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abstract

In this paper, Nikkei 225 Options are evaluated with the MS-EGARCH model, which is developed by combining the Markov switching model and the EGARCH model, and the effectiveness of the MS-EGARCH model in the option market is discussed. As a result of the empirical analysis, it is found that the bull and bear of the Nikkei 225 index, which is an underlying asset, cannot be captured with any models except the MS-EGARCH model, which assumes that the error term follows the t distribution. As for call option, whose period before expiration is long, it is found that the performance of this model is the best. In the case where volatility was increasing just after Lehman's fall, the performance of the conventional GARCH and BS models is quite poor, while the option evaluation with the MS-EGARCH and EGARCH models is extremely excellent. In addition, it is revealed that the evaluation of option prices is significantly influenced by the formulation considering the asymmetric relation between rates of return of underlying asset prices and volatility and the assumption that the distribution of the error term, expected rate of return, and volatility follow the Markov switching process.

1 Introduction

The volatility of expected rates of return plays an important role in option pricing theory and is a highly sensitive parameter for option price. The main assumptions when deriving the Black and Scholes (1973) option pricing model (hereinafter called "BS model") are that the expected rate of return is lognormally distributed and that volatility is constant over time. However, it is known as empirical fact that the volatility of expected rates of return

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fluctuate over time, and thus formulation and analysis of a model for fluctuating volatility is necessary. How to formulate volatility fluctuations and evaluate option pricing in such cases is an extremely important issue.

In order to understand the volatility variation clearly, Engle (1982) proposed the Autoregressive Conditional Heteroskedasticity (ARCH) model that formulates the volatility at each time as the linear function of the square of the past unexpected shock. In addition, Bollerslev (1986) added the past volatility values to the explanatory variables, and extended the GARCH (Generalized ARCH) model to a more general model. Empirical studies of options with such ARCH type models have been conducted by Engle and Mustafa (1992), Noh, Engle and Kane (1994), Saez (1997), Sabbatini and Linton (1998), Bauwens and Lubrano (1998), and Moriyasu (1999) [1]. In addition, empirical studies utilizing the GARCH model based on local risk neutrality, which was proposed by Duan (1995), have been conducted by Mitsui (2000), Duan and Zhang (2001), Bauwens and Lubrano (2002), Mitsui and Watanabe (2003), Watanabe (2003), Takeuchi and Watanabe (2008), and Watanabe and Ubukata (2009).

Incidentally, it is known that in the volatility variation model, including the ARCH model, the persistency of shock on volatility is extremely high. However, as Diabold (1986) and Lamoureux and Lastrapes (1990) pointed out, such persistency is considered to be caused by the structural change of volatility. Based on this fact, Hamilton and Susmel (1994) and Cai (1994) proposed the Morkov Switching ARCH (MS-ARCH) model by using a state variable that follows the Markov process in the formulation of the ARCH model, in order to take into account the structural change. Moreover, Gray (1996) proposed the Markov Switching GARCH (MS-GARCH) model by taking into account the structural change in the GARCH model, not the ARCH model [2]. The MSGARCH model was later modified by Klaassen (2002) and Haas et al. (2004).

Satoyoshi (2004) conducted an empirical analysis of TOPIX (Tokyo Stock Price Index) with the MS-GARCH model, and found that the rate of TOPIX change underwent switching and that this model is superior to the conventional GARCH model in forecasting volatility of daily data. On the basis of this result, Satoyoshi and Mitsui (2011) investigated the pricing of Nikkei 225 Options using the MS-GARCH model proposed by Gray (1996), and revealed that, for call options, the MS-GARCH model with Student's t-distribution gives

[1] In these studies, the risk neutrality of investors was assumed, and so risk premium was not taken into account. Therefore, risk assets are evaluated based on only the expectation of the asset return, and the expected return rate of risky assets becomes equal to that of risk-free assets.

[2] Gray (1996) named this model the regime-switching GARCH model. However, regime switching is induced by Markov chain. In this study, it is called the Markov switching GARCH model.

more accurate pricing results than GARCH models and the Black-Scholes model. The MS-GARCH model was developed by combining the Markov switching model and the GARCH model, and can describe sufficiently the characteristics of the thickness of the tail for the distribution of the rate of returns. Accordingly, it can be considered that the performance of option valuation was improved.

Here, the strain of the distribution was not considered at all, because the equation of rate of return was formulated with expected rate of return being a constant. Observing long-term data for the Nikkei 225 index, we can see that there are periods of upward trends (bulls) and periods of downwards trends (bears). By modeling these bull and bear periods through switching of expected rates of return, we can express the asymmetric distortions of rate of return distributions. Also, since the so-called asymmetry of volatility, whereby volatility is greater on days following falls in stock prices than on days following rises, has long been observed in the Nikkei 225 Index, it is possible that the EGARCH (Exponential GARCH) model, which inputs such variability characteristics, may be even better than the GARCH model when it comes to option valuation. Furthermore, when one considers that the asymmetry of volatility has been observed, it is possible that the Markov-Switching EGARCH (MS-EGARCH) model, which combines the EGARCH model with switching, may perform highly in option valuation.This study shall evaluate Nikkei 225 Options pricing, using an EGARCH model in which not only volatility in underlying asset, but also expected rates of return, induce Markov-switching - which is to say, an MS-EGARCH model - and shall be guessed the model performs highly as compared with other models. This MS-EGARCH model is based on the MS-GARCH model used by Haas et al. (2004) [3)]

The MS-GARCH model is expanded into the MS-EGARCH model, and the number of state variables following the Markov chain is also increased from one to two. With only one state variable, even if switching in expected rates of return is assumed, there is a constraint that it must occur at the same time as switching in volatility. Since switching in expected rates of return does not always occur at the same time as switching in volatility, it was decided to have two state variables, and a model that allowed for switching in expected rates of return and volatility to occur separately. Where investors are assumed to be risk-neutral, European option prices like the Nikkei 225 Options can easily be derived through Monte Carlo simulations. The call (put) option prices can be evaluated by averaging the present discount values of the maximum between the simulated price minus the exercise price and

[3)] Henry (2009) presents an MS-EGARCH model that expands upon the MS-GARCH model of Gray (1996), to analyze data from the British securities market.

zero (between the exercise price minus the simulated price and zero). We propose as a means of accelerating convergence in the simulation, two variance reduction methods: negative correlation and a control variate approach.

As a result of the empirical analysis, it was found that the bull and bear of the Nikkei 225 Index, which is an underlying asset, cannot be grasped with any models except the MS-EGARCH model, which is an underlying asset, cannot be captured with any models except the MS-EGARCH model, which assumes that the error term follows the t distribution. As for call option, whose period before expiration is long, it was found that the performance of this model is the best. In the case where volatility was increasing just after Lehman's fall, the performance of the conventional GARCH and BS models was quite poor, while the option evaluation with the MS-EGARCH and EGARCH models was extremely excellent. In addition, it was revealed that the evaluation of option prices is significantly influenced by the formulation considering the asymmetric relation between rates of return of underlying asset prices and volatility and the assumption that the distribution of the error term, expected rate of return, and volatility follow the Markov switching process.

The brief descriptions of the following sections are as follows: Section 2 describes the MS-EGARCH model and risk neutrality are assumed, and mentions a model for comparison in this study. Section 3 explains the method for evaluating European options using the Monte Carlo simulation. The results of the empirical analysis are summarized in Section 4. Section 5 contains conclusions and future study themes.

