1. Introduction

The advancement of technologies has been considered to be one of the most important factors in economic growth. When there are rapid technological advances, some of the existing capital stock becomes obsolete. The growth rate of GDP might fall since a faster pace of technological progress increases the rate of capital obsolescence.

Bresnahan and Trajtenberg (1995) introduce the concept of general purpose technologies (GPTs) in their analysis of technological advance and growth. They define GPTs as having the following three main characteristics: pervasiveness, continual innovations, and innovational complementarities. First, GPTs have the characteristic of pervasiveness in that they would be used in many sectors of the economy at the same time. Second, GPTs would induce continual innovations in various sectors. Third, GPTs are complementary in that the productivity of R&D in many sectors increases as a new GPT becomes available.

Howitt (1998) and Aghion and Howitt (1998) examine the effects of a new GPT on the growth rate of an economy in a creative-destruction model à la Aghion and Howitt (1992). These authors do not, however, consider fiscal policy in their model.

Effects of taxation and government spending on economic growth have been studied extensively in the economic growth literature (Barro (1990), Chamley (1986), Rebelo (1991), among others). These studies are, however, not based on settings in which innovation is the engine of long-run economic growth.

The purpose of this paper is to expand a model of Schumpeterian creative-destruction to account for fiscal policy and to analyze the effects of technological advances and fiscal policy on the growth rate of output. In particular, we analyze the effects of fiscal policy on the short-run and the long-run rate of growth when there are rapid technological progresses. We address this issue in two growth models. The first model is one with exogenous technological advances. The second model is an endogenous R&D model based on the Schumpeterian creative-destruction model à la Aghion and Howitt (1992, 1998). In the second model, technological advances are endogenized as quality improvements by research.
and development activities.

The rest of the paper is organized as follows. In Section 2, we analyze the effects of fiscal policy on the growth rate in a growth model with exogenous technological change. In Section 3, we analyze the effects of fiscal policy on the growth rate in the model with endogenous R&D. We extend the Schumpeterian creative-destruction model of Aghion and Howitt (1992, 1998) by considering government expenditures. In both models, we first examine the case of a constant saving rate. We then analyze the effects of fiscal policy on the growth rate in the setting where there is a representative household’s intertemporal utility maximization. Section 4 concludes.

2. Model with Exogenous Technological Change

2.1 Basic Setup

In this section, we describe the basic model with exogenous technological change. Let \( Y_t \) denote output, \( K_t \) the aggregate stock of capital, and \( L_t \) labor supply. Suppose that the production function is given by

\[
Y_t = A_t K_t^\alpha L_t^{1-\alpha},
\]

where \( A_t \) describes technology at time \( t \). We assume \( 0 < \alpha < 1 \).

Let \( g \) denote the rate of technological advance which is exogenously given. We assume that the growth rate of the labor force is \( n \), that is, \( L_t = L_0 e^{nt} \).

Existing capital becomes obsolete because of technological progress such as a new GPT. Let \( \beta \) be the rate of obsolescence caused by the arrival of a new GPT. Specifically we assume that \( \beta \) depends on \( g \), that is, \( \beta = \beta (g) \).

Let \( G_t \) denote the flow of government services. For example, we may think of \( G_t \) as public infrastructure expenditure providing a public good that is not subject to congestion. We assume that \( G_t = \tau Y_t \), where \( \tau \) is a proportional tax rate. Since public infrastructure can be thought of as a complementary factor in private production sectors, technological progress in private industries may also be enhanced by government spending on public infrastructure. Thus we suppose that government expenditures have an effect on the technological advances. In particular, we assume that

\[
A_t = B_t^{\alpha - \tau} = \left( e^{\frac{G_t}{Y_t}} \right)^{1-\alpha} \quad \text{and} \quad g' > 0.
\]

2.2 Effects of Fiscal Policy on Growth

As a reference, we will first analyze the case where there are no government
expenditures. Let us consider the case of a constant saving rate $s$. Then the resource constraint is

$$\dot{K}_t = sY_t - \delta K_t - \beta(g)K_t,$$

where $\delta$ denotes the rate of capital depreciation. Let $k_t = \frac{K_t}{B_tL_t}$ denote capital per efficiency unit of labor. Then the growth rate of capital is

$$\frac{\dot{K}_t}{K_t} = s\frac{Y_t}{K_t} - \delta - \beta(g) = sB_t^{1-\alpha} K_t^{\alpha - 1} L_t^{1-\alpha} - \delta - \beta(g).$$

