Authority and Conjectures in Organizations

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1. Introduction

Over the past three decades, corporate governance mechanisms have been a topic of growing interest in the organizational economics literature (see for instance, Grossman and Hart (1986), Hart and Moore (1990), and Baker et al. (1999)). In particular, the agency approach has been extensively explored in the related literature on corporate governance. For an authoritative survey on corporate governance, the reader is referred to Tirole (2001) among others.

This paper studies the allocation of authority in a hierarchical organization with a principal and an agent in an environment where complete contracts are not possible. Specifically, we study the principal-agent model of a contract game in which the principal and the agent choose one of profitable projects under an incomplete contract setting and examine equilibria with consistent conjectural variations for the contract game.

This paper is most closely related to an interesting paper by Aghion and Tirole (1997). Aghion and Tirole (1997) emphasize the distinction between formal authority and real authority and study the allocation of authority in a hierarchical organization. They find that the allocation of authority has significant effects on the choices of profitable projects. Rajan and Zingales (1998) argue that power comes from control over access to organizations' critical resources. Aghion et al. (2002) distinguish between transferable controls and contractible controls and examine the possibility of transferable controls in organizations. Dessein (2002) explores the role of power in organizations and in particular, examines how the allocation of authority affects the informational advantage of the agent.

This paper is also related to the literature on incomplete contracts. There are pioneering contributions to the incomplete contract theory by Grossman and Hart (1986), Aghion and Bolton (1992), and Hart and Moore (1999), among others. See also Hart (1995) and Tirole (1999) for important reviews on the theory of incomplete contracts.

The literature on corporate governance, however, has focused on Nash equilibrium.

Nash equilibrium assumes that players have zero conjectural variations. In this paper, we explicitly consider parties' non-zero conjectures and equilibrium with consistent conjectures as an equilibrium concept. Focusing on the concept of conjectural variations, we note that one particular feature of Nash equilibrium is that each player has zero conjectural variations with respect to the reactions of other players. If each player has non-zero conjectural variations, the outcome of competition among players may differ from that of Nash equilibrium. Thus, our primary focus is on non-zero conjectural variations in the contracting game. For a comprehensive study on conjectural variations in oligopoly, see for example, Takenaka and Kobayashi (2020).

The main results of this paper are as follows. Under the principal's formal authority, the values of consistent conjectural variations at equilibrium are negative. As a result, we obtain a more competitive equilibrium outcome compared to that of Nash equilibrium. Similarly, under the agent's formal authority, the values of consistent conjectural variations at equilibrium are negative.

The structure of this paper is as follows. In Section 2, we present the model. In Section 3, we consider two scenarios of the allocation of authority, we derive an equilibrium with consistent conjectures of the contract game for each of the two scenarios. Then we compare them with Nash equilibria. Section 3 also presents the results obtained by comparative statics analysis. In the final section, we state our conclusions.

2. The Model

In this section, we set up the model. Our model follows Aghion and Tirole (1997). Let us consider a contracting game in which a hierarchical organization with a principal and an agent determine a project. There are n projects indexed by $i = 1, \cdots, n$. The organization chooses one project from the n profitableprojects.

The principal is risk-neutral and has formal authority at the outset of the contracting game. Formal authority refers to the authority to make the organizational decisions, whereas real authority alludes to the party who has an effective control over decisions. We assume that the principal and the agent can get information on projects by exerting efforts.

When the principal has authority to choose a project, the chosen project gives benefit B to the principal and βb to the agent. We denote by parameter β the probability that the principal's preferred project is also the agent's preferred project.

When the principal delegates the decision to choose a project to the agent, the chosen project gives αB to the principal and b to the agent. We denote by parameter α the probability that the agent's preferred project is also the principal's preferred project. We assume that the parameters α and β satisfy

$$0 < \alpha \leq 1 \text{ and } 0 < \beta \leq 1 .$$

Before a contract is signed, the principal and the agent have no information about their own benefits. They can get information about the profitability of projects by exerting efforts. The principal obtains full information by incurring a cost $g_P(\mu)$ with probability μ. The agent obtains full information by incurring a cost $g_A(\rho)$ with probability ρ.

The timing of the contract game between the principal and the agent is as follows:

At stage 1, a contract between the principal and the agent is signed and the allocation of formal authority to one party, either the principal or the agent, is specified. At stage 2, they exert efforts to obtaining information. At stage 3, one of the projects is chosen.

Then, depending on the allocation of authority, we have the following payoffs of the parties.