2 Analytical Model

2.1 MS-EGARCH Model

Let R_t describe the rate of return for underlying asset prices, the MS-EGARCH model can be represented as follows:

$$R_t = \mu_a + \sqrt{V_{ab,t}} z_t, \tag{2.1}$$

$$z_t \sim i.i.d., E\left[z_t\right] = 0,\ V\left[z_t\right] = 1, \tag{2.2}$$

$$\ln\left(V_{ab,t}\right) = \omega_b + \beta \ln\left(V_{ab,t-1}\right) + \theta \left[\frac{R_{t-1} - \mu_a}{\sqrt{V_{ab,t-1}}}\right] + \gamma \left[\left|\frac{R_{t-1} - \mu_a}{\sqrt{V_{ab,t-1}}}\right| - E\left[\left|z_{t-1}\right|\right]\right], \tag{2.3}$$

$$\mu_a = \mu_1 \Delta_{1t} + \mu_2 \Delta_{2t}, \quad \mu_1 < \mu_2, \tag{2.4}$$

$$\omega_b = \omega_1 \Gamma_{1t} + \omega_2 \Gamma_{2t}, \quad \omega_1 < \omega_2. \tag{2.5}$$

Here, with equation (2.4), the variables $(\Delta_{1t}, \Delta_{2t})$ are such that where $\Delta_t = 1$ $(\Delta_{1t} = 1, \Delta_{2t} = 0)$ and where $\Delta_t = 2$ $(\Delta_{1t} = 0, \Delta_{2t} = 1)$. Similarly, with equation (2.5), the variables $(\Gamma_{1t}, \Gamma_{2t})$ are such that where $\Gamma_t = 1$ $(\Gamma_{1t} = 1, \Gamma_{2t} = 0)$ and when $\Gamma_t = 2$ $(\Gamma_{1t} = 0, \Gamma_{2t} = 1)$. Δ_t and Γ_t are state variables that jointly follow single Markov chains, and each take the values of 1 or 2. Volatility $V_{ab,t}$ is the conditional variance for R_t, $V_{ab,t} = V\left[R_t | \Delta_t = a, \Gamma_t = b, I_{t-1}\right]$ with the two state variables Δ_t and Γ_t, assuming the information set $I_{t-1} = \{R_{t-1}, R_{t-2}, \cdots\}$ until the time of $t-1$. The combinations of state variables at time t are: $(\Delta_t = 1, \Gamma_t = 1)$, $(\Delta_t = 1, \Gamma_t = 2)$, $(\Delta_t = 2, \Gamma_t = 1)$, and $(\Delta_t = 2, \Gamma_t = 2)$, and therefore the four combinations of expected rate of return and volatility are: low return, low volatility (μ_1, ω_1); low return, high volatility (μ_1, ω_2); high return, low volatility (μ_2, ω_1); and high return, high volatility (μ_2, ω_2). When there is one state variable, expected rate or return and volatility both switch simultaneously, but by introducing two state variables, Δ_t and Γ_t, the model allows for switching to be induced by expected rate of return and volatility at separate times. Also, if the expected rate of return is such that $\mu_1 < 0$, $\mu_2 > 0$, we can call the low-return phase "bear", and the high-return phase "bull". The transition probabilities can be expressed by the following equations:

$$
\begin{aligned}
&\Pr[\Delta_t = j | \Delta_{t-1} = i] = p_{ij}, \quad i, j = 1, 2, \\
&\Pr[\Gamma_t = l | \Gamma_{t-1} = k] = q_{kl}, \quad k, l = 1, 2, \\
&\sum_{j=1}^{2} p_{ij} = \sum_{l=1}^{2} q_{kl} = 1,
\end{aligned}
$$

where

$$
\mathbf{P} = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} q_{11} & q_{21} \\ q_{12} & q_{22} \end{pmatrix}.
$$

When estimating a volatility fluctuation model, the distribution of the error term z_t is often assumed to follow a standard normal distribution. The distribution for the rate of return of asset prices has been understood to follow a distribution with a thicker tail than a normal distribution, but even if the error term were to follow a normal distribution, if volatility fluctuates, then the kurtosis of the rate of return would exceed 3. However, the height of the kurtosis for the rate of return cannot always be explained by fluctuations in volatility alone, and in fact, in many previous studies, they have come to the conclusion that distribution with higher kurtosis fit error term distribution better than normal distribution do. Therefore, both normal distribution and t distribution will be considered for error term distribution. If the error term follows the normal distribution, z_t in equation (2.2) becomes as follows:

$$z_t \sim i.i.d.N\left(0,1\right). \tag{2.6}$$

If the error term follows the t-distribution, z_t becomes as follows:

$$z_t \sim i.i.d.t\left(0,1,\nu\right). \tag{2.7}$$

Here, ν represents degree of freedom, and the variance of z_t has been standardized to be one. In what follows, MS-EGARCH models in which error terms follow normal distributions shall be called MSEG-n models, while those in which error terms follow t distributions shall be called MSEG-t models.

In this study, in order to allow for maximum likelihood estimation in models, an idea on volatility formulation put forth by Haas et al. (2004) was employed. When state variables that follow Markov chains are directly introduced into GARCH and EGARCH models, parameters cannot be estimated through maximum likelihood methods. This is because volatility at time t is dependent on all state variables until time t, and therefore when state variables are assumed to take one of two states, one must consider 2^t combinations of state variables, making programming extremely difficult. Therefore, as with equation (2.3), when volatility at time t is $V_{ab,t}$, volatility at time $t-1$ is treated as $V_{ab,t-1}$. That is to say, when switching has occurred at time $t-1$, regardless of whether the state at time $t-1$ differs from that at time t, volatility is considered where the same state prevails at time $t-1$ as at time t, and volatility at time t is then determined according to this. For example, suppose that at time $t-1$ there is low return and low volatility; that is, the state variables are $V_{ab,t-1}$. If switching occurs at time t, and we have high return μ_2 and high volatility ω_2, then the state variables will change to $(\Delta_t = 2, \Gamma_t = 2)$. At this time, according to the equation (2.3), volatility will be

$$\ln\left(V_{22,t}\right) = \omega_2 + \beta \ln\left(V_{22,t-1}\right) + \theta\left[\frac{R_{t-1}-\mu_2}{\sqrt{V_{22,t-1}}}\right] + \gamma\left[\left|\frac{R_{t-1}-\mu_2}{\sqrt{V_{22,t-1}}}\right| - E\left[\left|z_{t-1}\right|\right]\right].$$

Looking at this formula, we see that volatility at time t is $V_{22,t}$, the constant term is ω_2, and that volatility and expected rate of return are $V_{22,t-1}$ and μ_2, respectively. That is to say, even though we have $(\Delta_{t-1} = 1, \Gamma_{t-1} = 1)$ at time $t-1$, it is almost as though it is the same state as at time t, $(\Delta_{t-1} = 2, \Gamma_{t-1} = 2)$. This means that we do not have to consider innumerous combinations of state variables, and can estimate parameters through the maximum likelihood method.