Let $\dot{k}_t$ denote capital per efficiency unit of labor. Then the growth rate of capital is

$$\frac{\dot{k}_t}{k_t} = \frac{sB_t^{1-\alpha} K_t^{\alpha - 1} L_t^{1-\alpha} - \delta - \beta(g)}{sk_t^{\alpha - 1}} = \delta + \beta(g) + g + n.$$

Note that we have

$$\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - \frac{\dot{B}_t}{B_t} - \frac{\dot{L}_t}{L_t} - \frac{\dot{K}_t}{K_t} - g - n.$$
If \( 1 - \alpha - \alpha \beta'(g) < 0 \), then an increase in \( g \) causes a decrease in the growth rate of per capita output; the growth rate of per capita output would then decrease owing to the arrival of the new GPT. This result is due to Aghion and Homitt (1998).

Next we analyze the case where there are government expenditures \( G_t \). Recall that \( G_t = \tau Y_t \). We have also assumed that when we consider government spending, \( B_t = e^{g(\tau)\tau} \). The change in capital stock is then given by

\[
\dot{K}_t = Y_t - C_t - G_t - \delta K_t - \beta(g(\tau))K_t.
\]

With a constant saving rate \( s \), the resource constraint is given by

\[
\dot{K}_t = (1 - \tau)sY_t - \delta K_t - \beta(g(\tau))K_t.
\]

It follows that

\[
\frac{\dot{K}_t}{K_t} = (1 - \tau)s k_t^{\alpha-1} - \delta - \beta(g(\tau)).
\]

The growth rate of per capita output is thus

\[
\frac{\dot{\hat{y}}}{\hat{y}} = \alpha \frac{\dot{k}}{k} + g(\tau) = \alpha \left[ (1 - \tau)s k^{\alpha-1} - \{ \delta + \beta(g(\tau)) + g(\tau) + n \} \right] + g(\tau).
\]

In the long run, we have \( \frac{\dot{\hat{y}}}{\hat{y}} = g(\tau) \). In the short run, given \( k_t \), by (2) we have

\[
\frac{\partial}{\partial \tau} \left( \frac{\dot{\hat{y}}}{\hat{y}} \right) = g'(\tau) \left[ 1 - \alpha \{ 1 + \beta'(g(\tau)) \} \right].
\]

We therefore obtain:

**Proposition** If \( 1 - \alpha \{ 1 + \beta'(g(\tau)) \} < 0 \), then \( \frac{\partial}{\partial \tau} \left( \frac{\dot{\hat{y}}}{\hat{y}} \right) < 0 \).

Thus an increase in the tax rate will reduce the short-run growth rate if the rate of capital obsolescence due to faster technological advances is sufficiently large.

Next we consider a representative consumer's utility maximization. Suppose that the representative consumer's utility function is given by
where $1/\theta$ is the elasticity of intertemporal substitution of consumption.

We assume that the representative consumer maximizes her intertemporal utility, which is given by

$$U = \int_0^\infty u(c)e^{-\rho t} dt = \int_0^\infty \left[ \frac{c^{1-\theta} - 1}{1-\theta} \right] e^{-\rho t} dt,$$

where $\rho$ is the discount rate.

The resource constraint of the economy is

$$\dot{K}_t = Y_t - C_t - G_t - \delta K_t - \beta(g(\tau))K_t.$$  \hspace{1cm} (3)

Then the current value Hamiltonian is given by

$$H(c,k,\lambda) = \frac{c^{1-\theta} - 1}{1-\theta} + \lambda \left[ (1-\tau)Y - c - \left\{ \delta + \beta(g(\tau)) + g(\tau) + n \right\} k \right],$$

where $\lambda$ is a costate variable.

