Public Pension Reform and Uncertainty

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1. Introduction

Compared to the populations of other developed countries, the population of Japan is ‘graying’ at a somewhat higher rate. In the year 2000, 17.4 percent of the Japanese population was over 65 years old. This figure is expected to climb to 22.5 percent by 2010, 29.6 percent by 2030, and 35.7 percent by 2050. Japan used to apply a complete Defined Benefit system for public pensions based on the pay-as-you-go system. The pay-as-you-go system becomes more vulnerable, however, as the composition of the elderly population rises. According to the Ministry of health, Labor and Welfare, the pension premium rate was expected to reach about 26 percent of monthly income (about 20% of annual income) by 2025.

The Japanese government reformed the public pension system in 2004 amid calls for a radical reform. The new system is said to adopt several features in common with the new pension system introduced in Sweden. The pension premium rate will be fixed at 18.3 percent of annual income after 2017. As of 2004 it was 13.934 percent. Benefits levels are adjusted according to social and economic conditions such as wages, longevity, and the number of workers. At the same time, however, the Japanese government guarantees that the pension benefit level will exceed 50 percent of the wage level of working generations1) (i.e., that a 50 percent replacement rate will be maintained). The pension premium rate could therefore move above 18.3 percent if social and economic conditions are exacerbated more severely than expected.

There are two main pension systems. One is the Defined Benefit pension system, the other is the Defined Contribution pension system. Each system has an advantage and a disadvantage. The advantage of Defined Benefit pension is that this system guarantees some amount, in other word replacement ratio, after retirement. Therefore, the volatility of assets or income after retirement is low. Moreover, Defined Benefit pension reduces the risk of increasing longevity on the benefit aspect because people can receive some amount before death. The disadvantage of this system is that the more the aging population increases, the more the working generation has to pay taxes (or premiums). Therefore, the rising longevity combined with low birth rate has a negative impact on the pension tax burden aspect. On the other hand, the advantage of Defined Contribution pension system is that this system is neutral to aging or a change of demographic structure. In other words, aging does not have a negative impact on the burden aspect under Defined Contribution pension system. The disadvantage of this system is that the volatility of assets or income after retirement is high. Moreover, Defined Contribution pension system is vulnerable to the risk of increasing longevity from the benefit aspect because people can not receive a benefit beyond their accumulated amount.

The purpose of this paper is to investigate the optimal level of replacement rate ratio in Japan by considering not only the risk of fluctuation of return but also of longevity.

2. Model description

We use the overlapping generation model to analyze the optimal level of replacement ratio. An individual lives a maximum of N periods and faces a possibility of death at each period. As we use the population projection published by the National Institute of Population and Social Security Research as demographic patterns[2]. An advantage of using those projected population is that simulation results are more realistic than in the case of assuming a population growth rate is constant. We assume than individual does not take into consideration his or her child. Thus there is no bequest motive. In addition, we assume that individual does not have any assets before he or she enters the economy.

2.1 Utility function

We consider a representative individual with the following utility function:

$$E\left[\sum_{j=1}^{N}\beta^{j-1}\pi_{j,t+j-1}u\left(c_{j,t+j-1}\right)\right] \tag{1}$$

where E is the expectation operator, β^{j-1} is the time discount factor at age j, $c_{j,t+j-1}$ is the consumption at age j in year $t+j-1$, and t is generation. $\pi_{j,t+j-1}$ is a probability that individual is alive at age j conditional on being alive at age 1. Survival rates of an individual differ at each generation. According to our assumption of population projection for Japan, the later individual enters the economy, the higher will be his or her survival rates at each time[3].

The period utility function $u(c)$ is of the constant relative risk aversion class

$$u(c) = \begin{cases} c^{1-\gamma}/(1-\gamma) & \gamma > 1 \\ log(c) & \gamma = 1 \end{cases}$$

where γ is the coefficient of relative risk aversion. The more the coefficient of relative risk aversion is large, the more individual is risk averse. If γ equals 1, the period utility function is $log(c)$.

2. 2 Budget constraints

An individual earns wage income during working periods and receives pension benefits after retirement. The individual consumes at each period and saves the difference between accumulated assets and consumption. We assume that individual consumes all accumulated assets at age N if he or she lives to the maximum period N. Thus the individual does not save at age N. Wage income is zero after retirement and pension benefits is zero before retirement. Pension premiums (or taxes) are proportionally imposed on wage income. Individual budget constraint is as follows:

$$c_{j,t+j-1}+s_{j,t+j-1}=\left(1+r_{t+j-1}\right)s_{j-1,(t+j-1)-1}+\left(1-\tau_{t+j-1}\right)w_{j,t+j-1}+b_{j,t+j-1} \tag{2}$$

where $s_{j,t+j-1}$ is savings, $w_{j,t+j-1}$ is wage income, and $b_{j,t+j-1}$ is pension benefits at age j in year $t+j-1$. τ_{t+j-1} is pension premiums in year $t+j-1$ and r_{t+l-1} is return of assets at initial of period $t+j-1$. The equation (2) becomes $c_{j,t+j-1}+s_{j,t+j-1}=(1+r_{t+j-1})s_{j-1,(t+j-1)-1}+(1-\pi_{t+j-1})w_{j,t+j-1}$ if individual is in working periods. As we assume assets of individual are zero before appearance in economy, it is $c_{1,t}+s_{1,t}=(1-\pi_t)w_{1,t}$ at age 1. On the other hand, it becomes $c_{j,t+j-1}+s_{j,t+j-1}=(1+r_{t+j-1})s_{j-1,(t+j-1)-1}+b_{j,t+j-1}$ if individual has retired. It is $c_{N,t+N-1}=(1+r_{t+N-1})s_{N-1,(t+N-1)-1}+b_{N,t+N-1}$ at age N if individual lives up to the maximum period N.

In this model, a public pension system is presumed as the Defined Benefit pension system based on pay-as-you-go. Under this pension system, pension benefits are defined by the replacement ratio multiplied by the average wage of working person. Therefore, pension benefits are described below:

$$b_{j,j+t-1}=\kappa\overline{w}_{j+t-1}\ , \qquad R+1\le j\le N \tag{3}$$

where κ is replacement rate ratio, $R+1$ is retirement age, $\overline{w}_{j+t-1}$ is average wage of workers in year $j+t-1$. If individual is in working periods $(1\le j\le R)$, $b_{j,j+t-1}$ equals zero. We assume that the retirement age is fixed for all generations. Therefore, the later individual enters the economy, the more he or she takes the risk of increasing longevity because his or her mortality rate is low. For simplicity, the wage growth rate is assumed to be constant at g. Thus the wage at next period is $w_{j+1,j+t}=(1+g)w_{j,j+t-1}$. If the individual has retired $(R+1\le i\le N)$, $w_{j,j+t-1}$ equals zero. In addition, labor supply is assumed to be exogenous. As we emphasize an advantage of the Defined Benefits pension, we assume there is no risk of a wage fluctuation.

