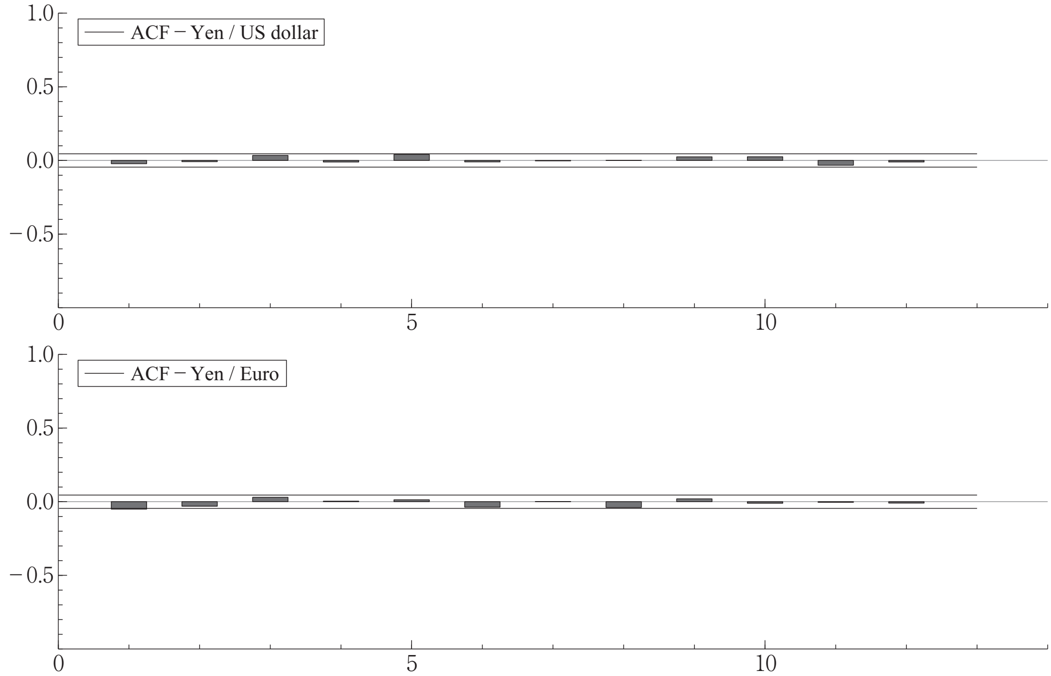


Figure 5. Spectral Density

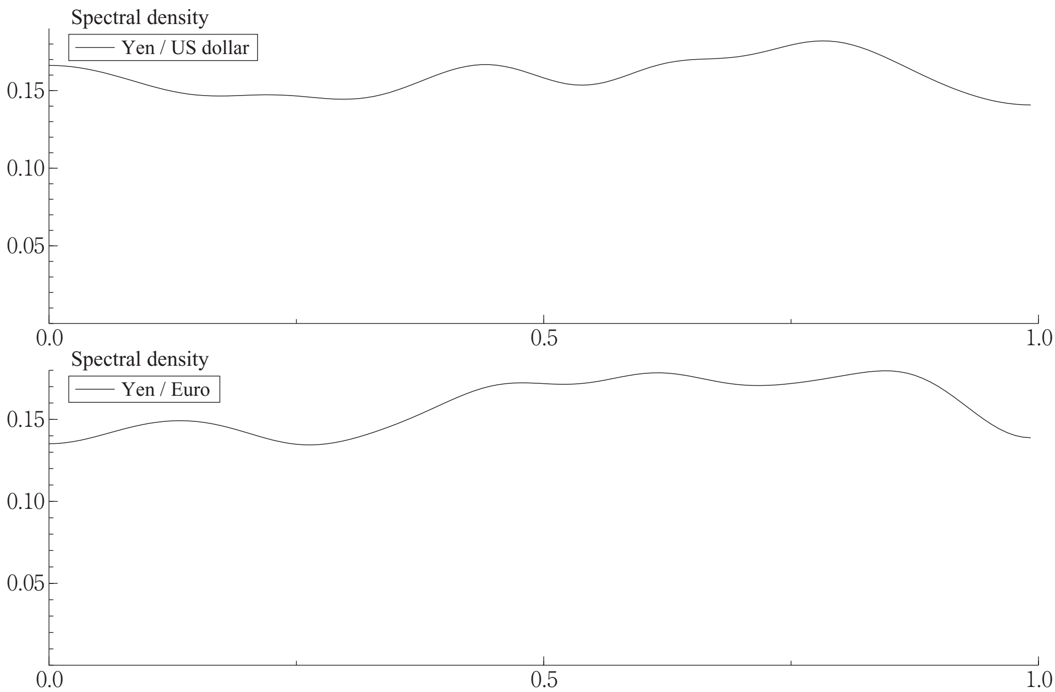
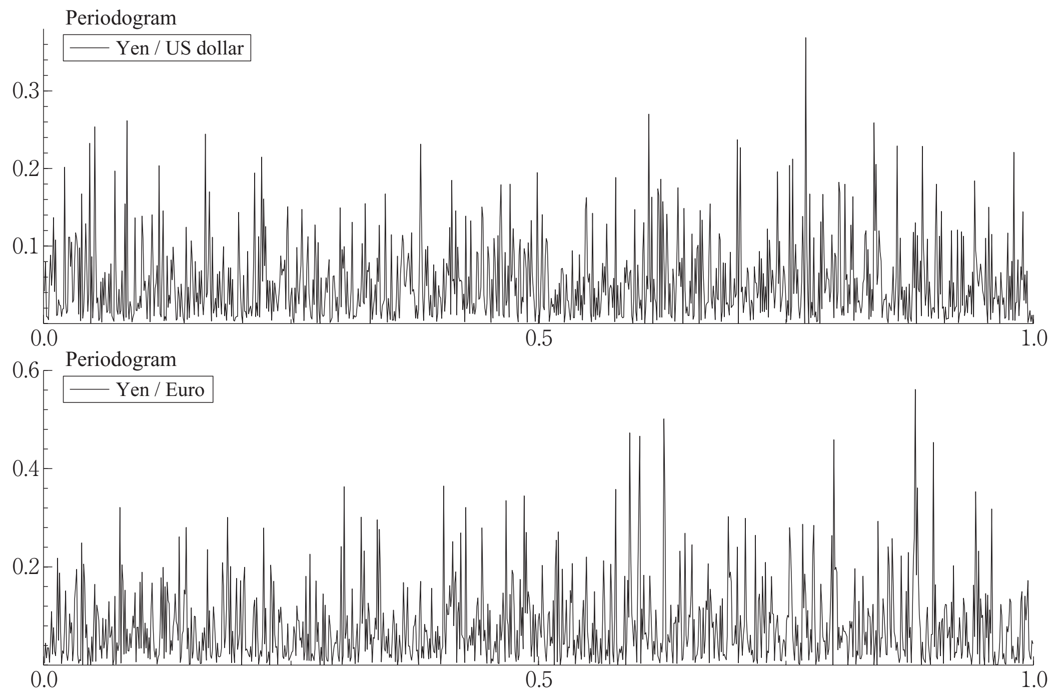


Figure 6. Periodogram



4.2 Empirical Results

The empirical results of asymmetric SV model are tabulated in Table 2. The values of β of the Yen / US dollar exchange rate and the Yen / Euro exchange rate are as follows: $\beta^{(US)}=0.954$, $\beta^{(Euro)}=0.995$, . These values are all statistically significant. The estimated values of σ_η and ρ are as follows: $\sigma_\eta^{(US)}=0.238$, $\sigma_\eta^{(Euro)}=0.153$, $\rho^{(US)}=-0.0006$, $\rho^{(Euro)}=-0.282$, but these values are not statistically significant. The fact that σ_η is statistically significant indicates that volatility fluctuates stochastically. In addition, the fact that β is near to 1 means that the shock of volatility has high persistence. The fact that the parameter ρ, which represents the correlation between volatility and the rate of return on exchange rates, is not statistically significant, indicates that there is no asymmetry in the Yen / US dollar or Yen / Euro exchange rate market.

Table 2. Estimation Results for Asymmetric SV Model (Yen / US dollar)

$$R_t = \sigma_t u_t$$
$$\ln \sigma_{t+1}^2 = \alpha + \beta \ln \sigma_t^2 + \eta_t$$
$$\begin{pmatrix} u_t \\ \eta_t \end{pmatrix} \sim NID\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho\sigma_\eta \\ \rho\sigma_\eta & \sigma_\eta^2 \end{bmatrix} \right), t = 1 \cdots T$$

	α	β	σ_η	ρ	$\ln L$
Estimate	0.121*	0.954*	0.238	−0.0006	−4310.071
	(0.026)	(0.049)	(0.178)	(0.004)	

∗ denotes statistical significance at the 5% level.