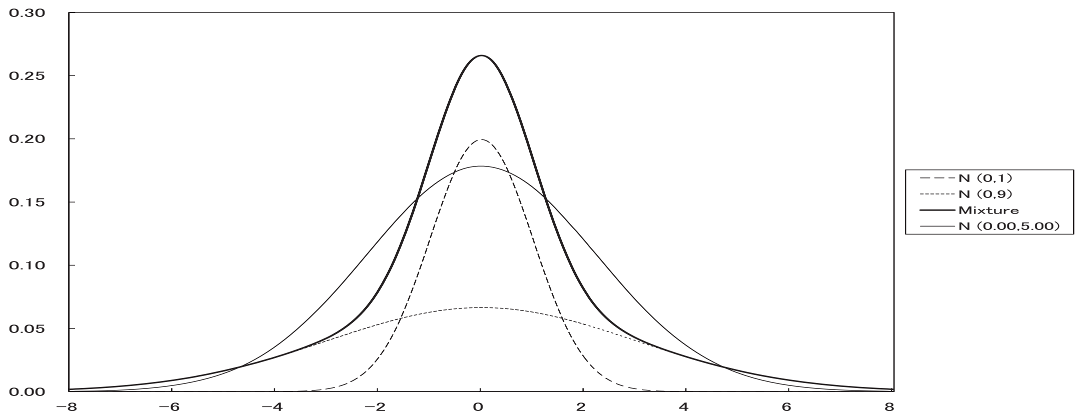

Figure 1: Only volatility follows Markov-switching processes

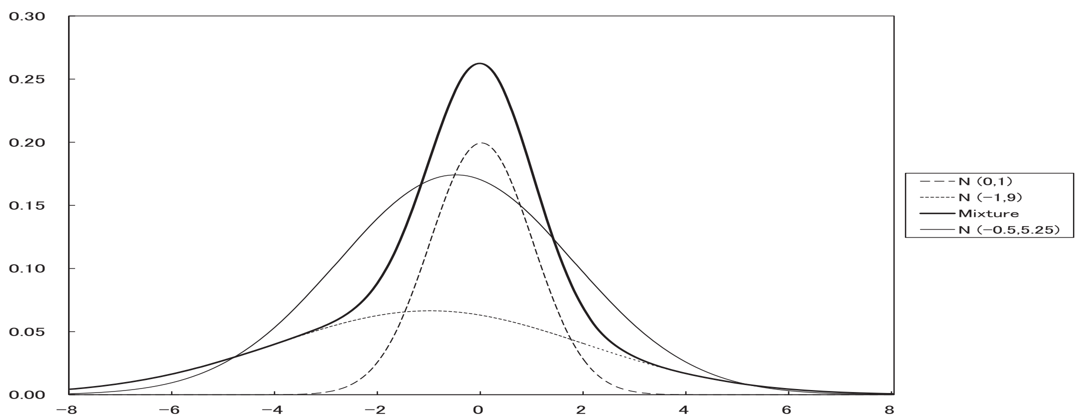

Figure 2: Both expected rates of return and volatility follow Markov-switching processes

In Satoyoshi and Mitsui (2011), expected rates of return for underlying asset prices are assumed to be constants, and option valuation is performed with an MS-GARCH model. In this study, this assumption is relaxed, and expected rates of return are allowed to switch at the same time as volatility. By including fluctuations in expected rates of return for underlying asset prices in this way, we are able to perceive not only the thickness of tails for expected rates of return distribution, but also distortion in distributions. As a result, option valuation performance can be expected to improve. Figures 1 and 2 show distortions arising in distributions through switching in expected rates of return. The heavy line in Figure 1 is a mixture normal distribution (same as weighted-value) graph drawn from two normal distributions, $N(0,1)$ and $N(-1,9)$, and a graph of a normal distribution, $N(-0.5,5.25)$ with the same mean and variance as the mixture normal distribution. We can see that unlike in the case of Figure 1, by choosing values that differ from the mean, we can express not only the thickness of the tail of the mixture normal distribution, but also distortions in the distribution.

2.2 Risk Neutrality of Investors and Formulation of Return

In this study, we define the simple return [4)]

$$R_t = \frac{S_t - S_{t-1}}{S_{t-1}}, \tag{2.8}$$

where S_t is the underlying asset price at time t. This paper assumes that investors are risk-neutral. Under this assumption, the conditional expected rate of return given I_{t-1} is equal to the simple risk-free interest rate r:

$$E\left[R_t \,|I_{t-1}\right] = r.$$

From equation (2.8),

$$E\left[\frac{S_t - S_{t-1}}{S_{t-1}} \,|I_{t-1}\right] = r,$$

therefore, $E\left[S_t \,|I_{t-1}\right]$ is given by

$$E\left[S_t \,|I_{t-1}\right] = S_{t-1}(1+r).$$

This equation means that the underlying asset price grows on average at the risk-free interest rate and investors require no compensation for risk.

In this study, option prices are evaluated, under the assumption that the rate of return of risk-free assets fluctuates in parallel with the expected rate of return of underlying asset prices. Namely, it is assumed that the interest rate of risk-free assets fluctuates according to the state variable Δ_t, so that $r_t = \mu_1$ when the expected rate of return of underlying assets at tiem t is μ_1 and $r_{t+1} = \mu_2$ when it becomes μ_2 through the switching of the expected rate of return at time $t+1$. With this assumption, it is possible to easily estimate the discounted option price as of maturity.

2.3 Model for Comparison

This study analyzed the following volatility variation models, as well as the MS-EGARCH model mentined in Section 2.1.

[4)] In the theory of financial engineering, including options, it is generally assumed that $R_t = \ln S_t - \ln S_{t-1}$, based on continuous compounding. However, in the model adopted for this study, option valuation cannot be conducted if rate of return is defined like this. For details, refer to Satoyoshi and Mitsui (2011).

(1) MS-EGARCH-c model (expected rate of return is constant)

$$R_t = \mu + \sqrt{V_{b,t}} z_t,$$
$$\ln(V_{b,t}) = \omega_b + \beta \ln(V_{b,t-1}) + \theta \left[\frac{R_{t-1} - \mu}{\sqrt{V_{b,t-1}}} \right] + \gamma \left[\left| \frac{R_{t-1} - \mu}{\sqrt{V_{b,t-1}}} \right| - E[|z_{t-1}|] \right],$$
$$\omega_b = \omega_1 \Gamma_{1t} + \omega_2 \Gamma_{2t}, \quad \omega_1 < \omega_2.$$

where when $\Gamma_t = 1$, it becomes $(\Gamma_{1t} = 1, \Gamma_{2t} = 0)$, and when $\Gamma_t = 2$, it becomes $(\Gamma_{1t} = 0, \Gamma_{2t} = 1)$ like the MS-EGARCH model. Γ_t is a state variable that follows the Markov chain, and equal to 1 or 2. "MSEG-c-n" implies that the error term follows the normal distribution, and "MSEG-c-t" means that the error term follows the t-distribution.

(2) MS-GARCH model

$$R_t = \mu_a + \sqrt{V_{ab,t}} z_t,$$
$$V_{ab,t} = \omega_b + \alpha(R_{t-1} - \mu_a)^2 + \beta V_{ab,t-1},$$
$$\mu_a = \mu_1 \Delta_{1t} + \mu_2 \Delta_{2t}, \quad \mu_1 < \mu_2,$$
$$\omega_b = \omega_1 \Gamma_{1t} + \omega_2 \Gamma_{2t}, \quad \omega_1 < \omega_2$$

where it is assumed that the switching of expected rate of return does not depend on the switching of volatility like the MS-EGARCH model. "MSG-n" implies that the error term follows the normal distribution, and "MSG-t" means that the error term follows the t-distribution.

(3) MS-GARCH-c model (expected rate of return is constant)

$$R_t = \mu + \sqrt{V_{b,t}} z_t,$$
$$V_{b,t} = \omega_b + \alpha(R_{t-1} - \mu)^2 + \beta V_{b,t-1},$$
$$\omega_b = \omega_1 \Gamma_{1t} + \omega_2 \Gamma_{2t}, \quad \omega_1 < \omega_2$$

"MSG-c-n" implies that the error term follows the normal distribution, and "MSG-c-t" means that the error term follows the t-distribution.

In this study, normal GARCH models, normal EGARCH models, and BS model [5] are applied for the pricing of options, and these models are compared. "-n" implies that the error term follows the normal distribution, and "-t" means that the error term follows the t-distribution.

[5] The European call option price C_T^{BS} and the European put option price P_T^{BS} at time T with an exercise

3 Method for pricing European options using Monte Carlo simulation

3.1 Option prices under the assumption of risk neutrality

The following is a brief explanation of the method for obtaining the option prices by means of the Monte Carlo simulation. When investors are risk neutral, the price of a European Option becomes the present discounted value that is calculated by discounting the expectation of the option price at the maturity with the interest rate of risk-free assets r. Namely, when it is assumed that the $T+\tau$ is maturity and that C_T is the price of the call option of the exercise price K at time T and that P_T is the put option price, the following expressions are obtained:

$$C_T = (1+r)^{-\tau} E\left[Max\left(S_{T+\tau} - K, 0\right)\right], \tag{3.1}$$

$$P_T = (1+r)^{-\tau} E\left[Max\left(K - S_{T+\tau}, 0\right)\right]. \tag{3.2}$$

Here, $S_{T+\tau}$ represents the underlying asset price at the maturity of the option. In this study, it is assumed that the interest rate of risk-free assets r fluctuates in parallel with the expected rate of return of underlying assets μ_a. Therefore, the interest rate from T to $T+\tau$, which is maturity, becomes μ_1 and μ_2 according to state variable Γ_t.