Another risk rises from the volatility of return of assets. We assume that return of assets is described by the following equations:

$$r_i=\mu+z_i \tag{4}$$

$$z_i=\rho z_{i-1}+\varepsilon \tag{5}$$

$$\varepsilon \sim N\left(0,\sigma^2\right) \tag{6}$$

where μ is the constant term and z_i is the permanent error term. For simplicity, this permanent error is assumed to follow AR(1) process that is either a unit-root or close to a unit-root to capture the persistence of return of assets over time. The term ρ controls the degree of persistence of previous error term. In addition, the transient error term ε is assumed to be normally distributed with zero mean and variance of σ^2.

2. 3 Individual decision problem

The decision problem of individual can be described as the dynamic programming problem. In this model, the state variable of generation t at age is expressed as $x_{j,j+t-1}=(s_{j,(j+t-1)-1}$, $z_{j+t-1})$ and the control variable is consumption $c_{j,j+t-1}$ or savings $s_{j,j+t-1}$. Let $V_{j,t+j-1}(x_{j,t+j-1})$ be the maximized value of the objective function of generation t at age j with the state variable $x_{j,j+t-1}$. $V_{j,t+j-1}(x_{j,t+j-1})$ is given as the solution to the following dynamic program.

$$V_{j,t+j-1}\left(x_{j.t+j-1}\right)=\max_{c_{j,t+j-1}}\left\{u\left(c_{j,t+j-1}\right)+\beta^{j-1}\frac{\pi_{j+1,t+j}}{\pi_{j,t+j-1}}E\left[V_{j+1,t+j}\left(x_{j+1.t+j}\right)\middle|z_{t+j-1}\right]\right\}$$
$$(i=1,\dots,N) \tag{7}$$

subject to equation (2) and $V_{j+1,t+j}\left(x_{j+1,t+j}\right)=0$

The value function at age N+1 is identically zero because death is certain beyond age $N(\pi_{N+1,t+N}=0)$. The decision rules and the value functions for each age $j=1,2,\cdots,N$ can be found by working the backward recursion from the last period of life[4].

2. 4 Government

A public pension system in this paper is described as the Defined Benefit pension system based on a pay-as-you-go. A characteristic of this system is that total benefits of retired persons and total taxes (or pension premium) of working persons are equal. Since total benefits is pension benefits multiplied by number of retire persons and total taxes is pension taxes multiplied by number of working persons, the budget balance is described as follows:

$$\sum_{j=R+1}^{N} b_{j,i}\times L_{j,i}=\sum_{j=1}^{R}\tau_i\times w_{j,i}\times L_{j,i} \tag{8}$$

where $L_{j,I}$ is population of age j agent in year i. Using equation (3) to substitute $b_{j,i}$ in equation (8), it exchange as follows:

$$\sum_{j=R+1}^{N}\kappa\overline{w}_i\times L_{j,i}=\sum_{j=1}^{R}\tau_i\times w_{j,i}\times L_{j,i} \tag{9}$$

The equation (9) implies that τ_i is determined if a replacement rate ratio κ settles. The degree of κ means the size of public pension because the degree of pension benefits and taxes (or premium) depend on κ.

The purpose of this paper is to find out what level of replacement ratio leads to the highest welfare of the individual. We can consider the optimal public pension size by finding that replacement ratio.

3. Simulation results

3. 1 Parameters setting

We explain how to select the parameters of the model. The parameters used by a benchmark simulation are listed in Table 1.

We regard one period of the model as 5 years. The time discount factor is set as 0.98. This value is the same as using in Feldstein and Ranguelova (2001). The coefficient of relative risk aversion is set as 2 in a benchmark case because it is plausible that this value would be less than 3 and probably 2 in Feldstein and Ranguelova (2001).

Table 1. Parameters

N = 16 (1 period = 5 years, nobody lives beyond 100 years old)
R+1 = 9 (Retirement age = 65 years)
β = 0.98
g = 2%
μ = 2%
ρ = 1
σ^2 =0.0 1
γ = 1, or 2, or 3
π : source is from life table in “Population Projection for Japan”(2002)

The demographic patterns follow the dates in the Population Projection for Japan (2002). Since one period of the model is assumed to be 5 years, we make an population projection in 5-year increments. Every 5 year represents an additional period. We use those dates taking the average of male and female population. In this model, we assume that individuals enter the economy at age 20 (model period 1) and die at age 100 (model period 17). Thus we ignore the population dates of age below 20 although the Population Projection for Japan shows these dates. In addition, retirement age is assumed to be age 65 (model period 9) and retirement age does not differ for all generations.

The survival rates follow the life table in the Population Projection for Japan. We also use the average dates of male and female survival rates. In Table 2, we find that the later individuals enter the economy, the higher the survival rates will be at every age. Thus, the later generations that enter the economy take more the risk of increasing longevity than generations that enter the economy at early age because retirement age does not differ for all generations. This means that the later generations gain more advantage of the Defined Benefits pension from the benefits aspect.

We assume that the wage growth rate g is 2 % and the expected return of assets μ is 2%. The coefficient of auto-regression ρ is assumed to be 1, and σ^2 is assumed to be 0.1.

Table 2. Survival Rates

	generation										
age	2000	5	10	15	20	25	30	35	40	45	50
0	1	1	1	1	1	1	1	1	1	1	1
5	0.99556	0.99602	0.99640	0.99671	0.99695	0.99716	0.99733	0.99747	0.99760	0.99770	0.99780
10	0.99494	0.99546	0.99590	0.99625	0.99653	0.99676	0.99695	0.99712	0.99725	0.99738	0.99748
15	0.99439	0.99493	0.99541	0.99579	0.99610	0.99635	0.99656	0.99674	0.99690	0.99703	0.99715
20	0.99283	0.99343	0.99400	0.99445	0.99482	0.99512	0.99538	0.99560	0.99579	0.99595	0.99609
25	0.99052	0.99125	0.99194	0.99248	0.99294	0.99331	0.99362	0.99390	0.99413	0.99433	0.99451
30	0.98806	0.98889	0.98970	0.99035	0.99089	0.99134	0.99171	0.99204	0.99232	0.99256	0.99278
35	0.98486	0.98587	0.98683	0.98761	0.98825	0.98879	0.98924	0.98964	0.98998	0.99027	0.99054
40	0.98047	0.98177	0.98294	0.98390	0.98469	0.98535	0.98592	0.98640	0.98683	0.98719	0.98752
45	0.97385	0.97553	0.97701	0.97822	0.97923	0.98008	0.98081	0.98144	0.98198	0.98246	0.98289
50	0.96333	0.96540	0.96734	0.96893	0.97026	0.97139	0.97235	0.97318	0.97391	0.97456	0.97512
55	0.94637	0.94919	0.95178	0.95393	0.95573	0.95726	0.95858	0.95972	0.96072	0.96160	0.96239
60	0.92190	0.92580	0.92941	0.93240	0.93492	0.93707	0.93892	0.94053	0.94194	0.94319	0.94430
65	0.88610	0.89156	0.89676	0.90109	0.90474	0.90786	0.91054	0.91287	0.91492	0.91674	0.91835
70	0.83146	0.83927	0.84696	0.85338	0.85880	0.86343	0.86741	0.87088	0.87393	0.87664	0.87904
75	0.75152	0.76303	0.77443	0.78397	0.79202	0.79891	0.80486	0.81003	0.81458	0.81861	0.82221
80	0.63335	0.64901	0.66538	0.67913	0.69082	0.70086	0.70954	0.71713	0.72381	0.72974	0.73503
85	0.46792	0.48657	0.50753	0.52540	0.54078	0.55411	0.56574	0.57597	0.58504	0.59312	0.60038
90	0.27758	0.29379	0.31500	0.33360	0.34995	0.36439	0.37720	0.38862	0.39885	0.40807	0.41641
95	0.11633	0.12581	0.14060	0.15412	0.16643	0.17763	0.18781	0.19709	0.20556	0.21332	0.22045
100	0.03071	0.03421	0.04058	0.04676	0.05267	0.05828	0.06356	0.06853	0.07319	0.07756	0.08166

