THE RUIN PROBABILITY AND THE RUIN TIME FOR THE RISK RESERVE PROCESS

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1. Introduction

In Japan, many companies are combined to avoid the ruin and it is a matter of common knowledge that a company has been ruined. Thus it is a pressing need to have the non-ruin probability and the non-ruin policy for them. Usually they reserve the fund for the ruin risk. Thus we constract the risk reserve process.

For the risk reserve process, in the steady state, the expected ruin time and the ruin probability have been studied by Doi (2000) and Doi (2006), respectively.

In this paper, we consider the risk reserve process with exponential or Erlang type claims and we find the non-ruin probability in finite time including the ruin probability.

Let us denote by $U(t)$ the reserve level at time t where $\{U(t)\}_{t\geq 0}$ is called the risk reserve process. If the reserve level is zero, the process ruins. The fluctuation of $U(t)$ is controlled by three elements: the claim inter-arrival time, the claim size and the premium rate. Let us assume that the claim inter-arrival time and the claim size are independent and identically distributed random variables, respectively. We also assume the premium rate is a constant. In the same way of Mikosch (2004) we introduce the notation :

$$\begin{cases} U(t) & : \text{ the reserve level at time } t, \\ u & : \text{ the initial reserve level } (u = U(0) > 0), \\ X_n & : \text{ the } n\text{-th claim size}, \\ T_n & : \text{ the } n\text{-th claim arrival time } (T_0 = 0), \\ W_n & : \text{ the claim inter-arrival time} \\ & \quad\quad \text{between } (n-1)\text{-th and } n\text{-th claim arrival times}, \\ c & : \text{ the premium rate}, \end{cases}$$

where X_n and W_n are independent.

The total claim amount process $\{S(t)\}_{t\geq 0}$ and the premium income $I(t)$ are defined as follows :

- The total claim amount process $\{S(t)\}_{t\geq 0}$ is define as

$$S(t) = \sum_{n=1}^{N(t)} X_n \quad (t \geq 0),$$

where $\{N(t)\}_{t\geq 0}$ is the claim number process defined by

$$N(\mathrm{t}) = \max\{n \geq 1 : T_n \leq t\} \quad (t \geq 0).$$

• We define the premium income $I(t)$ as

$$I(t) = ct,$$

which is the accumulated income by time t.

Therefore, we obtain the expression of risk reserve process $\{U(t)\}_{t\geq 0}$ as follows :

$$U(t) = u + I(t) - S(t), \quad (t \geq 0).$$

In the next section, we make a mathematical model to get the non-ruin probability in finite time.

2. MODEL-1 AND ANALYSIS

In the risk reserve process $\{U(t)\}_{t\geq 0}$, the ruin can occur only at the time $t = T_n$ for some $n \geq 1$, since $\{U(t)\}_{t\geq 0}$ linearly increases in the intervals $[T_n, T_{n+1}]$. We call the sequence $\{U(T_n)\}_{n\geq 0}$ the skeleton process of the risk reserve process $\{U(t)\}_{t\geq 0}$ (see Mikosch (2004)). By use of the skeleton process, we can express the event {ruin} in terms of the inter-arrival times W_n, the claim sizes X_n, the initial reserve level and the premium rate c, as follows :

$$\begin{aligned}\{\text{ruin}\} &= \left\{\inf_{t>0} U(t) < 0\right\} \\ &= \left\{\inf_{n\geq 1} [u + I(T_n) - S(T_n)] < 0\right\} \\ &= \left\{\inf_{n\geq 1} \left[u - \sum_{i=1}^{n}(X_i - cW_i)\right] < 0\right\}.\end{aligned}$$

Now, we define

$$\begin{aligned} Z_n &= X_n - cW_n, \quad (n \geq 1), \\ S_n &= Z_1 + \cdots + Z_n, \quad (n \geq 1,\ S_0 = 0). \end{aligned}$$

We propose R|Ex|Ex model where R means the risk reserve process, the first Ex means that the claim inter-arrival time W_n has an exponential distribution with rate λ, and the next Ex means that the claim size X_n has an exponential distribution with rate μ. We call it Model-1. We also discuss, in section 4, R|Ex|Er model (Model-2), where Er means Erlang type claim size with parameter μ and phase k. In what follows, we omit the subscript n.