In the case of the MS-EGARCH model, since there is no closed form analytical solution for equation (3.1) and (3.2), these expectations are estimated using Monte Carlo simulation. Let $\left\{S_{T+\tau}^{(i)}\right\}_{i=1}^{n}$ be the simulated values of $S_{T+\tau}$ and n be the number of sample paths. When n is sufficiently large, by the law of large numbers, these expectations can be approximated

price of K and a current maturity of τ can be obtained with the following BS model.

$$C_T^{BS} = S_T N(d_1) - K e^{-r^* \tau} N(d_2), \tag{2.9}$$

$$P_T^{BS} = -S_T N(-d_1) + K e^{-r^* \tau} N(-d_2), \tag{2.10}$$

$$d_1 = \frac{\ln\left(S_T/K\right) + (r^* + \sigma^2/2)\tau}{\sigma\sqrt{\tau}},$$

$$d_2 = d_1 - \sigma\sqrt{\tau},$$

$$N(d_i) = \int_{-\infty}^{d_i} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx, \quad i = 1, 2.$$

where, r^* is the continuously compounded risk-free interest rate and $N(\cdot)$ represents the distribution function of the standard normal distribution.

by the following equations:

$$E\left[Max\left(S_{T+\tau}-K,0\right)\right] \approx \frac{1}{n}\sum_{i=1}^{n} Max\left(S_{T+\tau}^{(i)}-K,0\right), \tag{3.3}$$

$$E\left[Max\left(K-S_{T+\tau},0\right)\right] \approx \frac{1}{n}\sum_{i=1}^{n} Max\left(K-S_{T+\tau}^{(i)},0\right). \tag{3.4}$$

3.2 Procedures of the Monte Carlo Simulation

The procedures for calculating an option price with the Monte Carlo simulation in this study's model are as follows, where it is assumed that the error term of the MSEG-n model follows the normal distribution.

1. Estimate the unknown parameters of the MSEG-n model with the maximum likelihood method, using the samples $\{R_1, R_2, \ldots, R_T\}$.

2. Sample $\left\{z_{T+1}^{(i)}, z_{T+2}^{(i)}, \ldots, z_{T+\tau}^{(i)}\right\}_{i=1}^{n}$ from independent standard normal distributions.

3. Sample $\left\{u_{1,T+1}^{(i)}, u_{1,T+2}^{(i)}, \ldots, u_{1,T+\tau}^{(i)}\right\}_{i=1}^{n}$ and $\left\{u_{2,T+1}^{(i)}, u_{2,T+2}^{(i)}, \ldots, u_{2,T+\tau}^{(i)}\right\}_{i=1}^{n}$ from independent standard rectangular distributions.

4. Obtain the state variables $\left\{\Delta_{T+1}^{(i)}, \Delta_{T+2}^{(i)}, \ldots, \Delta_{T+\tau}^{(i)}\right\}_{i=1}^{n}$, $\left\{\Gamma_{T+1}^{(i)}, \Gamma_{T+2}^{(i)}, \ldots, \Gamma_{T+\tau}^{(i)}\right\}_{i=1}^{n}$ following the Markov process using uniform random numbers obtained at Step 3. and the transition probabilities $\mathbf{P}$ and $\mathbf{Q}$ estimated with the maximum likelihood method.

5. Calculate $\left\{R_{T+1}^{(i)}, R_{T+2}^{(i)}, \ldots, R_{T+\tau}^{(i)}\right\}_{i=1}^{n}$ by substituting the values at Steps 2. and 4. into the MS-EGARCH model.

6. Obtain the underlying asset price $\left(S_{T+\tau}^{(1)}, S_{T+\tau}^{(2)}, \ldots, S_{T+\tau}^{(n)}\right)$ at the maturity time $T+\tau$ of the option with the following equation:

$$S_{T+\tau}^{(i)} = S_T \prod_{s=1}^{\tau}\left(1+R_{T+s}^{(i)}\right), \quad i=1,2,\ldots,n. \tag{3.5}$$

7. Calculate the call option's price C_T and the put option's price P_T with the following equation:

$$C_T \approx (1+r)^{-\tau}\frac{1}{n}\sum_{i=1}^{n} Max\left(S_{T+\tau}^{(i)}-K,0\right), \tag{3.6}$$

$$P_T \approx (1+r)^{-\tau}\frac{1}{n}\sum_{i=1}^{n} Max\left(K-S_{T+\tau}^{(i)},0\right). \tag{3.7}$$

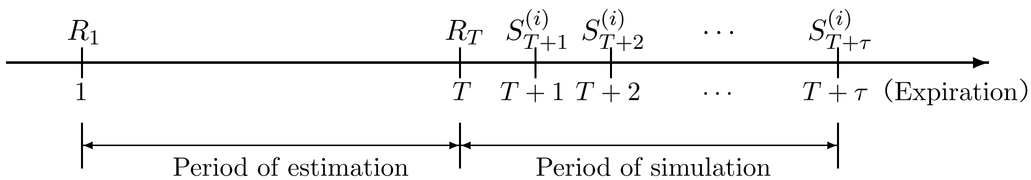

Figure 3: Period of estimation and simulation

Figure 3 shows the estimated period of model parameters and the simulation period. It is considered that the sufficient number of times of the Monte Carlo simulation is about 10,000. In order to reduce the variances of C_T and P_T, we propose the method of concurrently using the control variates and the antithetic variates, which are representative variance reduction techniques. In the case of the MSEG-t model, we can use Monte Carlo simulation in the same procedures.

Incidentally, at Step 4., the state variables that follow the Markov process are obtained using uniform random numbers and transition probabilities, but this method cannot be applied for the state variable Δ_{T+1}, Γ_{T+1} at time $T+1$, the starting point. This is because even after the maximum likelihood method at Step 1., the value of the state variable Δ_T, Γ_T at time T remains unknown, and it is impossible to calculate the state variable Δ_{T+1}, Γ_{T+1} from uniform random numbers and transition probabilities. Accordingly, with regard to Δ_{T+1}, the following calculation is conducted utilizing the probability $\Pr[\Delta_T = i | I_T]$ and the transition probability $\Pr[\Delta_{T+1} = j | \Delta_T = i]$ by means of the filtering technique of Hamilton (1989) (Hamilton Filter).

$$\Pr[\Delta_{T+1} = j | I_T] = \sum_{i=1}^{2} \Pr[\Delta_{T+1} = j | \Delta_T = i] \Pr[\Delta_T = i | I_T]$$

From this probability, sampling is carried out[6)] . We can obtain Γ_{T+1} in the same sampling.

4 Empirical results for Nikkei 225 Options

4.1 Data

The options used in empirical analysis for this study were Nikkei 225 call options and put options with contract months from June 2007 to January 2010 (32 contract months). Using a base of business days, closing prices were analyzed at 20 ($\tau = 20$) and 30 ($\tau = 30$) days

6) For details on Hamilton Filter, refer to Hamilton (1989), Kim and Nelson (1999).

before the option's maturity [7]. When $\tau = 20$, the number of samples for call and put options were 524 and 584, respectively, and when $\tau = 30$, the numbers were 495 and 539. Also, as a basic assumption, there were no transaction costs, tax charges or dividends, and no margins were required for options. For the interest-rate data for risk-free assets that was needed in calculations for the BS model, overnight unsecured call money was used. As parameter estimates for volatility fluctuation models in the MS-EGARCH-n and other models, closing prices for the Nikkei 225 Index at 20 or 30 days prior to maturity, as well as at 3,500 days prior to these dates, were used [8]. For example, where the closing price is taken 20 days prior to maturity, and the first contract month is June 2007, the option valuation date would be May 11, 2007, and 3,500 days prior to this would be February 25, 1993. By calculating the daily rate of return using the equation (2.8), the sample period would be from February 28, 1993 to May 11, 2007 (the size of the sample, $T = 3{,}500$), and the model's parameters would be estimated using the daily rate of return for this period. Given these estimated parameters, the option pricing would then be obtained through Monte Carlo simulation. Calculations would be performed in the same way from the next contract month onward. Where the closing price is taken 30 days prior to maturity, 3,500 days prior to the valuation date for the first contract month (June 2007) would be February 10, 1993, and data for the daily rate of return for this date would be used.