3. 2 Results

First of all, we show welfare levels of the generation who enter the economy in 2000 under every replacement rate ratio. We assume that the coefficient of relative risk aversion γ is 2 in Figure 1. We find that the welfare level in the case where a replacement ratio κ is over zero is higher than the case where κ equals zero. However the welfare level decreases if κ is too high. This reason is as follows. In the range of low replacement ratio, the individual gains from the advantage of the Defined Benefits pension which covers the risk of assets fluctuation and this individual lives longer after retirement. However the disadvantage of the Defined Benefits pension, in which the expected return of this pension is lower than the expected interest rate, exceeds the advantage of this pension. The optimal replacement ratio of the generation 2000 is 0.45.

Next, we show the welfare levels of the generation 2000 in the case where the coefficient of relative risk aversion γ equals 1 and 3. Figure 2 shows the case where γ is 1. We find that the optimal replacement ratio is 0.32 in this case. This result is consistent with the fact that the individual is less (more) sensitive to the fluctuation of the consumption if the coefficient

Figure 1.

$(\sigma^2=0.01, \gamma=2)$

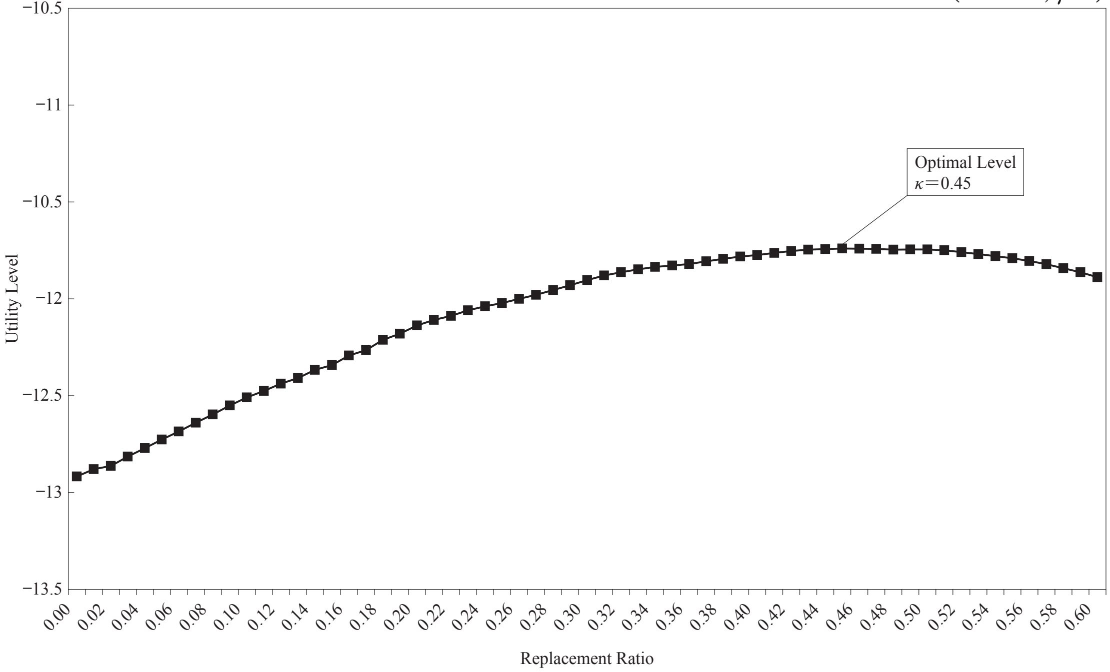


Figure 2.

$(\sigma^2=0.01, \gamma=1)$

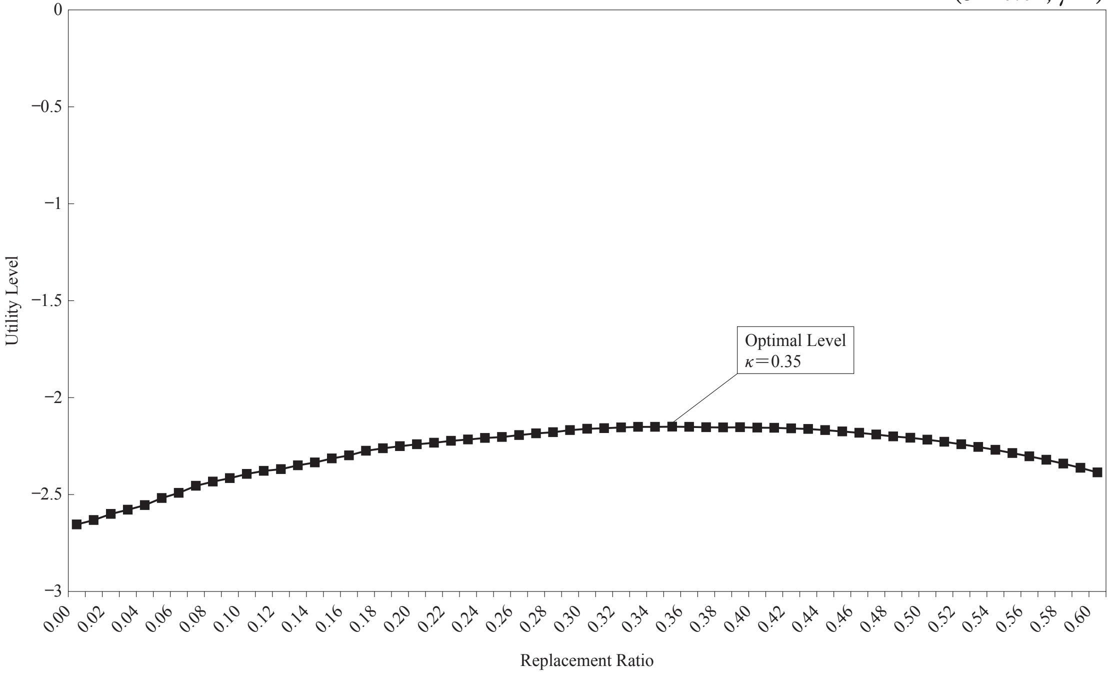
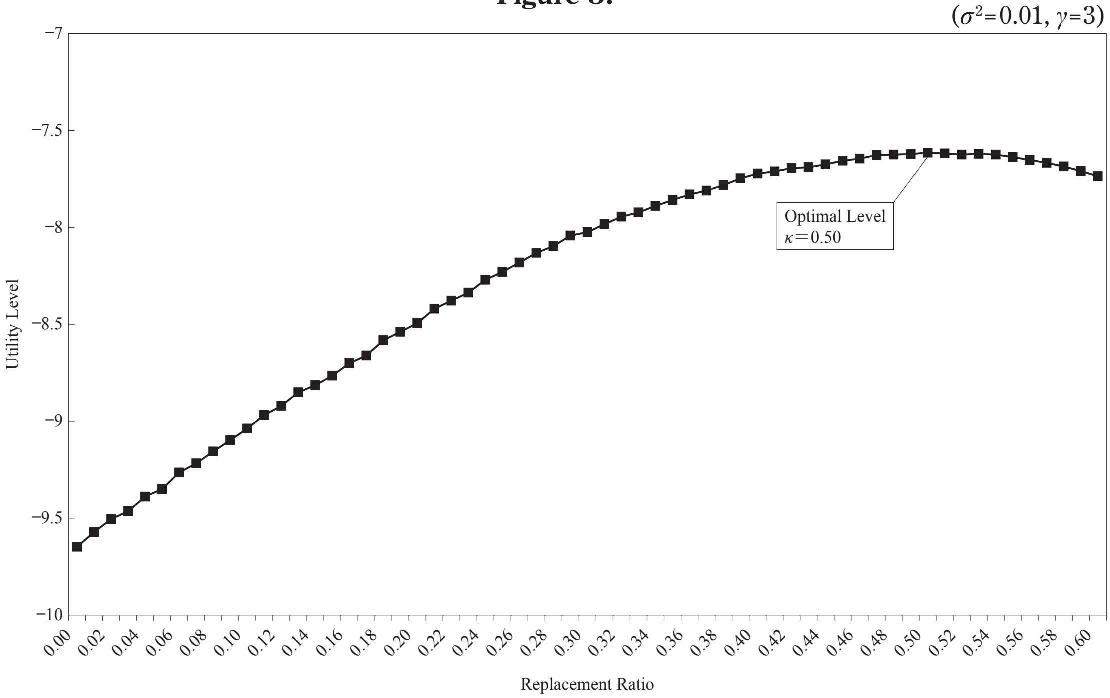