2.1 PROBABILITY DENSITY FUNCTION OF MODEL-1

We find the probability distribution of random variable Z for this model. First, let $Y = cW$, which has the probability density function as follows :

$$f_Y(y) = \begin{cases} \frac{\lambda}{c} e^{-\frac{\lambda}{c}y} & (y \geq 0) \\ 0 & (y < 0). \end{cases}$$

Next, let us denote

$$\begin{cases} Z = X - Y \\ V = Y. \end{cases}$$

Since X and Y are independent random variables, we obtain the joint probability density function with respect to Z and V

$$f_{ZV}(z, v) = \mu e^{-\mu(z+v)} \cdot \frac{\lambda}{c} e^{-\frac{\lambda}{c}v},$$

where the domain of v is

$$\begin{cases} 0 \leq v < \infty & (z \geq 0) \\ -z \leq v < \infty & (z < 0). \end{cases}$$

We obtain the probability density function $g(z)$ of Z as follows :

$$(1) \qquad g(z) = \begin{cases} \frac{\lambda\mu}{\lambda + c\mu} e^{-\mu z} & (z \geq 0) \\ \frac{\lambda\mu}{\lambda + c\mu} e^{\frac{\lambda}{c}z} & (z < 0). \end{cases}$$

2.2 NON-RUIN PROBABILITY IN FINITE TIME FOR MODEL-1

We define by $r_n(u, c)$ the non-ruin probability that the risk reserve process does not ruin till n-th claim arrival time given the initial reserve level u and the premium rate c, that is,

$$(2) \qquad r_n(u, c) = P(Z_1 < u, Z_2 < u - S_1, \cdots, Z_n < u - S_{n-1} | U(0) = u, T_1 < T_2 < \cdots < T_n < \infty).$$

For the general formula of $r_n(u, c)$, we have the following theorem, Kishikawa and Doi（2009）Preliminarily, we describe the following two lemmas.

LEMMA 1 *For any non-negative integer i, the following relation holds.*

$$\int_{-\infty}^{u} g(z)(u-z)^i e^{\mu z} dz = \frac{\lambda\mu}{\lambda + c\mu} \sum_{j=0}^{i+1} \frac{i!}{(i-j+1)!} \left(\frac{c}{\lambda + c\mu}\right)^j u^{i-j+1} . \tag{3}$$

LEMMA 2 *For any natural numbers m and n* $(m \leq n)$*, the following relation holds.*

$$\sum_{i=1}^{m} K_{n,i} = K_{n+1,m} . \tag{4}$$

THEOREM 1 *For R|Ex|Ex model, we obtain the probability* $r_n(u, c)$ *as follows :*

$$r_0(u,c) = 1 ,$$

$$r_n(u,c) = r_{n-1}(u,c) - \frac{1}{\mu}\left(\frac{\lambda\mu}{\lambda + c\mu}\right)^n e^{-u\mu} \sum_{i=0}^{n-1} K_{n,n-i} \frac{u^i}{i!} \left(\frac{c}{\lambda + c\mu}\right)^{n-i-1} \quad (n \geq 1), \tag{5}$$

where

$$\begin{cases} K_{n,1} = 1 , & (n \geq 1) \\ K_{n,n} = K_{n,n-1} , & (n \geq 2) \\ K_{n,l} = K_{n-1,l} + K_{n,l-1} , & (n \geq 3,\ 2 \leq l \leq n-1) \\ K_{n,l} = 0 , & (others). \end{cases} \tag{6}$$

2.3 NUMERICAL EXAMPLES FOR MODEL-1

In this section we consider the non-ruin policy. Assume the case that the mean claim inter-arrival time $1/\lambda = 1$ unit of time and the mean claim size $1/\mu = 1$ unit of amount. Then we need to decide the premium rate c under the condition that the probability $r_n(u, c)$ is greater than a certain value, where the number of claims n and the initial reserve level u are given. Now, we suppose n and u are equal to 100 and 10, respectively. From Theorem 1, we plot the graph of $r_{100}(10, c)$ against the premium rate c (Figure 1). In order to hold $r_{100}(10, c)$, the non-ruin probability that the process does not ruin till 100-th claim arrival time given the initial level 10 and the premium rate c, to be greater than 0.8, we must give the premium rate c the value of greater than 1.137.