Table 3. Estimation Results for Asymmetric SV Model (Yen / Euro)

$$R_t = \sigma_t u_t$$
$$\ln \sigma_{t+1}^2 = \alpha + \beta \ln \sigma_t^2 + \eta_t$$
$$\begin{pmatrix} u_t \\ \eta_t \end{pmatrix} \sim NID\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho\sigma_\eta \\ \rho\sigma_\eta & \sigma_\eta^2 \end{bmatrix} \right), t = 1 \cdots T$$

	α	β	σ_η	ρ	$\ln L$
Estimate	0.123	0.995*	0.153	−0.282	−4833.942
	(0.001)	(0.003)	(0.015)	(0.271)	

∗ denotes statistical significance at the 5% level.

The empirical results of "EGARCH(1,1)-*n*, " and "GARCH(1,1)-*GED* " are tabulated in Tables 3-6. The estimation results can be summarized as follows:

(1) Yen / US dollar exchange rate

(i) EGARCH(1,1)-*n*

μ, ω, and θ_1 were not statistically significant, while θ_2 and β_1 were statistically significant. The estimation result was $\theta_1 = -0.090$, but this value is not statistically significant, and so it can be concluded that there is no asymmetry in the Yen / US dollar exchange market. $\beta_1 = 0.920$ is near to 1. This indicates that the shock of the volatility in the rate of return on the Yen / US dollar exchange rate has a high persistence.

(ii) EGARCH(1,1)-*GED*

μ, ω, and θ_1 were not statistically significant, while θ_2, β_1, and ln $(\eta/2)$ were statistically significant. Since θ_1 is not statistically significant, there is no asymmetry in the Yen / US dollar exchange market, like EGARCH(1,1)-*n*. $\beta_1 = 0.934$ is near to 1. Since ln $(\eta/2) = -0.215$, it can be understood that the distribution of z_t has thicker tails than the standard normal distribution.

(2) Yen / Euro exchange rate

(i) EGARCH(1,1)-*n*

ω, and θ_1 were not statistically significant, while μ, θ_1 and β_1 were statistically significant. Since θ_1 is not statistically significant, it can be concluded that there is no asymmetry in the Yen / Euro exchange market. Since $\beta_1 = 0.986$, the shock of the volatility in the rate of return on the Yen / Euro exchange rate has a high persistence.

(ii) EGARCH(1,1)-*GED*

ω and θ_1 were not statistically significant, while μ, θ_2, β_1, and ln $(\eta/2)$ were statistically significant. Since θ_1 is not statistically significant, there is no asymmetry. In addition, $\beta_1 = 0.988$ is near to 1. Since ln $(\eta/2) = -0.340$, the distribution of z_t has thicker tails than the standard normal distribution.

Table 4. Estimation Results for EGARCH(1,1)-*n* Model (Yen / US dollar)

$$R_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t z_t, \quad \sigma_t > 0, \quad z_t \sim i.i.d.N(0,1),$$
$$\ln(\sigma_t^2) = \omega + \alpha_1 \left\{\theta_1 z_{t-1} + \theta_2 \left(|z_{t-1}| - \sqrt{\pi/2}\right)\right\} + \beta_1 \ln(\sigma_{t-1}^2).$$

	μ	ω	θ_1	θ_2	β_1
Estimate	0.008	−0.090	−0.047	0.150*	0.920*
Standard Error	(0.012)	(0.033)	(0.018)	(0.030)	(0.029)
Log-likelihood		−1659.482			

* denotes statistical significance at the 5% level.

Table 5. Estimation Results for EGARCH(1,1)-*GED* Model (Yen / US dollar)

$$R_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t z_t, \quad \sigma_t > 0, \quad z_t \sim GED(\eta),$$
$$\ln(\sigma_t^2) = \omega + \alpha_1 \{ \theta_1 z_{t-1} + \theta_2 (|z_{t-1}| - E|z_{t-1}|) \} + \beta_1 \ln(\sigma_{t-1}^2).$$

	μ	ω	θ_1	θ_2	β_1	$\ln(\eta/2)$
Estimate	0.015	-0.077^*	-0.032	0.139^*	0.934^*	-0.215^*
Standard Error	(0.012)	(0.029)	(0.018)	(0.030)	(0.025)	(0.044)
Log-likelihood		-1647.036				

$*$ denotes statistical significance at the 5% level.