4.2 Estimation results for the MS-EGARCH Models

Table 1 shows estimated results for an MS-EGARCH-t model. The sample period for daily rates of return is from February 10, 1993 until January 8, 2010, which is the maturity date in the final contract month (January 2010). Estimated values for expected rate of return, μ_1 and μ_2 are -0.052 and 0.115, and are thus split into negative and positive values. From this, when the state variable Δ_t is $\Delta_t = 1$, the Nikkei 225 Index can be called a bear, and when $\Delta_t = 2$, it can be called a bull. Estimated values for the transition probability, p_{11} and p_{22}, of Δ_t are 0.996 and 0.992, respectively, and are hence both are extremely close to 1. Therefore, we can see that if switching is induced in either the bull or the bear, the state would continue for a long time. Conversely, estimated values, q_{11} and q_{22}, for the transition probability of the second state variable, Γ_t, are 0.989 and 0.923. Of the sustainability in volatility, these parameters display sustainability in the part that can be explained by switching. Because

[7] Although closing prices may be given at different times for Nikkei 225 Options and Nikkei 225 stock index, this was not considered in this study.

[8] For data on Nikkei 225 Index (Nikkei Stock Average), Nikkei NEEDS-Financial QUEST was used. For parameter estimates, the programming language OxMetrics 5.00 (http://www.oxmetrics.net) was used.

$q_{11} > q_{22}$, it seems that if a switch is made to low volatility ($\Gamma_t = 1$), this state will continue for a long time, but the state of high volatility ($\Gamma_t = 2$) will not continue for long. The sustainability of volatility displayed in the EGARCH part of this model is β, and its estimated value is 0.986. Also, the estimated value for the parameter θ, which shows asymmetry, is -0.104, making it significant, and these results are the same as with the normal EGARCH model.

Table 1: Estimation results for the MSEG-t model

	p_{11}	p_{22}	q_{11}	q_{22}	μ_1	μ_2
Estimates	0.996	0.992	0.989	0.923	-0.052	0.115
Standard Errors	0.002	0.005	0.004	0.027	0.021	0.030

	ω_1	ω_2	β	θ	γ	ν
Estimates	0.001	0.038	0.986	-0.104	0.068	19.248
Standard Errors	0.003	0.009	0.003	0.012	0.016	7.276

Table 2: Log-likelihoods, AIC, SBIC

	Log-likelihoods	AIC	SBIC
MSEG-n	-7105.10	14232.19	14301.87
MSEG-t	-7103.13	14230.26	14306.26
MSEG-c-n	-7111.28	14238.57	14289.24
MSEG-c-t	-7107.62	14233.24	14290.24
MSG-n	-7156.12	14332.24	14395.58
MSG-t	-7148.63	14319.25	14388.92
MSG-c-n	-7159.27	14332.54	14376.87
MSG-c-t	-7149.79	14315.58	14366.25
GARCH-n	-7213.83	14435.65	14460.99
GARCH-t	-7154.48	14318.97	14350.64
EGARCH-n	-7163.89	14337.79	14369.46
EGARCH-t	-7115.94	14243.89	14281.89

Figure 4 shows the probability of becoming a bull ($\Delta_t = 2$). This probability is calculated once all data has been provided. The issue of which periods should be treated as being bulls, and which as being bears, is something that is not clearly defined. However, insofar as can be seen from the graph, when this model is used, it seems that we can generally perceive bulls and bears. Figure 5 is the probability of reaching high volatility ($\Gamma_t = 2$), and we know that states of high volatility do not continue for long. Also, it is clear that switching periods differ for the bulls and bears in Figure 4, and in order to perceive bulls and bears, as well as high volatility and low volatility, it seems that not one but two state variables following Markov chains are required.

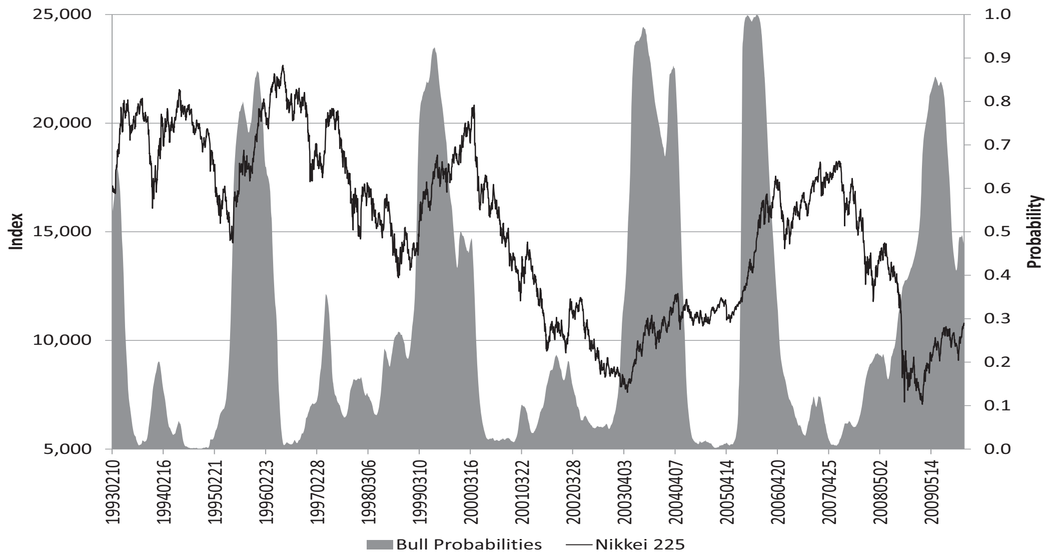

Figure 4: Probability of bulls

Table 2 shows the log likelihoods, AIC and SBIC, of all models used in this study. In terms of log likelihoods, the MS-EGARCH-t model, which assumed a t distribution for error terms, was the highest, while the MS-EGARCH-n model, which assumed a normal distribution for error terms, was the next highest. The MS-EGARCH-t model had the smallest AIC values, while the MS-EGARCH-c-n model, which only allowed switching for volatility, had the smallest SBIC values. From these results, it seems that the MS-EGARCH model, which combines the EGARCH model and the Markov Switching model, is exceptional for perceiving time-series fluctuations in Nikkei 225 Index. In order to determine whether the MS-EGARCH and MSG models, which include Markov switching, capture fluctuations in real data better than conventional GARCH and EGARCH models, we must perform tests that examine whether switching is occurring. However, as is widely known, under the null hypothesis that Markov switching does not exist, several of the model's parameters cannot be distinguished, and because test statistics do not follow normal asymptotic distributions, likelihood ratio tests are problematic. Hansen (1992, 1996) and Garcia (1998) offer test methods that give consideration to this problem, but because the aim of this study is option valuation, such tests will not be performed. Figure 6 shows changes in volatility for MS-EGARCH-n, EGARCH-n and GARCH-n.

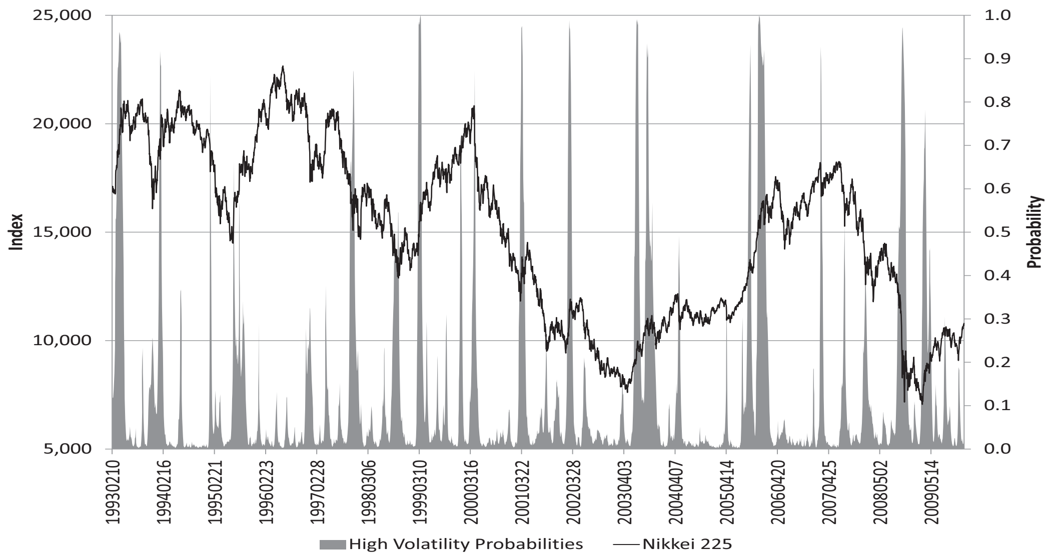

Figure 5: Probability of high volatility

4.3 Comparison of Option prices

Using the estimated values of option prices in the 12 kinds of models mentioned in Section 2.3 and actual market prices, the mean error rate (MER) and the root mean squared error rate (RMSER) are calculated, and each model is compared and discussed, as follows:

$$MER = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{\hat{X}_i^{\text{estimated}} - X_i^{\text{market price}}}{X_i^{\text{market price}}}\right), \tag{4.1}$$

$$RMSER = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(\frac{\hat{X}_i^{\text{estimated}} - X_i^{\text{market price}}}{X_i^{\text{market price}}}\right)^2}, \quad X = C, P. \tag{4.2}$$