Next we need to decide the initial reserve level u under the condition that the probability $r_n(u, c)$ is greater than a certain value, where the number of claims n and the premium rate c are given similarly above. Now, we suppose n and c are equal to 100 and 1.1, respectively. We plot the graph of $r_{100}(u, 1.1)$ against the initial reserve level u (Figure 2). In order to hold $r_{100}(u, 1.1)$ to be greater than 0.8, we must give the initial reserve level u the value of greater than 11.57.

Figure1.

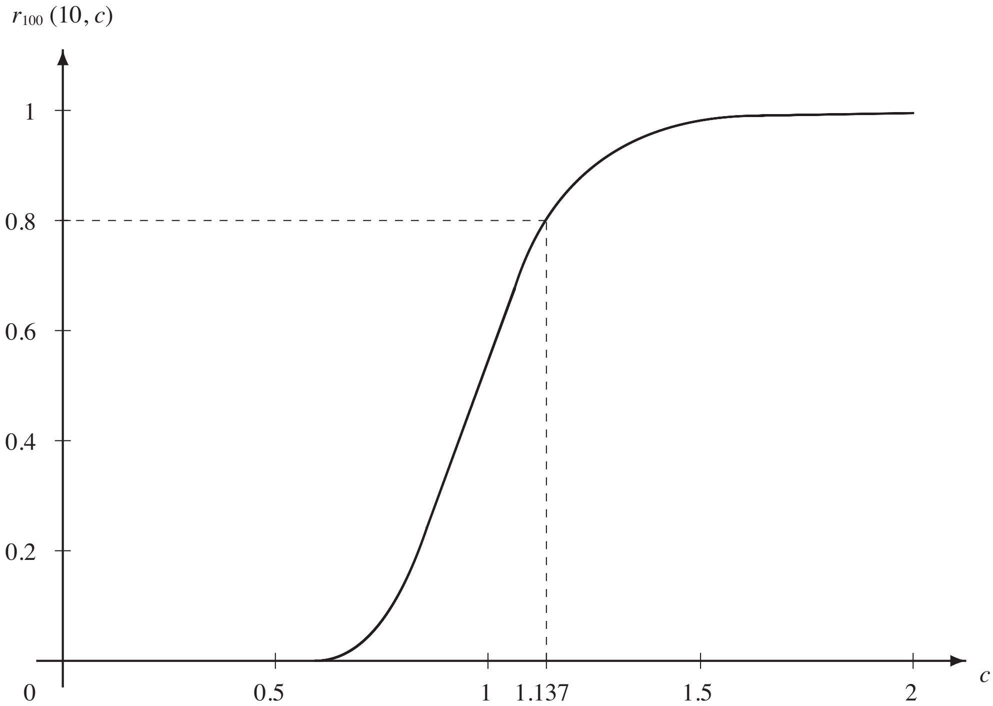
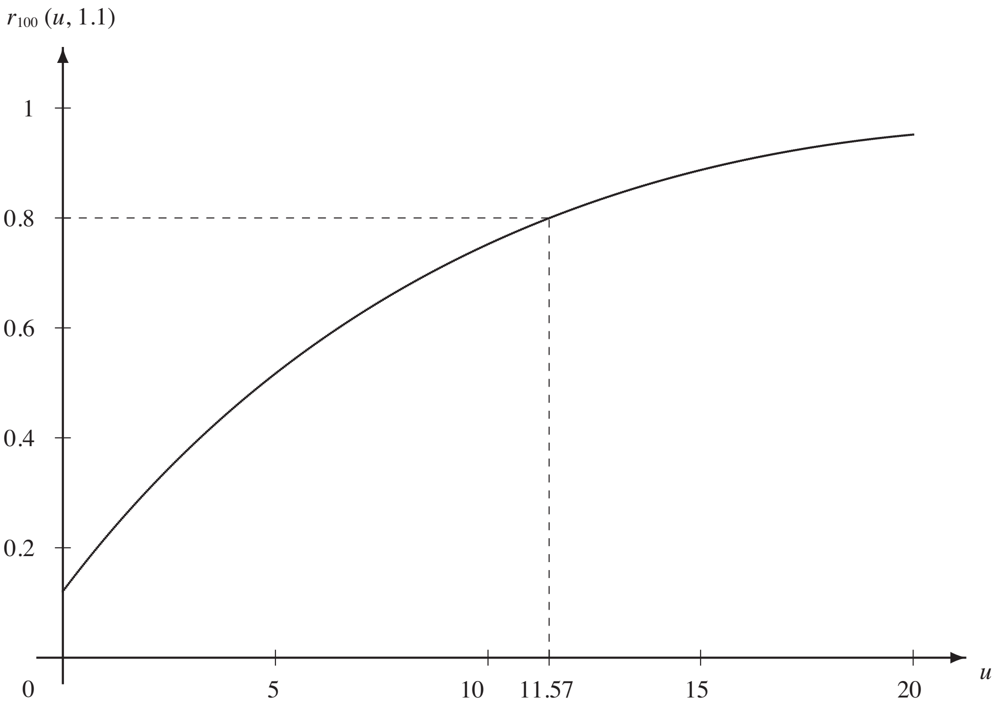