Figure 3.



of relative risk aversion is low (high). Therefore, the degree of the advantage of the Defined Benefits pension becomes small (large). In the case where γ is 3, the optimal replacement ratio is 5.0.

Next, table 3 shows welfare levels of all generations under every replacement ratio. We find that the optimal replacement ratio of all generations is below 5.0 in the case where γ is 1 and 2. In the case where γ equal to 3, the optimal level is also below 5.0 except for those who enter the economy in 2000.

Table 3. Simulation Result (δ^2=0.01, γ=1, All generations)

					generation						
κ	2000	5	10	15	20	25	30	35	40	45	50
0.05	−2.51848	−2.4502	−2.37658	−2.29189	−2.18033	−2.05602	−1.91777	−1.77557	−1.62618	−1.46602	−1.29315
0.06	−2.49129	−2.41664	−2.34994	−2.27721	−2.16496	−2.05089	−1.90916	−1.77115	−1.62065	−1.45829	−1.28441
0.07	−2.45521	−2.3938	−2.33304	−2.25094	−2.15623	−2.04076	−1.90203	−1.76213	−1.61692	−1.4525	−1.28063
0.08	−2.4333	−2.36419	−2.30744	−2.23533	−2.13692	−2.02985	−1.89653	−1.75747	−1.60638	−1.44747	−1.27039
0.09	−2.4159	−2.3513	−2.28643	−2.21285	−2.1195	−2.01385	−1.88817	−1.74813	−1.60256	−1.44446	−1.26662
0.10	−2.39375	−2.33837	−2.27386	−2.20359	−2.10342	−2.00337	−1.87598	−1.73983	−1.59349	−1.4334	−1.25807
0.11	−2.37828	−2.32415	−2.26525	−2.19418	−2.09807	−1.98896	−1.86497	−1.7334	−1.58754	−1.43464	−1.25341
0.12	−2.36903	−2.31024	−2.25788	−2.18662	−2.09149	−1.98724	−1.85701	−1.72578	−1.58982	−1.42604	−1.24737
0.13	−2.34966	−2.29914	−2.25497	−2.18466	−2.08532	−1.97745	−1.85385	−1.72747	−1.58464	−1.42598	−1.24476
0.14	−2.33431	−2.29199	−2.23935	−2.17771	−2.08715	−1.97824	−1.85254	−1.725	−1.58698	−1.43133	−1.24741
0.15	−2.31424	−2.27241	−2.23543	−2.17429	−2.08113	−1.97794	−1.85433	−1.72746	−1.5893	−1.42861	−1.25326
0.16	−2.29821	−2.25954	−2.22009	−2.16683	−2.08178	−1.97666	−1.85136	−1.72596	−1.59212	−1.43335	−1.25443
0.17	−2.27462	−2.24081	−2.20763	−2.16064	−2.08035	−1.98084	−1.85096	−1.7294	−1.5924	−1.4354	−1.25398
0.18	−2.26185	−2.23002	−2.1974	−2.15198	−2.07126	−1.97562	−1.85504	−1.7295	−1.59388	−1.43684	−1.26343
0.19	−2.25061	−2.21822	−2.18396	−2.14075	−2.0679	−1.9785	−1.85574	−1.7316	−1.60018	−1.44299	−1.26396
0.20	−2.24059	−2.21496	−2.181	−2.13707	−2.06659	−1.97337	−1.86056	−1.73689	−1.60123	−1.44604	−1.26748
0.21	−2.23273	−2.20615	−2.17746	−2.13253	−2.05871	−1.97062	−1.85714	−1.73834	−1.60362	−1.45194	−1.27401
0.22	−2.22337	−2.20302	−2.17665	−2.13453	−2.06114	−1.97512	−1.86334	−1.74861	−1.60715	−1.45279	−1.2749
0.23	−2.21638	−2.20303	−2.17913	−2.13798	−2.06443	−1.97539	−1.8663	−1.74997	−1.61675	−1.4609	−1.2823
0.24	−2.20801	−2.19488	−2.17674	−2.14308	−2.0708	−1.97934	−1.87015	−1.75549	−1.62566	−1.46772	−1.28792
0.25	−2.20378	−2.19136	−2.17624	−2.1449	−2.07596	−1.99086	−1.87985	−1.76423	−1.63593	−1.47709	−1.29432
0.26	−2.19369	−2.18436	−2.17582	−2.1456	−2.08326	−1.99907	−1.89242	−1.77973	−1.64919	−1.48914	−1.30514
0.27	−2.18444	−2.18152	−2.17473	−2.15023	−2.08797	−2.00494	−1.90426	−1.78934	−1.662	−1.50497	−1.31799
0.28	−2.17892	−2.17694	−2.17587	−2.15239	−2.09227	−2.01596	−1.91573	−1.80014	−1.67623	−1.51814	−1.33352
0.29	−2.16737	−2.17236	−2.17585	−2.15593	−2.10204	−2.02337	−1.92479	−1.81495	−1.68822	−1.53283	−1.3471
0.30	−2.1611	−2.16753	−2.17497	−2.15901	−2.1089	−2.03378	−1.93564	−1.82894	−1.70209	−1.54706	−1.3621
0.31	−2.15755	−2.1676	−2.17335	−2.16451	−2.11472	−2.04227	−1.94635	−1.84155	−1.71538	−1.56186	−1.37775
0.32	−2.15396	−2.16733	−2.17514	−2.17	−2.12267	−2.05028	−1.95743	−1.85291	−1.72998	−1.57734	−1.39145
0.33	−2.15094	−2.16649	−2.17911	−2.17656	−2.13099	−2.06272	−1.96701	−1.86418	−1.74499	−1.59125	−1.40534
0.34	−2.15148	−2.17142	−2.18413	−2.18429	−2.14057	−2.07392	−1.98247	−1.88138	−1.76026	−1.60609	−1.41919
0.35	−2.15045	−2.17388	−2.19178	−2.20536	−2.15208	−2.08717	−1.99959	−1.89926	−1.77946	−1.62535	−1.43872
0.36	−2.15147	−2.17992	−2.20293	−2.20536	−2.16787	−2.10216	−2.01711	−1.91603	−1.7986	−1.64368	−1.45681
0.37	−2.15283	−2.18491	−2.21124	−2.21721	−2.18291	−2.12042	−2.0346	−1.93695	−1.82051	−1.66593	−1.47795
0.38	−2.15372	−2.18934	−2.21938	−2.23056	−2.19851	−2.13757	−2.05619	−1.95884	−1.8442	−1.6893	−1.50113
0.39	−2.15326	−2.19089	−2.22723	−2.24371	−2.21397	−2.15522	−2.07589	−1.98129	−1.86821	−1.71434	−1.52679
0.40	−2.15531	−2.19847	−2.23704	−2.2551	−2.22798	−2.17227	−2.09559	−2.00455	−1.89232	−1.7386	−1.55041