Under the principal's formal authority, the principal's payoff F_P is

$$F_P = \mu B + (1-\mu)\ \rho\alpha B - g_P(\mu).$$

The agent's payoff F_A is

$$F_A = \mu\beta b + (1-\mu)\ \rho b - g_A(\rho).$$

Under the delegation of authority, the principal's payoff G_P is

$$G_P = \rho\alpha B + (1-\rho)\mu B - g_P(\mu).$$

The agent's payoff G_A is

$$G_A = \rho b + (1-\rho)\mu\beta b - g_A(\rho).$$

To derive equilibria explicitly, we assume that the cost functions take the following forms:

$$g_P(\mu) = \frac{c\mu^2}{2} \quad \text{and} \quad g_A(\rho) = \frac{c\rho^2}{2}, c > 0.$$

The principal chooses an effort level μ to maximize her payoff, while the agent chooses an effort level ρ to maximize his payoff. We consider non-zero conjectural variations in the game. Particularly, we derive equilibrium with consistent conjectural variations. Let ϕ_μ denote the principal's conjecture about the agent's response when the principal changes her strategies. Let ϕ_ρ denote the agent's conjecture about the principal's response when the agent changes his strategies.

3. Equilibrium and Consistent Conjectures

In this section, we examine equilibrium with consistent conjectural variations. The related literature has focused on considering Nash equilibrium. Nash conjectures assume that the values of conjectural variations are zero. In this paper, we consider consistent conjectural variations (CCVs).

The conditions of consistency are given as follows:

Given the two reaction functions, $\mu = R_1(\rho)$ and $\rho = R_2(\mu)$, conjectural variations are consistent if the following conditions are satisfied:

$$\frac{dR_1(\mu)}{d\rho} = \phi_\mu \quad \text{and} \quad \frac{dR_2(\rho)}{d\mu} = \phi_\rho$$

3.1 Principal's formal authority

First, we consider the scenario in which the principal has formal authority on organizational decisions.

The principal's pay off F_P is

$$F_P = \mu B + (1-\mu)\ \rho\alpha B - \frac{c\mu^2}{2}.$$

The agent's payoff F_A is

$$F_A = \mu\beta b + (1-\mu)\ \rho b - \frac{c\rho^2}{2}.$$

The first order conditions of each party's maximization problem are

$$\frac{dF_P(\mu)}{d\mu} = B - \rho\alpha B + (1-\mu)\alpha B\phi_\mu - c\mu = 0$$

and

$$\frac{dF_A(\rho)}{d\rho} = \phi_\rho\beta b - \phi_\rho\rho b + (1-\mu)b - c\rho = 0.$$

Thus, we have the following reaction functions:

$$\mu = \frac{B+\alpha\beta\phi_\mu}{\alpha B\phi_\mu + c} - \frac{\alpha B}{\alpha B\phi_\mu + c}\ \rho$$

and

$$\rho = \frac{b+\beta b\phi_\rho}{b\phi_\rho + c} - \frac{b}{b\phi_\rho + c}\ \mu.$$

For conjectural variations to be consistent at equilibrium, we must have

$$\phi_\rho = -\frac{\alpha B}{\alpha B\phi_\mu + c}$$

and

$$\phi_\mu = -\frac{b}{b\phi_\rho + c}.$$

Thus, we have

$$\phi_\mu = \frac{b(\alpha B\phi_\mu + c)}{\alpha bB - c(\alpha B\phi_\mu + c)}.$$

It follows that we have the following consistent conjectural variations.

$$\phi^*_\mu = -\frac{c \mp \sqrt{c^2 - 4\alpha bB}}{2\alpha B} \tag{1}$$

and

$$\phi^*_\rho = -\frac{c \mp \sqrt{c^2 - 4\alpha bB}}{2b}. \tag{2}$$

Therefore, the consistent conjectural variations at equilibrium are negative:

$$\phi^*_\mu < 0 \quad \text{and} \quad \phi^*_\rho < 0.$$

Thus, we have the following result.

Proposition 1.

The equilibrium efforts (μ_I^*, ρ_I^*) are

$$\mu_I^* = \frac{det\left(\begin{bmatrix} B+\alpha B\phi_\mu^* & \alpha B \\ b+bB\phi_\rho^* & b\phi_\rho^*+c \end{bmatrix}\right)}{det\left(\begin{bmatrix} \alpha B\phi_\mu^*+c & \alpha B \\ b & b\phi_\rho^*+c \end{bmatrix}\right)}$$

and

$$\rho_I^* = \frac{det\left(\begin{bmatrix} \alpha B\phi_\mu^*+c & B+\alpha B\phi_\mu^* \\ b & b+bB\phi_\rho^* \end{bmatrix}\right)}{det\left(\begin{bmatrix} \alpha B\phi_\mu^*+c & \alpha B \\ b & b\phi_\rho^*+c \end{bmatrix}\right)},$$

where ϕ_μ^* and ϕ_ρ^* are given by (1) and (2) respectively.

3.2 Agent's formal authority

Next, we consider the scenario in which the principal delegates the decision rights to the agent.