Here, $\hat{X}_i^{\text{estimated}}$ is the option price estimated through the Monte Carlo simulation, or the theoretical price of the B-S model, and $X_i^{\text{market price}}$ represents the market option price. m is the number of samples.

Following Bakshi et al. (1997), we divide the option data into five different categories of moneyness. These are shown in Table 3. (1) If $S/K < 0.91$, a call option is deep-out-of-the-money (DOTM), and a put option is deep-in-the-money (DITM); (2) If $0.91 \leq S/K < 0.97$,

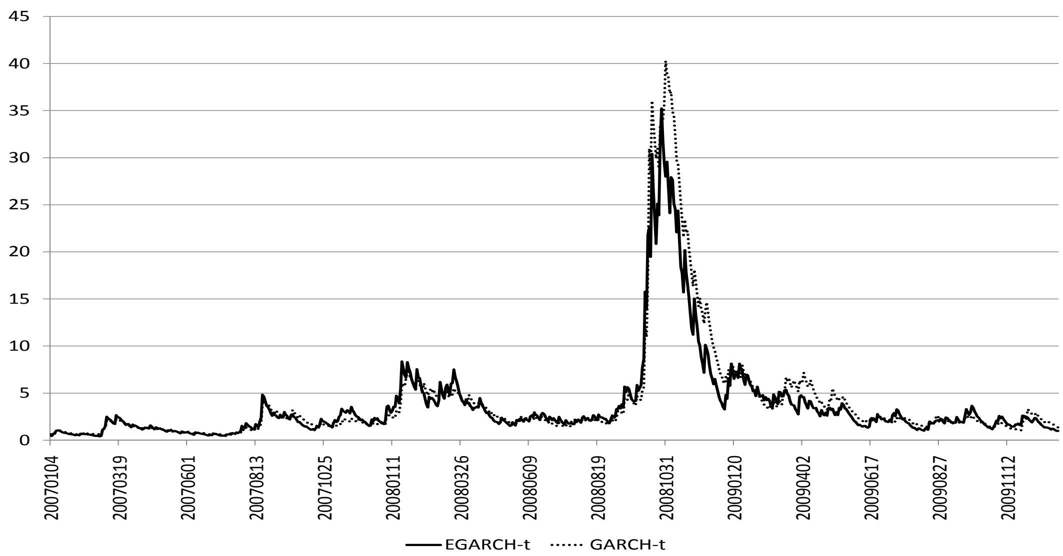

Figure 6: Volatility

Table 3: Moneyness

Moneyness	Call Option	Put Option
$S/K < 0.91$	deep-out-of-the-money (DOTM)	DITM
$0.91 \leq S/K < 0.97$	out-of-the-money (OTM)	ITM
$0.97 \leq S/K \leq 1.03$	at-the-money (ATM)	ATM
$1.03 < S/K \leq 1.09$	in-the-money (ITM)	OTM
$1.09 < S/K$	deep-in-the-money (DITM)	DOTM

a call option is out-of-the-money (OTM), and a put option is in-the-money (ITM); (3) If $0.97 \leq S/K \leq 1.03$, both a call option and a put option are at-the-money (ATM); (4) If $1.03 < S/K \leq 1.09$, a call option is ITM, and a put option is OTM; (5) If $S/K > 1.09$, a call option is DITM, and a put option is DOTM.

4.3.1 Estimation Results for Call Options

Calculated MER and RMSER results for call options 20 days prior to maturity are contained in Table 4. Firstly, looking at MER results, in Total the MSEG-n model value is -0.092, and the downward and upward bias of its option valuation estimated values is the lowest among all models. Next is the MSEG-tmodel, and we can see that these MSEG models are superior to other models in terms of their MER standards. Also, the values obtained with the MSG,

MSG-c, GARCH and BS models are extremely poor, and even without including switching, the EGARCH model's values are relatively good. According to RMSER standards, the rate of divergence between estimated values and market prices is lowest when the EGARCH-n model is used, followed by the EGARCH-t model. The MSEG and MSEG-c model's values are relatively good, but do not rival those of the normal EGARCH model. Also, even according to these standards, values obtained with the MSG, MSG-c, GARCH and BS models are extremely large. From the MER and RMSER results detailed above, we can see that for valuations of call options 20 days prior to maturity, before considering switching we must at least include an EGARCH model in formulas expressing fluctuations in volatility.

Table 5 shows results for 30 days prior to maturity. Looking at the MER results, in Total the MS-EG-t model value is -0.116, and as with results for 20 days prior to maturity, the downward and upward bias of its option valuation estimated values is the lowest among all models, followed by the MSEG-n model, while the MSEG model is superior to other models according to MER standards. For each moneyness, it can be considered that since the values of DOTM and OTM for the MSEG model were small, the total value became small.Also, values obtained with the MSG, MSG-c, GARCH and BS models are extremely poor. According to RMSER standards, the rate of divergence between estimated values and market prices is lowest when the MSEG-t model is used. Under either MER or RMSER, the superiority of the MSEG-t model was indicated. This result differs from that 20 days before expiration. The MSEG-t model is the only model that can capture bull and bear. There is a possibility that there was the effect of the prediction of bull and bear, because the period from expiration lengthened from 20 to 30 days. Accordingly, it is better to evaluate call option, whose period before expiration is long, with a model that can capture bull and bear.

4.3.2 Estimation Results for Put Options

Table 6 shows results for 20 days prior to maturity. Differing from the case of call option, values are nearly equal, but compared with the total values, the MSEG-c-n model indicated -0.384 for MER and 0.528 for RMSER, which are the smallest under respective standards. 30 days before the expiration shown in Table 7, MER was the smallest in the MSEG-t and MSEG-c-t models, and RMSER was the smallest in the MSEG-c-t model. Like the case of 20 days before expiration, values do not vary significantly among models.

For 30 days before in the call option, the performance of the MSEG model including bull and bear was the best. However, in the case of put option, values are slightly better in the MSEG-c model in which only volatility undergoes switching, regardless of the period before expiration. Accordingly, it considered unnecessary to consider the bull and bear of underlying asset prices in the evaluation of put option.

4.3.3 Estimation Results by Period

Tables 8 to 11 show MER and RMSER results sorted by year, for the period from 2007 to 2009. The results of call option in 2008 indicate that option evaluation is quite overpricing in the MSG, MSG-c, GARCH, and BS models for both 20 and 30 days before expiration. On the other hand, the values in the MSEG, MSEG-c, and EGARCH models in 2008 are nearly equal to those in other years.

Figure 6 shows the variation in volatility in the EGARCH-t and GARCH-t models from 2007. Due to Lehman's fall in Sep. 2008, volatility increased steeply, and the value of the GARCH-t model is much larger than that of the EGARCH-t model. This is considered to cause the difference in option evaluation. Namely, it can concluded that since the volatility in the GARCH-t model was high, call option was overestimated in the MSG, MSG-c, and GARCH models. Meanwhile, put option did not show the effect of the variation in volatility due to Lehman's fall.