Figure 2.



3. REMARKS 1

We have found the general formula that derives the non-ruin probability for the R|Ex|Ex model in finite time. We assume that the claim inter-arrival time has the exponential distribution. Also, we assume that the claim size has the exponential distribution. The claim size, however, may have the other distribution.

Although the premium rate is constant in this paper, actually it is variable according to the number of claim arrivals per unit time and so on.

Furthermore, since the expected ruin time in the steady state has been obtained by Doi（2000）, it is also a future subject to get the expected ruin time for our models.

4. MODEL-2 AND ANALYSIS

We propose R|Ex|Er model where R and Ex stand for the same as model-1 otherwise Er means that the claim size X_n has the Erlang distribution with parameter μ and phase k. We call it Model-2.

4.1 PROBABILITY DENSITY FUNCTION OF Z FOR MODEL-2

In this section we find the probability distribution of random variable Z for this model. In the same way of R|Ex|Ex model, Kishikawa and Doi（2009）), we obtain the joint probability density function with respect to Z and $V = cW$

$$(7) \qquad f_{ZV}(z,v) = \frac{(k\mu)^k}{(k-1)!}(z+v)^{k-1}e^{-k\mu(z+v)} \cdot \frac{\lambda}{c}e^{-\frac{\lambda}{c}v}$$

where the domain of v is obtained as follows:

$$\begin{cases} 0 \le v < \infty & (z \ge 0) \\ -z < v < \infty & (z < 0). \end{cases}$$

For the general formula of $r_n^{(k)}(u,c)$, we have the following theorem, Doi and Kishikawa（2011）Preliminarily, we describe the two lemmas.

LEMMA 3 *For any natural number k, the following relation holds.*

$$(8) \qquad \int_0^{\infty}(y+x)^k e^{-rx}\,dx = \sum_{i=0}^{k}\frac{k!}{(k-i)!}r^{-i-1}y^{k-i}, \quad r > 0,\ y \ge 0.$$

LEMMA 4 *For any natural number k, the following relation holds.*

(9)
$$\int_{-y}^{\infty} (y+x)^k e^{-rx}\,dx = k!r^{-k-1}e^{ry}, \quad r>0,\ y<0.$$

For $z \geq 0$, from Lemma 3, we obtain the probability density function of Z as follows:

(10)
$$\begin{aligned} g_k(z) &= \int_0^{\infty} \frac{(k\mu)^k}{(k-1)!}(z+v)^{k-1}e^{-k\mu(z+v)} \cdot \frac{\lambda}{c}e^{-\frac{\lambda}{c}v}\,dv \\ &= \frac{\lambda(k\mu)^k}{c(k-1)!}e^{-k\mu z}\sum_{i=0}^{k-1}\frac{(k-1)!}{(k-i-1)!}\left(k\mu+\frac{\lambda}{c}\right)^{-i-1} \cdot z^{k-i-1} \\ &= \frac{\lambda}{c}(k\mu)^k e^{-k\mu z}\left(k\mu+\frac{\lambda}{c}\right)^{-k}\sum_{i=0}^{k-1}\frac{1}{(k-i-1)!}\left(k\mu+\frac{\lambda}{c}\right)^{k-i-1} \cdot z^{k-i-1} \\ &= \frac{\lambda}{c}(k\mu)^k\left(k\mu+\frac{\lambda}{c}\right)^{-k} \cdot e^{-k\mu z}\sum_{i=0}^{k-1}\frac{1}{i!}\left(k\mu+\frac{\lambda}{c}\right)^{i} z^i. \end{aligned}$$

Also, for $z \geq 0$, from Lemma 4, we obtain

(11)
$$\begin{aligned} g_k(z) &= \int_{-z}^{\infty} \frac{(k\mu)^k}{(k-1)!}(z+v)^{k-1}e^{-k\mu(z+v)} \cdot \frac{\lambda}{c}e^{-\frac{\lambda}{c}v}\,dv \\ &= \frac{\lambda(k\mu)^k}{c(k-1)!}e^{-k\mu z}\,(k-1)!\left(k\mu+\frac{\lambda}{c}\right)^{-k} \cdot e^{(k\mu+\frac{\lambda}{c})z} \\ &= \frac{\lambda}{c}(k\mu)^k\left(k\mu+\frac{\lambda}{c}\right)^{-k} \cdot e^{\frac{\lambda}{c}z}. \end{aligned}$$

Now, let us set

(12)
$$\begin{cases} \alpha = k\mu + \dfrac{\lambda}{c} \\ \beta = k\mu \\ A = \alpha^{-1}\beta, \end{cases}$$

then we have

(13)
$$g_k(z) = \begin{cases} \dfrac{\lambda}{c}A^k e^{-\beta z}\displaystyle\sum_{i=0}^{k-1}\frac{\alpha^i}{i!}z^i & (z \geq 0) \\ \dfrac{\lambda}{c}A^k e^{\frac{\lambda}{c}z} & (z < 0). \end{cases}$$

4.2 PROPOSITIONS FOR MODEL-2

We define by $r_n^{(k)}(u,c)$ the non-ruin probability that the risk reserve process does not ruin till n-th claim arrival time given the initial reserve level u and the premium rate c, that is,

$$(14)\quad r_n^{(k)}(u,c) = P(Z_1 < u,\ Z_2 < u - S_1, \cdots, Z_n < u - S_{n-1} | U(0) = u,\ T_1 < T_2 < \cdots < T_n < \infty),$$

where the claim size X_m, $(m = 1, 2, \cdots n)$ has the Erlang distribution with parameter μ and phase k.

By use of $g_k(z)$ above, we can obtain $r_n^{(k)}(u,c)$. First, for $r_1^{(k)}(u,c)$, we describe the following three lenmas.

LEMMA 5 *For any non-negative integer i, the following relation holds.*

$$(15)\quad \int_0^u z^i e^{-\beta z} dz = \frac{i!}{\beta}\left(\beta^{-i} - e^{-u\beta}\sum_{j=0}^{i}\frac{\beta^{-j}}{(i-j)!}u^{i-j}\right).$$

LEMMA 6 *For any natural number k and n, the following relation holds.*

$$(16)\quad \sum_{i=0}^{n}\binom{k+i}{i} = \binom{k+n+1}{n}.$$

We have the following lemma with respect to β and A of (12).