The necessary conditions are then

$$\frac{\partial H}{\partial c} = c^{-\theta} - \lambda = 0$$

and

$$\dot{\lambda} = \rho \lambda - \frac{\partial H}{\partial k}$$

$$= \left\{ \rho + \delta + \beta(g(\tau)) + g(\tau) + n - \alpha(1-\tau)k^{\alpha-1} \right\} \lambda.$$ 

The transversality condition is $\lim\limits_{t\to\infty} e^{-\rho t} \cdot \lambda k_t = 0$.

Thus we obtain
The steady state equilibrium values of consumption and of the stock of capital are obtained from (3) and (4).

Suppose that a new GPT becomes available. Then in the long-run, we obtain
\[ \frac{\partial}{\partial \tau} \left( \frac{\hat{y}}{\hat{y}} \right) = g'(\tau) > 0. \]
In the short run, given \( k_t \), we have
\[ \frac{\partial}{\partial \tau} \left( \frac{\hat{y}}{\hat{y}} \right) = -\alpha k^{\alpha-1} + g'(\tau) \left[ 1 - \alpha \{1 + \beta'(g)\} \right] - \frac{\alpha}{k} \cdot \frac{\partial c}{\partial g} \cdot g'(\tau). \]
From (4), we obtain \( \frac{\partial c}{\partial g} < 0 \). Suppose that \( 1 - \alpha \{1 + \beta'(g)\} < 0 \). Then it is possible that
\[ \frac{\partial}{\partial \tau} \left( \frac{\hat{y}}{\hat{y}} \right) > 0 \] holds. This is in contrast to the case of a fixed saving rate above.

3. Endogenous R&D

3.1 R&D and Obsolescence

In the previous section, we have analyzed the effects of fiscal policy on the growth rate of output in a setting where there is exogenous technological change. In this section, we analyze a setting where technological progress is endogenous. The model in this section extends the basic model in Aghion and Howitt (1992) to the case where there are government expenditures.

Suppose that the introduction of a new GPT causes an increase in technological advance. We assume that old capital becomes obsolete because of technological progress caused by the new GPT.

Let \( Y_{it} = A_{it} \cdot x_{it}^{a} \), where \( A_{it} \) is a productivity parameter, \( Y_{it} \) output, and \( x_{it} \) an intermediate product of sector \( i \) at time \( t \). Let \( Y_t \) denote gross output at time \( t \). The production function for final output is given by
\[ Y_t = L_t \int_{0}^{t} Y_u du = L_t \int_{0}^{t} A_y \cdot x_{u}^{a} du, \]
where \( L_t \) is labor input (for simplicity, it is normalized to \( L = 1 \)).

We assume that the final good sector is perfectly competitive. Then the inverse demand function for intermediate goods is \( P_{it} = \alpha A_{it} x_{it}^{\alpha - 1} \), where \( P_{it} \) is the price of intermediate good \( i \).

We model R&D as follows. Let \( m_t \) denote the productivity-adjusted amount of input used in research, that is, \( m_t = \frac{M_t}{A_{t \text{max}}} \), where \( A_{t \text{max}} \) denotes the leading edge technology. Note that \( A_{t \text{max}} \) is the maximal value obtainable by the productivity parameters in the economy. Then the flow of innovations per unit of time, that is, the Poisson arrival rate of innovations, is \( \nu_t = \mu m_t \), where \( \mu \) is a parameter representing the productivity of R&D.

We assume that growth in the leading edge parameter \( A_{t \text{max}} \) comes from knowledge spillovers of innovations. Specifically, the rate of growth of technology is

\[
g_A = \frac{\dot{A}_{t \text{max}}}{A_{t \text{max}}} = \mu m_t \varphi, \tag{5}
\]

where \( \varphi \) is the magnitude of innovation.

We assume that \( \varphi \) depends on the ratio of government expenditures to national income, \( \varphi = \varphi \left( \frac{G}{Y} \right) \). This assumption captures the idea that government expenditures on infrastructure would possibly increase the opportunity of private innovative activities and hence have effects on the magnitude of innovation.

### 3.2 Innovations with Capital

Final output is allocated among consumption, government spending, R&D expenditures, and investment. So final output is \( Y_t = C_t + G_t + M_t + I_t \), where \( I_t \) denotes investment at time \( t \).