Table 4: Estimation results for call options (20 days prior to maturity)

MER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
DOTM	−0.098	−0.233	−0.316	−0.292	1.335	2.015	241
OTM	−0.146	−0.041	−0.188	−0.194	0.319	0.342	74
ATM	−0.115	−0.076	−0.132	−0.123	0.027	0.023	64
ITM	−0.074	−0.040	−0.079	−0.072	−0.001	−0.008	48
DITM	−0.032	−0.008	−0.030	−0.026	0.012	0.002	97
Total	−0.092	−0.127	−0.201	−0.188	0.665	0.978	524

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
DOTM	1.361	1.522	3.079	3.209	−0.538	−0.457	2.378	241
OTM	0.402	0.383	0.408	0.386	−0.167	−0.165	0.111	74
ATM	0.045	0.043	0.069	0.053	−0.118	−0.116	−0.067	64
ITM	0.006	0.007	0.015	0.012	−0.079	−0.070	−0.055	48
DITM	0.009	0.010	0.016	0.015	−0.032	−0.026	−0.016	97
Total	0.690	0.762	1.487	1.541	−0.299	−0.259	1.093	524

RMSER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
DOTM	1.021	0.804	0.797	0.906	2.544	3.985	241
OTM	0.304	0.351	0.309	0.318	0.709	0.724	74
ATM	0.164	0.241	0.172	0.169	0.155	0.148	64
ITM	0.105	0.145	0.108	0.104	0.090	0.089	48
DITM	0.060	0.095	0.060	0.058	0.055	0.053	97
Total	0.705	0.570	0.558	0.630	1.747	2.717	524

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
DOTM	2.541	2.988	8.255	7.495	0.758	0.776	7.164	241
OTM	0.812	0.759	0.741	0.743	0.303	0.300	0.655	74
ATM	0.165	0.155	0.184	0.164	0.169	0.165	0.250	64
ITM	0.091	0.090	0.095	0.092	0.114	0.107	0.119	48
DITM	0.054	0.054	0.057	0.056	0.064	0.060	0.059	97
Total	1.751	2.047	5.606	5.091	0.532	0.543	4.866	524

Table 5: Estimation results for call options（30 days prior to maturity）

MER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
DOTM	−0.257	−0.289	−0.463	−0.338	1.524	2.193	260
OTM	−0.110	−0.022	−0.167	−0.158	0.230	0.289	77
ATM	−0.092	−0.073	−0.125	−0.117	0.018	0.047	66
ITM	−0.039	−0.025	−0.063	−0.054	0.008	0.023	42
DITM	−0.011	0.013	−0.027	−0.021	0.023	0.031	50
Total	−0.169	−0.166	−0.294	−0.224	0.841	1.208	495

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
DOTM	1.544	1.916	3.021	4.143	−0.489	−0.434	3.358	260
OTM	0.370	0.359	0.405	0.409	−0.114	−0.123	0.066	77
ATM	0.078	0.078	0.095	0.097	−0.111	−0.103	−0.033	66
ITM	0.045	0.047	0.059	0.061	−0.063	−0.049	−0.026	42
DITM	0.040	0.042	0.046	0.050	−0.030	−0.019	0.006	50
Total	0.887	1.081	1.672	2.263	−0.298	−0.267	1.768	495

RMSER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
DOTM	0.926	0.714	0.762	0.833	3.551	4.209	260
OTM	0.350	0.373	0.336	0.333	0.522	0.568	77
ATM	0.168	0.215	0.178	0.174	0.148	0.156	66
ITM	0.092	0.148	0.099	0.096	0.090	0.098	42
DITM	0.061	0.098	0.064	0.062	0.063	0.067	50
Total	0.689	0.546	0.573	0.622	2.583	3.060	495

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
DOTM	2.658	3.187	6.620	8.714	0.814	0.819	9.471	260
OTM	0.655	0.624	0.714	0.685	0.366	0.331	0.552	77
ATM	0.171	0.167	0.206	0.189	0.183	0.165	0.224	66
ITM	0.097	0.099	0.113	0.116	0.101	0.091	0.143	42
DITM	0.065	0.068	0.077	0.082	0.068	0.061	0.099	50
Total	1.945	2.324	4.807	6.322	0.612	0.612	6.868	495

Table 6: Estimation results for put options (20 days prior to maturity)

MER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
DOTM	−0.712	−0.690	−0.672	−0.709	−0.858	−0.833	291
OTM	−0.348	−0.308	−0.314	−0.333	−0.448	−0.435	54
ATM	−0.162	−0.132	−0.138	−0.149	−0.204	−0.198	64
ITM	−0.061	−0.054	−0.041	−0.047	−0.074	−0.076	67
DITM	−0.013	0.005	0.002	−0.001	−0.006	−0.016	108
Total	−0.414	−0.392	−0.384	−0.406	−0.501	−0.489	584

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
DOTM	−0.864	−0.872	−0.864	−0.842	−0.773	−0.755	−0.850	291
OTM	−0.483	−0.490	−0.494	−0.477	−0.330	−0.354	−0.410	54
ATM	−0.243	−0.240	−0.237	−0.224	−0.142	−0.164	−0.127	64
ITM	−0.108	−0.105	−0.105	−0.091	−0.050	−0.059	−0.010	67
DITM	−0.032	−0.030	−0.028	−0.020	−0.005	−0.007	0.014	108
Total	−0.520	−0.524	−0.519	−0.503	−0.438	−0.435	−0.474	584

RMSER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
DOTM	0.757	0.747	0.725	0.756	0.876	0.857	291
OTM	0.393	0.377	0.363	0.382	0.485	0.478	54
ATM	0.203	0.218	0.186	0.194	0.252	0.251	64
ITM	0.090	0.139	0.081	0.083	0.102	0.106	67
DITM	0.040	0.056	0.038	0.038	0.042	0.049	108
Total	0.553	0.547	0.528	0.551	0.642	0.629	584

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
DOTM	0.880	0.890	0.885	0.868	0.812	0.793	0.898	291
OTM	0.514	0.524	0.533	0.523	0.378	0.395	0.545	54
ATM	0.283	0.282	0.276	0.272	0.193	0.205	0.261	64
ITM	0.129	0.126	0.125	0.118	0.090	0.092	0.105	67
DITM	0.053	0.053	0.055	0.051	0.041	0.040	0.054	108
Total	0.649	0.656	0.654	0.641	0.589	0.577	0.662	584

Table 7: Estimation results for put options (30 days prior to maturity)

MER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
DOTM	−0.691	−0.650	−0.660	−0.643	−0.836	−0.832	307
OTM	−0.282	−0.242	−0.253	−0.272	−0.388	−0.428	51
ATM	−0.147	−0.116	−0.124	−0.140	−0.179	−0.212	65
ITM	−0.059	−0.057	−0.047	−0.056	−0.078	−0.098	56
DITM	−0.014	−0.030	−0.011	−0.014	−0.026	−0.035	60
Total	−0.446	−0.416	−0.421	−0.416	−0.545	−0.554	539

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
DOTM	−0.858	−0.849	−0.845	−0.793	−0.714	−0.698	−0.810	307
OTM	−0.477	−0.475	−0.474	−0.436	−0.264	−0.289	−0.352	51
ATM	−0.259	−0.258	−0.266	−0.232	−0.131	−0.151	−0.106	65
ITM	−0.141	−0.137	−0.142	−0.117	−0.056	−0.067	−0.003	56
DITM	−0.064	−0.061	−0.062	−0.048	−0.017	−0.022	0.017	60
Total	−0.587	−0.580	−0.580	−0.538	−0.455	−0.453	−0.506	539

RMSER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
DOTM	0.728	0.701	0.704	0.687	0.859	0.855	307
OTM	0.317	0.309	0.293	0.306	0.421	0.454	51
ATM	0.198	0.226	0.185	0.193	0.227	0.248	65
ITM	0.106	0.151	0.100	0.104	0.118	0.130	56
DITM	0.056	0.087	0.054	0.055	0.066	0.073	60
Total	0.563	0.546	0.544	0.533	0.667	0.668	539