κ	55	60	65	70	75	80	85	90	95	2100
0.05	−1.0991	−0.90046	−0.70151	−0.49718	−0.2978	−0.10135	0.107397	0.304934	0.502283	0.70243
0.06	−1.08762	−0.88602	−0.68768	−0.49569	−0.29317	−0.08619	0.114167	0.315705	0.516491	0.714978
0.07	−1.07799	−0.87678	−0.67752	−0.47801	−0.27583	−0.08148	0.119638	0.329748	0.527863	0.729351
0.08	−1.06988	−0.86776	−0.66333	−0.46658	−0.26676	−0.06557	0.134949	0.333837	0.536569	0.738554
0.09	−1.06266	−0.8592	−0.65507	−0.4574	−0.25081	−0.04948	0.151482	0.347619	0.552159	0.747242
0.10	−1.05163	−0.84979	−0.64657	−0.44297	−0.23733	−0.03951	0.156799	0.365749	0.562945	0.763716
0.11	−1.04578	−0.83962	−0.63279	−0.43938	−0.23075	−0.02702	0.173721	0.381985	0.574251	0.770651
0.12	−1.03966	−0.83641	−0.62745	−0.42395	−0.21523	−0.0141	0.19313	0.38933	0.587296	0.783841
0.13	−1.03935	−0.83106	−0.62552	−0.41618	−0.20644	−0.00156	0.198105	0.397132	0.599295	0.796173
0.14	−1.03936	−0.83141	−0.62049	−0.41345	−0.20402	−0.00052	0.202296	0.407363	0.603666	0.805244
0.15	−1.04399	−0.83235	−0.62191	−0.41504	−0.20792	0.000381	0.206903	0.408254	0.607292	0.802912
0.16	−1.04756	−0.83739	−0.62691	−0.41596	−0.20696	0.000275	0.205952	0.411798	0.608748	0.807711
0.17	−1.05159	−0.84044	−0.63039	−0.41539	−0.20893	0.000249	0.204066	0.409891	0.611978	0.816
0.18	−1.05738	−0.84203	−0.63123	−0.41976	−0.20817	−0.0017	0.206689	0.413932	0.615318	0.814568
0.19	−1.05619	−0.84542	−0.63057	−0.41754	−0.20861	0.001465	0.208646	0.413442	0.615715	0.813981
0.20	−1.05803	−0.84807	−0.6347	−0.4244	−0.21286	−0.00111	0.206166	0.412908	0.616435	0.817675
0.21	−1.06496	−0.85045	−0.6348	−0.42328	−0.21025	−0.00277	0.206174	0.415602	0.61545	0.816928
0.22	−1.06881	−0.85477	−0.64102	−0.42455	−0.20851	0.000321	0.207067	0.414523	0.615944	0.816855
0.23	−1.06986	−0.85557	−0.63872	−0.42186	−0.20993	−0.00014	0.208288	0.414259	0.614932	0.8186
0.24	−1.07412	−0.85808	−0.64001	−0.42397	−0.21241	−0.00276	0.20487	0.414041	0.616477	0.819201
0.25	−1.07816	−0.86159	−0.64504	−0.42895	−0.21584	−0.00587	0.201823	0.411643	0.616567	0.815762
0.26	−1.09081	−0.86956	−0.65343	−0.43793	−0.22224	−0.00905	0.198711	0.406497	0.611847	0.809572
0.27	−1.10282	−0.88202	−0.66206	−0.44819	−0.23208	−0.01714	0.19366	0.401214	0.606553	0.804989
0.28	−1.1179	−0.89631	−0.67696	−0.46043	−0.2421	−0.02674	0.185967	0.393428	0.597717	0.79876
0.29	−1.13217	−0.9084	−0.68815	−0.47065	−0.25381	−0.038	0.176961	0.385607	0.589897	0.790866
0.30	−1.14678	−0.92058	−0.69937	−0.4806	−0.26318	−0.04873	0.165732	0.377051	0.580244	0.784255
0.31	−1.15944	−0.9346	−0.71036	−0.49151	−0.27455	−0.05775	0.158552	0.367195	0.571683	0.774862
0.32	−1.17321	−0.94763	−0.7242	−0.5032	−0.284	−0.06705	0.148181	0.358422	0.563814	0.766745
0.33	−1.18759	−0.96087	−0.73681	−0.51514	−0.29546	−0.07636	0.1376	0.346343	0.555694	0.757645
0.34	−1.20222	−0.97545	−0.74882	−0.52689	−0.30493	−0.08848	0.126115	0.338477	0.54665	0.74737
0.35	−1.21867	−0.99084	−0.76229	−0.53819	−0.31755	−0.09995	0.116112	0.327742	0.536784	0.739053
0.36	−1.23522	−1.00607	−0.7796	−0.55428	−0.33337	−0.11408	0.103662	0.315946	0.524107	0.726796
0.37	−1.25714	−1.02534	−0.7968	−0.57144	−0.34816	−0.12807	0.088372	0.302066	0.51034	0.712555
0.38	−1.28059	−1.04595	−0.81784	−0.59083	−0.36652	−0.14544	0.071564	0.284911	0.494714	0.696408
0.39	−1.30428	−1.07083	−0.84075	−0.61211	−0.38746	−0.16605	0.052267	0.26813	0.476627	0.680462
0.40	−1.32826	−1.09391	−0.8629	−0.63609	−0.40975	−0.18636	0.03369	0.248396	0.459067	0.662062