Table 6. Estimation Results for EGARCH(1,1)-*n* Model (Yen / Euro)

$$R_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t z_t, \quad \sigma_t > 0, \quad z_t \sim i.i.d.N(0,1),$$
$$\ln(\sigma_t^2) = \omega + \alpha_1 \left\{ \theta_1 z_{t-1} + \theta_2 \left(|z_{t-1}| - \sqrt{\pi/2} \right) \right\} + \beta_1 \ln(\sigma_{t-1}^2).$$

	μ	ω	θ_1	θ_2	β_1
Estimate	0.029^*	-0.008	0.0005	0.152^*	0.986^*
Standard Error	(0.013)	(0.005)	(0.011)	(0.023)	(0.005)
Log-likelihood		-1872.254			

$*$ denotes statistical significance at the 5% level.

Table 7. Estimation Results for EGARCH(1,1)-*GED* Model (Yen / Euro)

$$R_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t z_t, \quad \sigma_t > 0, \quad z_t \sim GED(\eta),$$
$$\ln(\sigma_t^2) = \omega + \alpha_1 \{ \theta_1 z_{t-1} + \theta_2 (|z_{t-1}| - E|z_{t-1}|) \} + \beta_1 \ln(\sigma_{t-1}^2).$$

	μ	ω	θ_1	θ_2	β_1	$\ln(\eta/2)$
Estimate	0.051^*	-0.011	0.0007	0.138^*	0.988^*	-0.340^*
Standard Error	(0.011)	(0.005)	(0.014)	(0.027)	(0.005)	(0.042)
Log-likelihood		-1838.090				

$*$ denotes statistical significance at the 5% level.

5. Conclusion and Future Themes

This paper focused on the asymmetry between volatility and the rate of return on foreign exchange rates, and empirically discussed it by inputting the data of the Yen / US dollar exchange rate and the Yen / Euro exchange rate into the asymmetric SV model and the EGARCH-M model. The major findings in this paper can be summarized as follows:

1. The shock of volatility in the inter-bank spot Yen / US dollar exchange rate and Yen / Euro exchange rate in the Tokyo foreign exchange market is highly persistent.
2. There is no asymmetry between volatility and the rate of return on the inter-bank spot Yen / US dollar exchange rate in the Tokyo foreign exchange market.
3. There is no asymmetry between volatility and the rate of return on the inter-bank spot Yen / Euro exchange rate in the Tokyo foreign exchange market.

We can enumerate the following issues as themes of the future studies: (i) We can nominate the Markov Switching (MS) model[16] instead of the asymmetric SV model for describing the characteristics of the volatility fluctuation in the foreign exchange market. So, Lam and Li [1998] proposed the MS-SV model integrating MS into the SV model. Then, it is necessary to combine the asymmetric SV model with MS and formulate this combined model[17]. (ii) We should compare the QML method with the other models[18] such as the Bayes procedure[19], which utilizes Markov-Chain Monte Carlo (MCMC)[20] and is adopted often in the recent financial econometrics. (iii) We can include the empirical analysis using other major currencies, such as sterling pond, Hong Kong dollar, and Australian dollar.

(Associate Professor, College of Economics, Nihon University)

Notes

1) In the stock market, as a relation between rate of return and volatility, there exists such asymmetry that when rate of return decreases, volatility increases in the next term, and when rate of return increases, volatility decreases in the next term. For details, see Black [1976], Nelson [1991], and Bekaert and Wu [2000].

2) For details about the SV model, see Taylor [1994], Ghysels, Harvey and Renault [1996], Shephard [1996], Jiang [2002] and Shephard [2005]. For the information on its application to foreign exchange rates, refer to Hol [2003] and Mitsui [2004].

3) With regard to such ARCH models, see Bollerslev, Chou and Kroner [1992], Bera and Higgins [1993], or Bollerslev, Engle and Nelson [1994], for checking statistical properties and analyses. To check empirical studies on finance, refer to Taylor [1994] and Shephard [1996].