We assume that intermediate sectors are monopolistically competitive, whereas, we suppose that the final output sector is perfectly competitive. Then the marginal product is given by \( \alpha A_{it} x_{it}^{\alpha - 1} \). Thus the inverse demand function for intermediate goods is

\[
p_{it} = \alpha A_{it} x_{it}^{\alpha - 1}.
\]

We assume that each intermediate product is produced using capital according to the following production function:
\(x_t^* = \frac{K_{it}}{A_{it}}\), where \(K_{it}\) is the amount of capital allocated to the production of intermediate product \(i\). Then \(x_{it}\) is efficiency unit of capital. Let \(\xi_t\) denote the cost of capital. Then the profit for an intermediate sector \(i\) is

\[
\pi_{it} = p_{it}x_{it} - \xi_t K_{it} = A_{it} \alpha x_{it}^{\alpha-1} - \xi_t A_{it} x_{it}.
\]

Maximizing this profit yields

\[
x_{it} = \left(\frac{\alpha^2}{\xi_t}\right)^{\frac{1}{\alpha-1}} = \left(\frac{\xi_t}{\alpha^2}\right)^{\frac{1}{\alpha-1}}.
\]

Thus all intermediate sectors will produce the same quantity, that is, \(x_{it} = x_t\) for all \(i\). It follows that each monopolist in intermediate products obtains a flow of profits, that is, given by

\[
\pi_{it} = A_{it} \pi_t = A_{it} \alpha (1-\alpha) x_t^\alpha.
\]

Let \(A_t = \int_0^\infty A_{it} \, di\) be the average productivity parameter across all sectors at time \(t\). Then the aggregate capital stock is

\[
K_t = \int_0^\infty K_{it} \, di = \int_0^\infty x_{it} A_{it} \, di = \int_0^\infty x_t A_{it} \, di = x_t \int_0^\infty A_{it} \, di = x_t A_t.
\]

Hence the common output of all intermediate products is \(x_t = \frac{K_t}{A_t}\).

Let \(k_t = \frac{K_t}{A_t}\), which expresses the capital stock per efficiency unit. Then the profits for each monopolist are \(\pi_{it} = A_{it} \alpha (1-\alpha) k_t^\alpha\). The expected value of an innovation at time \(t\) is given by

\[
V_t = \int_0^\infty \exp\left[-\int_\varepsilon^\infty \left(r_\varepsilon + \psi\right) \, d\varepsilon\right] \pi_{it} \, dz,
\]

where \(r_\varepsilon\) is an instantaneous interest rate at time \(\varepsilon\). The instantaneous discount rate
is the rate of interest plus the rate of creative destruction. Then expected value of an innovation at time \( t \) is

\[
V_t = \frac{\alpha (1-\alpha) A_t^{\text{max}} k_t^\alpha}{r_t + \mu m_t}.
\]

Equating the marginal cost and the marginal benefit of R&D yields \( A_t^{\text{max}} = \mu V_t \).

Hence the research arbitrage equation becomes

\[
1 = \frac{\mu \alpha (1-\alpha) k_t^\alpha}{r_t + \mu m_t}.
\]

### 3.3 Effects of Fiscal Policy

We now proceed to analyze the effects of fiscal policies on the rate of growth both in the short run and in the long run. Let \( G_t \) denote the flow of government services. Note again that \( G_t \) is taken to be a public good not subject to congestion. We assume that \( G_t = \tau Y_t \), where \( \tau \) is a proportional tax rate.

We first analyze the case of a constant saving rate \( s \). The cost of capital is \( \xi_t = r_t + \delta + \psi_t \). We have \( r_t + \psi_t = \xi_t - \delta = \alpha^2 k_t^{\alpha-1} - \delta \).