Table 4. Simulation Result (δ^2=0.01, γ=2, All generations)

	generation																				
κ	2000	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	2100
0.20	–12.1385	–12.219	–12.2875	–12.3532	–12.3513	–12.3097	–12.2145	–12.1162	–12.0146	–11.8724	–11.7024	–11.4606	–11.2135	–10.9759	–10.7549	–10.5283	–10.3046	–10.087	–9.87876	–9.67729	–9.48406
0.21	–12.1101	–12.1994	–12.2621	–12.3189	–12.3229	–12.3081	–12.2122	–12.0948	–12.0006	–11.8651	–11.6924	–11.4487	–11.2079	–10.9713	–10.7406	–10.5116	–10.2904	–10.0742	–9.86526	–9.66594	–9.4762
0.22	–12.0898	–12.1825	–12.2429	–12.3012	–12.2951	–12.2881	–12.1978	–12.0878	–11.9949	–11.8606	–11.6792	–11.4406	–11.2001	–10.9584	–10.7297	–10.5035	–10.2783	–10.0627	–9.8533	–9.65373	–9.46147
0.23	–12.0612	–12.1623	–12.2318	–12.2785	–12.2794	–12.2668	–12.189	–12.0916	–11.9794	–11.846	–11.6665	–11.4304	–11.1919	–10.9518	–10.7196	–10.4918	–10.2733	–10.0561	–9.84949	–9.64337	–9.44859
0.24	–12.04	–12.1455	–12.2263	–12.275	–12.2769	–12.252	–12.1837	–12.082	–11.973	–11.8335	–11.656	–11.4233	–11.1794	–10.9425	–10.7098	–10.4811	–10.262	–10.043	–9.83608	–9.62942	–9.43821
0.25	–12.0231	–12.1294	–12.2151	–12.2727	–12.2697	–12.2457	–12.182	–12.0854	–11.9751	–11.8266	–11.6471	–11.4139	–11.169	–10.9332	–10.7019	–10.4758	–10.2513	–10.0331	–9.82389	–9.62396	–9.42752
0.26	–12.0014	–12.1056	–12.2043	–12.2653	–12.2757	–12.2372	–12.1707	–12.0863	–11.9727	–11.8311	–11.6465	–11.409	–11.1639	–10.9263	–10.6949	–10.4647	–10.2439	–10.025	–9.81699	–9.61622	–9.42624
0.27	–11.9805	–12.0837	–12.1829	–12.2637	–12.2702	–12.2322	–12.1664	–12.0849	–11.9811	–11.8327	–11.6467	–11.4063	–11.1613	–10.922	–10.6886	–10.4586	–10.2412	–10.0231	–9.81081	–9.61304	–9.4208
0.28	–11.9556	–12.0684	–12.1597	–12.2485	–12.2676	–12.2346	–12.1652	–12.0817	–11.9896	–11.8367	–11.6469	–11.4077	–11.1625	–10.9184	–10.6846	–10.4568	–10.2365	–10.0212	–9.808	–9.60896	–9.41884
0.29	–11.9311	–12.0471	–12.1415	–12.2333	–12.2595	–12.2349	–12.1649	–12.0814	–11.9925	–11.8411	–11.6518	–11.4085	–11.1625	–10.9199	–10.6843	–10.4568	–10.2345	–10.0187	–9.81058	–9.60763	–9.41441
0.30	–11.9045	–12.033	–12.1341	–12.2172	–12.2491	–12.2328	–12.1679	–12.0819	–11.9988	–11.8529	–11.657	–11.4128	–11.1632	–10.9219	–10.684	–10.4547	–10.2333	–10.0191	–9.80976	–9.60467	–9.41223
0.31	–11.881	–12.0169	–12.1264	–12.2037	–12.2366	–12.2278	–12.1697	–12.0855	–12.0009	–11.8581	–11.6641	–11.4107	–11.1629	–10.9192	–10.6852	–10.4562	–10.2319	–10.0172	–9.80887	–9.60428	–9.41095
0.32	–11.8647	–12.0041	–12.1175	–12.2072	–12.2343	–12.2302	–12.1694	–12.0852	–12.0061	–11.8602	–11.6724	–11.4138	–11.1655	–10.9174	–10.684	–10.4569	–10.2306	–10.0134	–9.80212	–9.59912	–9.40817
0.33	–11.8495	–11.9913	–12.114	–12.2039	–12.2341	–12.2235	–12.1716	–12.0885	–12.0088	–11.8663	–11.6769	–11.4203	–11.1671	–10.9163	–10.6826	–10.4548	–10.2321	–10.0112	–9.8009	–9.59721	–9.40525
0.34	–11.8369	–11.9833	–12.1055	–12.2026	–12.2396	–12.2313	–12.1735	–12.092	–12.0074	–11.8746	–11.6858	–11.4279	–11.1674	–10.9226	–10.6871	–10.4566	–10.2288	–10.0081	–9.79852	–9.59889	–9.40413
0.35	–11.83	–11.9694	–12.0969	–12.1993	–12.2337	–12.2323	–12.1812	–12.1039	–12.0187	–11.8787	–11.6911	–11.4376	–11.1758	–10.9292	–10.6872	–10.4542	–10.2297	–10.0075	–9.79921	–9.5963	–9.40107
0.36	–11.8216	–11.9634	–12.0927	–12.2015	–12.2397	–12.2402	–12.1898	–12.1122	–12.0333	–11.8895	–11.7007	–11.4423	–11.1852	–10.9335	–10.6882	–10.4588	–10.2326	–10.0124	–9.80095	–9.5962	–9.40441
0.37	–11.8077	–11.9611	–12.0904	–12.203	–12.2437	–12.2503	–12.2049	–12.1286	–12.0475	–11.9078	–11.7142	–11.4558	–11.1946	–10.9405	–10.6985	–10.4664	–10.2397	–10.0184	–9.80355	–9.6013	–9.40791
0.38	–11.7951	–11.9566	–12.0956	–12.2058	–12.255	–12.2562	–12.2155	–12.1485	–12.0656	–11.9221	–11.7297	–11.4709	–11.2087	–10.9546	–10.7124	–10.4757	–10.2459	–10.0251	–9.81354	–9.60905	–9.41735
0.39	–11.7835	–11.9531	–12.0985	–12.2094	–12.2664	–12.274	–12.2326	–12.1616	–12.0867	–11.9424	–11.7499	–11.4877	–11.2232	–10.9685	–10.721	–10.4844	–10.2571	–10.0354	–9.82268	–9.61703	–9.42401
0.40	–11.7755	–11.9447	–12.0929	–12.218	–12.2703	–12.2842	–12.2462	–12.1796	–12.1025	–11.9586	–11.7687	–11.504	–11.2414	–10.9826	–10.7343	–10.4975	–10.2678	–10.0449	–9.83189	–9.62647	–9.43317
0.41	–11.7647	–11.9356	–12.0913	–12.2179	–12.2763	–12.2923	–12.2609	–12.1949	–12.1222	–11.9784	–11.787	–11.5215	–11.253	–10.9964	–10.7505	–10.5096	–10.2782	–10.056	–9.84317	–9.63596	–9.44054
0.42	–11.7542	–11.9267	–12.0934	–12.2254	–12.2879	–12.3041	–12.2741	–12.2127	–12.1379	–11.9943	–11.8029	–11.5405	–11.2726	–11.011	–10.763	–10.5247	–10.2911	–10.0664	–9.85231	–9.64498	–9.44887
0.43	–11.7476	–11.9328	–12.0972	–12.2357	–12.3004	–12.3203	–12.2906	–12.2333	–12.1615	–12.0166	–11.8273	–11.5634	–11.2906	–11.0313	–10.7805	–10.5373	–10.305	–10.0787	–9.86071	–9.65401	–9.45979
0.44	–11.7449	–11.9368	–12.1052	–12.25	–12.319	–12.3414	–12.3128	–12.2579	–12.1882	–12.046	–11.855	–11.5876	–11.3143	–11.0519	–10.8018	–10.5555	–10.3211	–10.0937	–9.87502	–9.66573	–9.47001
0.45	–11.7423	–11.937	–12.1158	–12.2609	–12.3374	–12.3679	–12.3417	–12.2843	–12.2163	–12.077	–11.8866	–11.6196	–11.3444	–11.0782	–10.8245	–10.5782	–10.3414	–10.1113	–9.8916	–9.68287	–9.48409
0.46	–11.7424	–11.9434	–12.1235	–12.2772	–12.3608	–12.3899	–12.3664	–12.3126	–12.2489	–12.1115	–11.9208	–11.6544	–11.3745	–11.1077	–10.8519	–10.6054	–10.3646	–10.1346	–9.91085	–9.70067	–9.50222
0.47	–11.7434	–11.9483	–12.136	–12.2958	–12.3809	–12.4128	–12.3916	–12.3425	–12.2834	–12.1491	–11.9584	–11.6888	–11.4091	–11.14	–10.8817	–10.6307	–10.39	–10.1573	–9.93248	–9.72201	–9.52207
0.48	–11.7476	–11.9566	–12.151	–12.3136	–12.4003	–12.4375	–12.4216	–12.3747	–12.3232	–12.19	–11.9986	–11.7282	–11.445	–11.1742	–10.913	–10.6617	–10.4169	–10.182	–9.95606	–9.74319	–9.54417
0.49	–11.7464	–11.9621	–12.1608	–12.3329	–12.4265	–12.4665	–12.4555	–12.4142	–12.369	–12.2357	–12.0434	–11.7679	–11.4839	–11.21	–10.9466	–10.6921	–10.446	–10.2083	–9.98145	–9.76632	–9.56578
0.50	–11.7467	–11.9698	–12.1783	–12.3552	–12.4553	–12.5002	–12.4932	–12.4596	–12.4191	–12.2865	–12.0908	–11.8152	–11.528	–11.2514	–10.9864	–10.7273	–10.4791	–10.2384	–10.0096	–9.79445	–9.59289