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
DOTM	0.874	0.866	0.865	0.821	0.752	0.732	0.868	307
OTM	0.496	0.498	0.502	0.475	0.308	0.321	0.470	51
ATM	0.287	0.288	0.301	0.278	0.200	0.203	0.242	65
ITM	0.162	0.161	0.172	0.157	0.107	0.110	0.130	56
DITM	0.087	0.086	0.094	0.087	0.058	0.058	0.084	60
Total	0.687	0.681	0.682	0.646	0.581	0.567	0.678	539

Table 8: Estimation results for call options（by year, 20 days prior to maturity）

MER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
2007	0.004	−0.259	−0.154	−0.219	0.552	0.491	101
2008	0.116	−0.054	−0.059	0.018	1.185	1.974	203
2009	−0.329	−0.135	−0.354	−0.364	0.236	0.282	220
all periods	−0.092	−0.127	−0.201	−0.188	0.665	0.978	524

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
2007	0.607	0.440	0.655	0.526	−0.216	−0.232	−0.040	101
2008	1.208	1.489	3.211	3.351	−0.237	−0.146	2.977	203
2009	0.251	0.239	0.277	0.336	−0.394	−0.376	−0.125	220
all periods	0.690	0.762	1.487	1.541	−0.299	−0.259	1.093	524

RMSER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
2007	0.588	0.457	0.480	0.485	1.459	1.264	101
2008	0.939	0.677	0.654	0.795	2.427	4.134	203
2009	0.461	0.506	0.491	0.504	0.926	1.037	220
all periods	0.705	0.570	0.558	0.630	1.747	2.717	524

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
2007	1.594	1.235	1.531	1.272	0.502	0.499	1.081	101
2008	2.417	3.043	8.893	8.075	0.530	0.573	7.761	203
2009	0.864	0.861	0.896	0.904	0.547	0.534	0.519	220
all periods	1.751	2.047	5.606	5.091	0.532	0.543	4.866	524

Table 9: Estimation results for call options（by year, 30 days prior to maturity）

MER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
2007	0.069	−0.133	−0.114	−0.044	0.930	1.332	101
2008	−0.059	−0.241	−0.243	−0.132	1.568	2.035	206
2009	−0.417	−0.102	−0.447	−0.422	−0.002	0.237	188
all periods	−0.169	−0.166	−0.294	−0.224	0.841	1.208	495

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
2007	1.410	1.468	1.838	1.812	0.005	−0.046	0.408	101
2008	1.266	1.615	2.849	4.082	−0.325	−0.235	4.162	206
2009	0.190	0.287	0.294	0.512	−0.431	−0.421	−0.124	188
all periods	0.887	1.081	1.672	2.263	−0.298	−0.267	1.768	495

RMSER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
2007	0.678	0.539	0.514	0.563	1.935	2.435	101
2008	0.801	0.552	0.599	0.704	3.751	4.373	206
2009	0.548	0.543	0.574	0.553	0.370	0.712	188
all periods	0.689	0.546	0.573	0.622	2.583	3.060	495

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
2007	2.398	2.521	3.401	3.556	0.645	0.613	2.725	101
2008	2.452	3.069	7.023	9.412	0.634	0.656	10.460	206
2009	0.537	0.694	0.761	1.161	0.569	0.559	0.581	188
all periods	1.945	2.324	4.807	6.322	0.612	0.612	6.868	495

Table 10: Estimation results for put options (by year, 20 days prior to maturity)

MER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
2007	−0.438	−0.352	−0.368	−0.430	−0.520	−0.532	113
2008	−0.287	−0.240	−0.267	−0.280	−0.394	−0.379	216
2009	−0.511	−0.538	−0.490	−0.502	−0.583	−0.562	255
all periods	−0.414	−0.392	−0.384	−0.406	−0.501	−0.489	584

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
2007	−0.538	−0.559	−0.565	−0.566	−0.429	−0.439	−0.555	113
2008	−0.412	−0.409	−0.390	−0.357	−0.333	−0.321	−0.304	216
2009	−0.603	−0.606	−0.609	−0.597	−0.531	−0.530	−0.582	255
all periods	−0.520	−0.524	−0.519	−0.503	−0.438	−0.435	−0.474	584

RMSER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
2007	0.566	0.523	0.506	0.568	0.651	0.654	113
2008	0.430	0.421	0.421	0.432	0.556	0.537	216
2009	0.633	0.644	0.613	0.627	0.703	0.687	255
all periods	0.553	0.547	0.528	0.551	0.642	0.629	584

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
2007	0.655	0.678	0.687	0.688	0.579	0.580	0.706	113
2008	0.561	0.563	0.546	0.519	0.495	0.469	0.548	216
2009	0.713	0.717	0.719	0.709	0.663	0.654	0.727	255
all periods	0.649	0.656	0.654	0.641	0.589	0.577	0.662	584

Table 11: Estimation results for put options (by year, 30 days prior to maturity)

MER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
2007	−0.369	−0.241	−0.300	−0.298	−0.521	−0.585	110
2008	−0.380	−0.349	−0.371	−0.356	−0.465	−0.467	195
2009	−0.536	−0.555	−0.519	−0.522	−0.623	−0.612	234
all periods	−0.446	−0.416	−0.421	−0.416	−0.545	−0.554	539

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
2007	−0.590	−0.601	−0.595	−0.571	−0.322	−0.337	−0.619	110
2008	−0.513	−0.494	−0.491	−0.420	−0.433	−0.416	−0.337	195
2009	−0.647	−0.643	−0.647	−0.621	−0.536	−0.537	−0.594	234
all periods	−0.587	−0.580	−0.580	−0.538	−0.455	−0.453	−0.506	539

RMSER

	MSEG		MSEG-c		MSG		m
	n	t	n	t	n	t	
2007	0.477	0.413	0.425	0.398	0.642	0.677	110
2008	0.494	0.466	0.485	0.466	0.604	0.602	195
2009	0.648	0.652	0.633	0.631	0.726	0.713	234
all periods	0.563	0.546	0.544	0.533	0.667	0.668	539

	MSG-c		GARCH		EGARCH		BS	m
	n	t	n	t	n	t		
2007	0.675	0.686	0.686	0.663	0.460	0.446	0.749	110
2008	0.630	0.613	0.613	0.552	0.548	0.526	0.574	195
2009	0.736	0.732	0.734	0.708	0.655	0.645	0.722	234
all periods	0.687	0.681	0.682	0.646	0.581	0.567	0.678	539

5 Conclusions and Future Study Themes

This study focused on the bulls and bears of underlying asset price fluctuations, and evaluated Nikkei 225 option prices using MS-EGARCH models, in order to verify the effectiveness of the MS-EGARCH models in the Nikkei 225 Options market. The main findings of this study are summarized below.

1. The bull and bear of underlying asset prices cannot be captured with any models except the MSEG-t model.

2. For the call options of 20 days before expiration, the MSEG-n model is selected under MER, and the EGARCH-n model is selected under RMSER. When the period before

expiration is 30 days, the performance of the MSEG-t model is the best in either case. Accordingly, for call options, whose period before expiration is long, it is better to adopt a model that can capture bull and bear for evaluation.

3. On the other hand, for put options, values are slightly better in the MSEG-c model in which only volatility undergoes switching, regardless of the period before expiration. Accordingly, it is unnecessary to consider the bull and bear of underlying asset prices.

4. In the phase of the increase in volatility just after Lehman's fall, call option evaluation in the MSG, MSG-c, GARCH, and BS models is quite overpricing, but the performance of the MSEG, MSEG-c, and EGARCH models is extremely excellent.

5. Formulation based on asymmetry between rates of return of underlying asset prices and volatility, and the assumption that expected rates of return, the distribution of the error term, and volatility follow Markov-switching processes, are both of crucial importance in evaluating option prices.

The future study subjects include the following three:

1. Make a comparison with the option prices and performance based on the stochastic volatility model, which is another representative volatility-changing model.

2. Conduct formulation, taking into account risk premium in the process of the underlying asset return rate, rather than assuming risk neutrality of investors.

3. Analyze the option prices in the volatility-changing model in detail. Study particularly implied volatility and volatility smile, etc.

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