The change in the stock of capital is

\[
\dot{K} = Y - C - M - G - \delta K_t - \mu m_t K_t = (1-\tau)s Y - M - \delta K_t - \mu m_t K_t.
\]

Then we have

\[
\dot{k}_t = (1-\tau) s k_t^\alpha - m(1+\varphi) - \left\{ \delta + \psi_t + g_s \right\} k_t
\]

\[
= (1-\tau) s k_t^\alpha - m(1+\varphi) - \left\{ \delta + \psi_t + \psi_s \varphi \right\} k_t.
\]

The research arbitrage equation is given by

\[
1 = \mu \cdot \frac{(1-\tau)\alpha (1-\alpha) k_t^\alpha}{\alpha^2 k_t^{\alpha-1} - \delta + \mu m}.
\]

Note that we have
Then the rate of growth of per capita output is

$$\frac{\dot{y}_t}{y_t} = \alpha \frac{\dot{k}_{t}}{k_t} + \frac{\dot{A}_t}{A_t}.$$  

By (5) and (6), we obtain

$$\frac{\dot{y}_t}{y_t} = \alpha \left\{ (1 - \tau) s k_{t}^{\alpha - 1} - m(1 + \phi) - \delta - \psi_{t} - \psi_{t} \phi \right\} + \mu m_{t} \phi.$$  

In the long run, we have $\frac{\dot{y}_t}{y_t} = \mu m_{t} \phi$. From (7), we may show that $\frac{\partial m_{t}}{\partial \tau} < 0$. Thus we conclude that

$$\frac{\partial}{\partial \tau} \left\{ \frac{\dot{y}_t}{y_t} \right\} = \mu \left[ \frac{\partial m_{t}}{\partial \tau} + m \phi'(\tau) \right] \geq 0.$$  

In the short run, given $k_{t}$, we have

$$\frac{\partial}{\partial \tau} \left\{ \frac{\dot{y}_t}{y_t} \right\} = -\alpha s k_{t}^{\alpha - 1} + \alpha m_{t} \phi' \left( \tau \right) \left[ \mu - \alpha (1 + \mu) \right] + \frac{\partial m_{t}}{\partial \tau} \left[ \mu \phi - \alpha (1 + \mu) (1 + \phi) \right].$$  

From (7), we may show that $\frac{\partial m_{t}}{\partial \tau} < 0$. Hence, given $k_{t}$, we obtain $\frac{\partial}{\partial \tau} \left( \frac{\dot{y}_t}{y_t} \right) \geq 0$.

We have assumed thus far that $\phi$ depends on the ratio of government expenditures to national income. Instead, we could assume that $\mu$ depends on the ratio of government expenditures to national income. Then we have

$$\frac{\partial}{\partial \tau} \left\{ \frac{\dot{y}_t}{y_t} \right\} = -\alpha s k_{t}^{\alpha - 1} + \left[ \phi - \alpha (1 + \phi) \right] m_{t} \mu' \left( \tau \right) + \frac{\partial m_{t}}{\partial \tau} \left[ \alpha (1 + \phi) + \mu \{ \phi - \alpha (1 + \phi) \} \right].$$  

Hence we obtain $\frac{\partial}{\partial \tau} \left( \frac{\dot{y}_t}{y_t} \right) \geq 0$.

Next we consider a representative consumer’s utility maximization. Suppose again that the representative consumer’s utility function is given by

$$Y_t = \int_0^x x^\alpha A_u du = x^\alpha \int_0^A A_u du = x^\alpha A_t = A_t k_t^\alpha.$$
\begin{align*}
u(c) &= \frac{c^{1-\theta} - 1}{1 - \theta},
\end{align*}

where \( \frac{1}{\theta} \) is the elasticity of intertemporal substitution of consumption. Let \( \rho \) be the discount rate. We assume that the representative consumer maximizes her intertemporal utility subject to the following budget constraint.

\begin{align*}
\dot{k} &= (1 - \tau) y - c - m(1 + \varphi) - \{ \delta + \psi + g_A \} k.
\end{align*}

The rate of interest will be \( r_i = \rho + \theta \mu m, \varphi \). Then the problem is to maximize

\begin{align*}
U &= \int_0^\infty e^{-\rho t} \left[ \frac{c^{1-\theta} - 1}{1 - \theta} \right] dt,
\end{align*}

subject to the differential equation for capital intensity per sector,

\begin{align*}
\dot{k}_i &= (1 - \tau) k_i^\alpha - c - m(1 + \varphi) - \{ \delta + \psi + g_A \} k_i.
\end{align*}

The current value Hamiltonian is

\begin{align*}
H(c, k, \lambda) &= \frac{c^{1-\theta} - 1}{1 - \theta} + \lambda \left[ (1 - \tau) k^\alpha - c - m(1 + \varphi) - \{ \delta + \psi + g_A \} k \right],
\end{align*}

where \( \lambda \) is a costate variable.