LEMMA 7 *For any natural number k, the following relation holds.*

$$(17)\quad A^k + \frac{\lambda}{c\beta}\sum_{i=1}^{k} A^i = 1\ .$$

Using these Lemmas, we obtain the following proposition for $r_1^{(k)}(u,c)$

PROPOSITION 1 *For R|Ex|Er model, the following relation holds.*

$$(18)\quad r_1^{(k)}(u,c) = 1 - \frac{\lambda}{c\beta}e^{-u\beta}\sum_{i_1=1}^{k} A^{i_1}\sum_{i_2=0}^{k-i_1}\frac{\beta^{i_2}}{i_2!}u^{i_2}.$$

LEMMA 8 *For any non-negative integer i, the following relation holds.*

$$(19)\quad \int_{-\infty}^{0} e^{\alpha z}(u-z)^i dz = \alpha^{-i-1} i! \sum_{j=0}^{i}\frac{(u\alpha)^j}{j!},\quad \alpha > 0.$$

LEMMA 9 *For any non-negative integer i and j, the following relation holds.*

(20) $$\int_0^u (u-z)^i z^j \, dz = \frac{i!j!}{(i+j+1)!} u^{i+j+1} .$$

For the general formula of $r_n^{(k)}(u,c)$, we have the following proposition by use of lemmas above.

PROPOSITION 2 *For R | Ex | Er model, we obtain the probability $r_n^{(k)}(u,c)$ as follows:*

(21) $$r_0^{(k)}(u,c) = 1,$$
$$r_n^{(k)}(u,c) = r_{n-1}^{(k)}(u,c) - \left(\frac{\lambda}{c}\right)^n \alpha^{-n} A^{k(n-1)-1} e^{-u\beta} \cdot \sum_{i_1=1}^{k} A^{i_1} \sum_{i_2=0}^{k-i_1} A^{i_2} \sum_{i_3=0}^{k+i_2} \sum_{i_4=0}^{k+i_3} \cdots \sum_{i_{n+1}=0}^{k+i_n} \frac{(u\alpha)^{i_{n+1}}}{i_{n+1}\,!}, \quad n \geq 1,$$

where

(22) $$\begin{cases} \alpha = k\mu + \dfrac{\lambda}{c} \\ \beta = k\mu \\ A = \alpha^{-1}\beta . \end{cases}$$

4.3 REDUCTION OF MULTI-SUMMATION

Although we obtained the general formula of $r_n^{(k)}(u,c)$, this general formula is not suitable for numerical calculations because of too many summations. In order to reduce the number of summation, let us denote the summation part of (21) by

(23) $$\phi_k(n) = \sum_{i_1=1}^{k} A^{i_1} \sum_{i_2=0}^{k-i_1} A^{i_2} \sum_{i_3=0}^{k+i_2} \sum_{i_4=0}^{k+i_3} \cdots \sum_{i_{n+1}=0}^{k+i_n} \frac{(u\alpha)^{i_{n+1}}}{i_{n+1}\,!}, \quad n \geq 1.$$

We have the following lemma on the summation.

LEMMA 10 *If k is given, for any natural number m and n ($m \leq kn$), the following relation holds.*

(24) $$\sum_{i=1}^{m} K_{n,i}^{(k)} = K_{n+1,m}^{(k)} .$$

The following proposition is obtained by use of this lemma.

PROPOSITION 3 *If we define the following, for any natural number k,*

$$
(25)\qquad \begin{cases} K_{n,1}^{(k)} = A^{k-1}, & (n \geq 1) \\ K_{1,m}^{(k)} = \sum_{j=1}^{m} A^{k-j}, & (2 \leq m \leq k) \\ K_{n,kn-j}^{(k)} = K_{n,k(n-1)}^{(k)}, & (n \geq 2,\ 1 \leq j \leq k) \\ K_{n,m}^{(k)} = K_{n-1,m}^{(k)} + K_{n,m-1}^{(k)}, & (n \geq 2,\ 2 \leq m \leq k(n-1)) \\ K_{n,m}^{(k)} = 0, & (others), \end{cases}
$$

then (23) *is reduced to*

$$
(26)\qquad \phi_k(n) = A \sum_{i=0}^{kn-1} K_{n,kn-i}^{(k)} \frac{(u\alpha)^i}{i!}, \quad n \geq 1.
$$

4.4 NON-RUIN PROBABILITY IN FINITE TIME FOR MODEL-2

From *Proposition 2* and *3,* we have the following theorem.