Table 5. Simulation Result

($\delta^2=0.01$, $\gamma=3$, All generations)

	generation																				
κ	2000	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	2100
0.25	−8.22962	−8.25038	−8.24583	−8.21006	−8.11458	−8.02543	−7.88246	−7.70601	−7.51505	−7.30617	−7.05899	−6.76596	−6.48072	−6.21615	−5.95311	−5.69411	−5.45646	−5.22608	−5.01498	−4.81519	−4.62699
0.26	−8.1816	−8.21001	−8.21085	−8.19543	−8.10058	−7.99197	−7.86995	−7.70282	−7.50886	−7.28646	−7.04381	−6.75297	−6.47433	−6.20081	−5.9386	−5.68723	−5.44178	−5.21576	−5.00129	−4.7995	−4.61302
0.27	−8.13138	−8.15708	−8.18271	−8.16504	−8.08742	−7.9747	−7.8436	−7.68281	−7.49548	−7.27915	−7.02811	−6.73996	−6.4601	−6.18866	−5.92883	−5.67668	−5.43011	−5.20146	−4.98939	−4.78528	−4.5998
0.28	−8.09715	−8.10921	−8.13742	−8.13652	−8.06809	−7.95803	−7.80368	−7.66779	−7.48575	−7.26935	−7.01146	−6.72641	−6.4445	−6.17452	−5.91309	−5.6638	−5.41804	−5.18878	−4.97634	−4.7782	−4.58745
0.29	−8.04274	−8.07793	−8.10269	−8.1115	−8.0502	−7.94787	−7.78575	−7.65487	−7.47575	−7.25875	−7.0036	−6.71163	−6.42376	−6.15913	−5.89813	−5.65041	−5.40616	−5.17655	−4.96547	−4.76906	−4.579
0.30	−8.02465	−8.05597	−8.06938	−8.08539	−8.02761	−7.93444	−7.78028	−7.63977	−7.47548	−7.24674	−6.99648	−6.70542	−6.41244	−6.14695	−5.88224	−5.63838	−5.39889	−5.16273	−4.94645	−4.74765	−4.56253
0.31	−7.98336	−8.02887	−8.06121	−8.06481	−8.00771	−7.91864	−7.77385	−7.61935	−7.4729	−7.24442	−6.98607	−6.69143	−6.40373	−6.12866	−5.86961	−5.61544	−5.3806	−5.14458	−4.93249	−4.73636	−4.55344
0.32	−7.94476	−8.01115	−8.05005	−8.03818	−7.99054	−7.90197	−7.78379	−7.61172	−7.47078	−7.23923	−6.98591	−6.68441	−6.38964	−6.11261	−5.85252	−5.6037	−5.36418	−5.13657	−4.92121	−4.72949	−4.54472
0.33	−7.92345	−7.98269	−8.0328	−8.03924	−7.99141	−7.89385	−7.76791	−7.60958	−7.46259	−7.24903	−6.98712	−6.68199	−6.38525	−6.10998	−5.84951	−5.59899	−5.3614	−5.13722	−4.91953	−4.71876	−4.53406
0.34	−7.88907	−7.95956	−8.01128	−8.03742	−7.98217	−7.8961	−7.77322	−7.61737	−7.4543	−7.24991	−6.99305	−6.68754	−6.38758	−6.1087	−5.85055	−5.59995	−5.35972	−5.13432	−4.91493	−4.71255	−4.52544
0.35	−7.85887	−7.9354	−7.99267	−8.02155	−7.97753	−7.8968	−7.76117	−7.62994	−7.45769	−7.25485	−6.99849	−6.68982	−6.39276	−6.1126	−5.84939	−5.59896	−5.35647	−5.13191	−4.91477	−4.70828	−4.52279
0.36	−7.82976	−7.90246	−7.96399	−8.00788	−7.97309	−7.89261	−7.76804	−7.62684	−7.46697	−7.26003	−7.00497	−6.69594	−6.40009	−6.1105	−5.84704	−5.59695	−5.35647	−5.13201	−4.91442	−4.70996	−4.52211
0.37	−7.8107	−7.87692	−7.94241	−7.98922	−7.95803	−7.88499	−7.77137	−7.62344	−7.47002	−7.27059	−7.00917	−6.7056	−6.3977	−6.1127	−5.84642	−5.59408	−5.35629	−5.129	−4.9138	−4.70936	−4.5188
0.38	−7.78136	−7.86227	−7.9351	−7.98116	−7.94887	−7.87379	−7.76973	−7.63178	−7.47583	−7.26658	−7.01305	−6.70841	−6.40435	−6.1183	−5.84998	−5.59525	−5.35492	−5.12837	−4.91651	−4.71143	−4.51821
0.39	−7.74766	−7.84378	−7.92619	−7.97211	−7.94865	−7.87779	−7.77006	−7.63886	−7.48281	−7.28094	−7.0269	−6.71627	−6.40892	−6.1193	−5.85128	−5.59872	−5.35532	−5.12857	−4.91259	−4.71129	−4.51747
0.40	−7.72292	−7.83713	−7.92168	−7.97205	−7.95471	−7.89027	−7.77847	−7.64435	−7.49441	−7.28412	−7.02983	−6.72061	−6.41415	−6.1243	−5.85372	−5.59657	−5.35676	−5.12897	−4.91516	−4.70925	−4.51746
0.41	−7.71214	−7.82609	−7.91896	−7.97912	−7.95442	−7.8962	−7.78602	−7.66311	−7.51425	−7.30165	−7.04198	−6.73343	−6.42168	−6.13361	−5.86225	−5.60348	−5.35915	−5.13028	−4.91531	−4.7121	−4.52048
0.42	−7.69493	−7.80506	−7.90842	−7.97513	−7.96598	−7.9049	−7.80345	−7.682	−7.53418	−7.324	−7.06048	−6.74988	−6.43924	−6.14689	−5.87374	−5.61353	−5.36757	−5.13642	−4.919	−4.71808	−4.52739
0.43	−7.68983	−7.79792	−7.90492	−7.9873	−7.97482	−7.9176	−7.82265	−7.70433	−7.55678	−7.34362	−7.08076	−6.77213	−6.4611	−6.16521	−5.88891	−5.6266	−5.37979	−5.14743	−4.92841	−4.72569	−4.53629
0.44	−7.6748	−7.79805	−7.91389	−7.98899	−7.98857	−7.93532	−7.83605	−7.71832	−7.57707	−7.36202	−7.10624	−6.79322	−6.481	−6.18307	−5.90565	−5.64139	−5.39213	−5.15792	−4.93979	−4.73404	−4.54419
0.45	−7.65669	−7.79531	−7.91235	−7.9904	−7.99692	−7.94673	−7.8577	−7.74147	−7.60027	−7.38657	−7.13244	−6.81886	−6.50361	−6.20459	−5.92167	−5.65788	−5.40722	−5.17176	−4.95077	−4.74447	−4.55328