The necessary conditions are

\begin{align*}
\frac{\partial H}{\partial c} &= c^{-\theta} - \lambda = 0.
\end{align*}

and

\begin{align*}
\dot{\lambda} &= \rho \lambda - \frac{\partial H}{\partial k} = \left\{ (\rho - (1 - \tau) \alpha k^{\alpha - 1} + \delta + \mu \mu m + \mu m \varphi \} \lambda. \right.
\end{align*}

The transversality condition is \( \lim_{t \to \infty} e^{-\rho t} \cdot \lambda k = 0 \).

Thus we have
\[ \frac{\dot{c}}{c} = \frac{1}{\theta} \left[ (1-\tau)\alpha k^{\sigma-1} - \{\delta + \rho + \mu m + \mu \varphi \} \right] \]  \hspace{1cm} (8)

and

\[ \frac{\dot{y}}{y} = \alpha \frac{k}{k} + \frac{\dot{A}}{A} = \alpha \left[ (1-\tau)\alpha k^{\sigma-1} - \frac{c}{\theta} \frac{m(1+\varphi)}{k} - \{\delta + \psi + g_A\} \right] + g_A \]

The research arbitrage equation is given by

\[ 1 = \mu \frac{(1-\tau)\alpha(1-\alpha)k^\alpha}{\rho + \theta \mu m \varphi + \mu m} \cdot \] \hspace{1cm} (9)

In the long-run, we have \( \frac{\partial}{\partial \tau} \left( \frac{\dot{y}}{y} \right) = \mu \left[ \frac{\partial m}{\partial \tau} \varphi + m \varphi' (\tau) \right]. \) From (9), we may show that \( \frac{\partial m}{\partial \tau} \) is negative. Hence we obtain \( \frac{\partial}{\partial \tau} \left( \frac{\dot{y}}{y} \right) > 0. \)

In the short run, given \( k_t \), we have

\[ \frac{\partial}{\partial \tau} \left( \frac{\dot{y}}{y} \right) = -\alpha^2 k^{\sigma-1} - \frac{\alpha}{k} \frac{\partial c}{\partial \tau} - \frac{\alpha m}{k} \varphi' (\tau) - \alpha \mu m \varphi' (\tau) \] \[ - \frac{\alpha \partial m}{k \partial \tau} (1+\varphi) \]
\[ = -\alpha^2 k^{\sigma-1} - \frac{\alpha}{k} \frac{\partial c}{\partial \tau} - \frac{\alpha m}{k} \varphi' (\tau) + (1-\alpha) \mu m \varphi' (\tau) - \frac{\alpha \partial m}{k \partial \tau} (1+\varphi). \]

By (8) and (9), we have \( \frac{\partial c}{\partial \tau} < 0 \) and \( \frac{\partial m}{\partial \tau} < 0. \) Hence, given \( k_t \), we obtain \( \frac{\partial}{\partial \tau} \left( \frac{\dot{y}}{y} \right) > 0. \)

Therefore it is possible that whereas an increase in government spending financed by an increase in the tax rate raises the long-run growth rate, it decreases the short-run growth rate.

4. Conclusion

Technological progress has been considered as one of the most important factors in economic growth. When new technologies are available, old technologies will generally become obsolete. In this paper, we have addressed the question of how the introduction of a new technology affects the growth rate of output and what effects government expenditures
have on the growth rate of output.

We have first analyzed the effects of fiscal policies on the growth rate in a neoclassical growth model with exogenous technological progresses. Moreover we have extended the Schumpeterian creative destruction model of Aghion and Howitt (1992, 1998) by considering government expenditures to examine the effects of the arrival of a new GPT and fiscal policies on the growth rate of output.

We have shown that if technological advances are large enough, then it would be possible that the growth rate might be negative. We have also shown that fiscal policy can be effective in that it would increase the long run growth rate, but could reduce the short run growth rate through capital obsolescence due to the creative destruction effect of R & D enhanced by government spending.

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