4) Assumed $\psi^2 \exp(\mathrm{ht}) \equiv \sigma_t^2$, we can obtain other expressions of Equations (2.1) and (2.2) as follows:

$$R_t = \psi \exp\left(\frac{h_t}{2}\right) u_t,$$

$$h_t = \phi h_{t-1} + \eta_t,$$

$$\begin{pmatrix} u_t \\ \eta_t \end{pmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix}\right), t = 1, \cdots, T.$$

where ψ represents a scale parameter. These equations are frequently-used expressions in empirical analyses.

5) With regard to random variable X, when (i) $E(X) = \mu$, (ii) $Var(X) = \sigma^2$, (iii) X is independent and identically distributed ($Cov(Xt, Xs) = 0$, $t \neq s$), and (iv) X follows the normal distribution with a mean of μ and a variance of σ^2, X is expressed by $X \sim NID(\mu, \sigma^2)$.
6) For details, refer to Ruiz [1994], Harvey, Ruiz and Shephard [1994].
7) For details, refer to Harvey and Shephard [1996].
8) For details on the linear state space model, refer to Harvey [1981], Hamilton [1994b], Brockwell and Davis [1991, Chapter 12], and Brockwell and Davis [2002, Chapter 8].
9) For further details on Kalman Filter, refer to Hamilton [1994a, Chapter 13], Brockwell and Davis [1991, Chapter 12], and Brockwell and Davis [2002, Chapter 8].
10) For details, refer to Harvey and Shephard [1996].
11) Set the estimate to be $\hat{\theta}_T$, $\hat{\theta}_T$ asymptotically approaching a distribution as follows:

$$\sqrt{T}(\hat{\theta}_T - \theta) \xrightarrow{d} N(0, D^{-1}SD^{-1}).$$

where

$$\frac{1}{T}\frac{\partial^2 \ln L}{\partial\theta\partial\theta'} \xrightarrow{p} D > 0, \quad \frac{1}{\sqrt{T}}\frac{\partial \ln L}{\partial\theta} \xrightarrow{d} N(0, S),$$

in which "$\xrightarrow{d}$" represents convergence in distribution and "$\xrightarrow{p}$" represents convergence in probability.

12) In general, the degree of order for the GARCH model is determined based on the two information criteria: Akaike's Information Criterion (AIC) and Schwart's Information Criterion (SIC). When parameters are estimated with the maximum-likelihood method, AIC and SIC can be expressed by the following equations:

$$\mathrm{AIC} = -2\ln L + 2n,$$
$$\mathrm{SIC} = -2\ln L + n\ln T,$$

where $\ln L$ represents the log likelihood calculated from estimated parameters, and n represents the number of estimated parameters, and T denotes the number of samples.

13) A general EGARCH(p, q) can be expressed by the following equation:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{p}\alpha_i\{\theta_1 z_{t-i} + \theta_2(|z_{t-i}| - E|z_{t-i}|)\} + \sum_{j=1}^{q}\beta_j \ln(\sigma_{t-j}^2).$$

14) At this time, EGARCH(1,1) can be expressed by the following equation:

$$\ln(\sigma_t^2) = \omega + \alpha_1\left\{\theta_1 z_{t-1} + \theta_2\left(|z_{t-1}| - \sqrt{\pi/2}\right)\right\} + \beta_1\ln(\sigma_{t-1}^2).$$

15) Nikkei NEEDS-Financial Quest was used.
16) For further information on the Markov Switching model, see Kim and Nelson [1999] and Hamilton and Raj (eds.) [2002].
17) For details on the MS model, refer to Kim and Nelson (eds.) [1999] and Hamilton and Raj (eds.) [2002].

18) For more information regarding estimators other than QML, see Campbell, Lo and Mackinlay [1997], Gourieroux and Jasiak [2001], and Tsay [2002].

19) For details about theory, methods, and a glossary of Bayes statistics, refer to Berger [1985] and Bauwens and Lubrano (eds.) [1995].

20) For further discussion on MCMC, refer to Gilks, Richardson and Spiegelhalter [1996], Tanner [1996], Bauwens, Lubrano and Richard [1999], Robert and Casella [1999], Chen, Shao and Ibrahim [2000], and Chib [2001].

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