0.46	−7.64519	−7.78459	−7.90781	−7.999	−8.01083	−7.9683	−7.88168	−7.76847	−7.63037	−7.41985	−7.16521	−6.84862	−6.52857	−6.22696	−5.94411	−5.6767	−5.42413	−5.18534	−4.96329	−4.75571	−4.56143
0.47	−7.62695	−7.77372	−7.91312	−8.0166	−8.03136	−7.98929	−7.91087	−7.8007	−7.66817	−7.46292	−7.20385	−6.88646	−6.56126	−6.25546	−5.96921	−5.70019	−5.4441	−5.20379	−4.97859	−4.76662	−4.57452
0.48	−7.62444	−7.77944	−7.92542	−8.03775	−8.05385	−8.02113	−7.94635	−7.83899	−7.70988	−7.51074	−7.25034	−6.92894	−6.60042	−6.28954	−6.00161	−5.72826	−5.47034	−5.22522	−4.99755	−4.78488	−4.59014
0.49	−7.62153	−7.78547	−7.93816	−8.05318	−8.08177	−8.05145	−7.97815	−7.87584	−7.75939	−7.56266	−7.30136	−6.975	−6.64245	−6.32937	−6.0378	−5.76118	−5.49974	−5.25216	−5.02053	−4.8061	−4.61161
0.50	−7.61508	−7.79494	−7.95253	−8.07889	−8.10993	−8.08481	−8.01769	−7.92283	−7.81601	−7.61827	−7.35529	−7.02647	−6.6893	−6.37329	−6.07782	−5.79691	−5.5317	−5.28059	−5.04493	−4.83043	−4.63292
0.51	−7.61854	−7.80064	−7.96913	−8.09938	−8.14278	−8.12511	−8.06414	−7.98011	−7.88139	−7.68362	−7.41707	−7.0833	−6.74269	−6.42091	−6.12078	−5.83496	−5.56535	−5.3099	−5.07394	−4.85529	−4.65736
0.52	−7.62442	−7.80894	−7.98549	−8.13126	−8.18179	−8.17145	−8.11608	−8.04679	−7.95172	−7.75556	−7.4866	−7.14885	−6.80212	−6.47472	−6.1697	−5.87824	−5.60483	−5.34681	−5.10587	−4.88591	−4.68626
0.53	−7.62046	−7.82035	−8.01354	−8.16892	−8.22614	−8.219	−8.17752	−8.11823	−8.02926	−7.83284	−7.56226	−7.22042	−6.86628	−6.53356	−6.22307	−5.92848	−5.64948	−5.38729	−5.14331	−4.92119	−4.71922
0.54	−7.62465	−7.84109	−8.04291	−8.20677	−8.27447	−8.27783	−8.25118	−8.19917	−8.11465	−7.91857	−7.64479	−7.29618	−6.93673	−6.59801	−6.28178	−5.98082	−5.6974	−5.43128	−5.18408	−4.95905	−4.75536
0.55	−7.63739	−7.85868	−8.07276	−8.25174	−8.33102	−8.34548	−8.3309	−8.28629	−8.20788	−8.01255	−7.73534	−7.38087	−7.01354	−6.66791	−6.3458	−6.03886	−5.75039	−5.47994	−5.22969	−5.00127	−4.79525

4. Conclusion

We simply conclude as follows. If $\gamma=1$, or $\gamma=2$, reducing the replacement ratio below 50% increases the welfare of all future generations. If $\gamma=3$, reducing the replacement ratio below 50% increases the welfare of future generations except for a generation in 2000.

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Appendix 1

We describe how to calculate the optimal decision rules of the individual. We use the grid method to solve the individual decision rules. First of all, we put a grid on the state variable $x_{k,l}=(s_{k,l},z_l)$. We start in the last period of N of an individual's life and solve for the control variable $s_{j.i}$ for grid point in the state variable $x_{j+1,i+1}=(s_{j+1,i+1},z_{i+1})$, setting $V_{j+1,i+1}(x_{j+1,i+1})=0$. Given the maximizing control variable $s_{j.i}$ on gridpoints, we can determine values for the value function $V_{j,i}(x_{j,i})$ on gridpoints. We repeat this procedure to solve for value functions and decision rules for all earlier periods.

In this paper, we assigned a value of 500 points and 5 points $(s_{k,l},z_l)$ on the grid. The spacing between points on the asset grid increases with asset levels. More specifically, asset gridpoints are placed according to $s^1=0$, $s^m=d\times m^{2.35}$, $m=2,\dots,500$, where $d=\overline{s}/500^{2.35}$ and $\overline{s}$ is an upper bound imposed on the asset grid.

We run simulations by using the optimal decision rules which are described above. We generate the pseudorandom member and run the simulations 10,000 times for all generations in this paper. The expected utility is calculated by averaging the 10,000 simulations.

Notes

1) See Ministry of Health, Labor and Welfare (2005).

2) In The Population Projection for Japan (2002), it is assumed that a population growth rate is constant after 2100.

3) However, it is presumed that survival rates after 2050 is same as that of 2050 in Population Projection for Japan (2002).

4) Using the budget constraint (2) to substitute for $c_{j,t+j-1}$ in Bellman's equation (7), the problem reduces to choosing the decision control variable $s_{j,j+t-1}$.

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