The principal's payoff G_P is

$$G_P = \rho\alpha B + (1-\rho)\mu B - g_P(\mu).$$

The agent's payoff G_A is

$$G_A = \rho b + (1-\rho)\mu\beta b - g_A(\rho).$$

Then, the first order conditions are

$$\frac{dG_P(\mu)}{d\mu} = \Psi_\mu \alpha B - \Psi_\mu \mu B + (1-\rho)B - c\mu = 0$$

and

$$\frac{dG_A(\rho)}{d\rho} = b - \mu\beta b + (1-\rho)\Psi_\rho \beta b - c\rho = 0.$$

Thus, we have the following reaction functions:

$$\mu = \frac{B(\Psi_\mu \alpha + 1)}{\Psi_\mu B + c} - \frac{B}{\Psi_\mu B + c}\rho$$

and

$$\rho = \frac{b+\beta b\Psi_\rho}{\beta b\Psi_\rho + c} - \frac{b\beta}{\beta b\Psi_\rho + c}\mu.$$

For conjectural variations to be consistent at equilibrium, we must have

$$\Psi_\rho = -\frac{B}{B\Psi_\mu + c}$$

and

$$\Psi_{\mu} = -\frac{\beta b}{\beta b \Psi_{\rho} + c}.$$

We have the following consistent conjectural variations at equilibrium.

$$\phi^*_{\mu} = -\frac{c \mp \sqrt{c^2 - 4b\beta B}}{2B} < 0 \quad (3)$$

and

$$\phi^*_{\rho} = -\frac{c \mp \sqrt{c^2 - 4b\beta B}}{2\beta b} < 0. \quad (4)$$

Thus, we have the following result.

Proposition 2.

The equilibrium efforts (μ^*_D, ρ^*_D) are

$$\mu^*_D = \frac{det\left(\begin{bmatrix} B(\Psi^*_{\mu}\alpha + 1) & B \\ b + \beta b \phi^*_{\rho} & \beta b \Psi^*_{\rho} + c \end{bmatrix}\right)}{det\left(\begin{bmatrix} \Psi^*_{\mu} B + c & B \\ b\beta & \beta b \Psi^*_{\rho} + c \end{bmatrix}\right)}$$

and

$$\rho^*_D = \frac{det\left(\begin{bmatrix} \Psi^*_{\mu} B + c & B(\Psi^*_{\mu}\alpha + 1) \\ b\beta & b + \beta b \Psi^*_{\rho} \end{bmatrix}\right)}{det\left(\begin{bmatrix} \Psi^*_{\mu} B + c & B \\ b\beta & \beta b \Psi^*_{\rho} + c \end{bmatrix}\right)},$$

where Ψ^*_{μ} and Ψ^*_{ρ} are given by (3) and (4) respectively.

3.3 Nash equilibrium

Next, we compare the equilibrium with consistent conjectures with Nash equilibrium for each of the two regimes.

Under the principal's authority, we have

$$\mu = \frac{B - \rho\alpha B}{c}$$

and

$$\rho = \frac{(1-\mu)b}{c}.$$

Thus, the Nash equilibrium efforts are

$$\mu_I^N = \frac{B(\alpha b - c)}{\alpha Bb - c^2}$$

and

$$\rho_I^N = \frac{b(B - c)}{\alpha Bb - c^2}\,.$$

Under the agent's formal authority, we have the following reaction functions:

$$\mu = \frac{(1-\rho)B}{c}$$

and

$$\rho = \frac{(1-\beta\mu)b}{c}\,.$$

Thus, the Nash equilibrium efforts (μ_D^N, ρ_D^N) are

$$\mu_D^N = \frac{B(b - c)}{B\beta b - c^2}$$

and

$$\rho_D^N = \frac{b(\beta B - c)}{B\beta b - c^2}\,.$$

Note that by comparing with Nash equilibrium (μ^N, ρ^N), we get

$$\mu_I^* + \rho_I^* > \mu_I^N + \rho_I^N,$$

and

$$\mu_D^* + \rho_D^* > \mu_D^N + \rho_D^N.$$

3.4 Comparative statics analysis

We obtain the following results by implementing comparative statics analysis. Hereafter, for simplicity, we assume that ϕ_μ^*, ϕ_ρ^*, Ψ_μ^* and Ψ_ρ^* take the larger roots.

Corollary 1.

The values of consistent conjectural variations are increasing in cost. That is,

$$\frac{\partial \phi_\mu^*}{\partial c} > 0,\ \frac{\partial \phi_\rho^*}{\partial c} > 0,\ \frac{\partial \Psi_\mu^*}{\partial c} > 0, \text{ and } \frac{\partial \Psi_\rho^*}{\partial c} > 0.$$

Corollary 2.

The values of consistent conjectural variations are decreasing in the principal's benefit parameter B. That is,

$$\frac{\partial \phi^*_\rho}{\partial B} < 0 \text{ and } \frac{\partial \Psi^*_\rho}{\partial B} < 0.$$

Corollary 3.

The values of consistent conjectural variations are decreasing in the agent's benefit parameter b. That is,

$$\frac{\partial \phi^*_\mu}{\partial b} < 0 \text{ and } \frac{\partial \Psi^*_\mu}{\partial b} < 0.$$

4. Conclusion

This paper has examined equilibria with consistent conjectural variations in a model of the allocation of authority in an organization with a principal and an agent. We have shown that under the principal's formal authority, the values of consistent conjectural variations at the equilibrium are negative. It implies that the outcome is more competitive than that of Nash equilibrium. We have also shown that under the agent's formal authority, the values of consistent conjectural variations at the equilibrium are negative.

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