THEOREM 2 *For R|Ex|Er model, we obtain the non-ruin probability in finite time* $r_n^{(k)}(u,c)$ *as follows:*

$$
r_0^{(k)}(u,c) = 1,
$$

$$
(27)\qquad r_n^{(k)}(u,c) = r_{n-1}^{(k)}(u,c) - \left(\frac{\lambda}{c}\right)^n \alpha^{-n} A^{k(n-1)} e^{-u\beta} \sum_{i=0}^{kn-1} K_{n,kn-i}^{(k)} \frac{(u\alpha)^i}{i!}, \quad n \geq 1,
$$

Where

4.5 NUMIRICAL EXAMPLES FOR MODEL-2

We set the mean claim inter-arrival time as $1/\lambda = 1$ unit of time and the mean main claim size as $1/\mu = 1$ unit of amount. We need to decide the premium rate c under the condition that the probability $r_n^{(k)}(u,c)$ is greater than a certain value, where the number of claims n, the initial reserve level u and the phase k are given. We assume that the claim consists of the sum of main and two optional ones. Therefore we suppose that $n = 100$, $u = 10$ and $k = 3$. We plot the graph of $r_{100}^{(3)}(10,c)$ against the premium rate c (Figure 3). In order to hold $r_{100}^{(3)}(10,c)$ to be greater than 0.8,we must give the premium rate c the value of greater than 1.068 from Figure 3.

Next we need to decide the initial reserve level u under the condition that the probability $r_n^{(k)}(u,c)$ is greater than a certain value, where the number of claims n, the premium rate c and phase k are given similarly above. Now, we suppose n, c and k is equal to 100, 1.1 and 3, respectively. Then we plot the graph of $r_{100}^{(3)}(u,1.1)$ against the initial reserve level u (Figure 4). In order to hold $r_{100}^{(3)}(u,1.1)$ to be greater than 0.8, we must give the initial reserve level u the value of greater than 8.555 from Figure 4.

Figure 3.

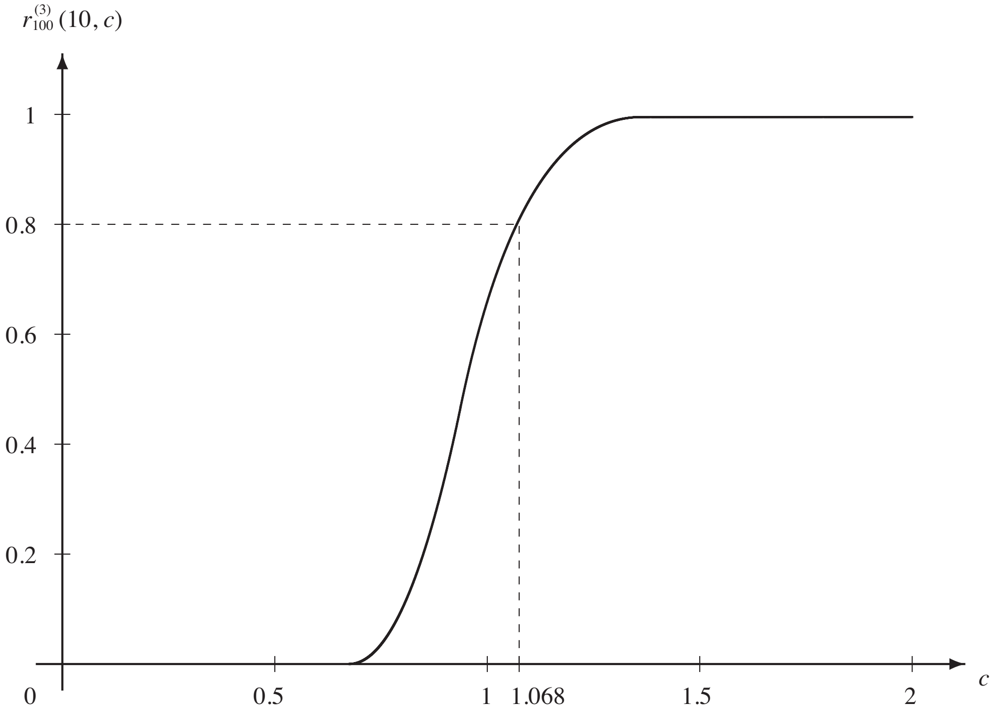
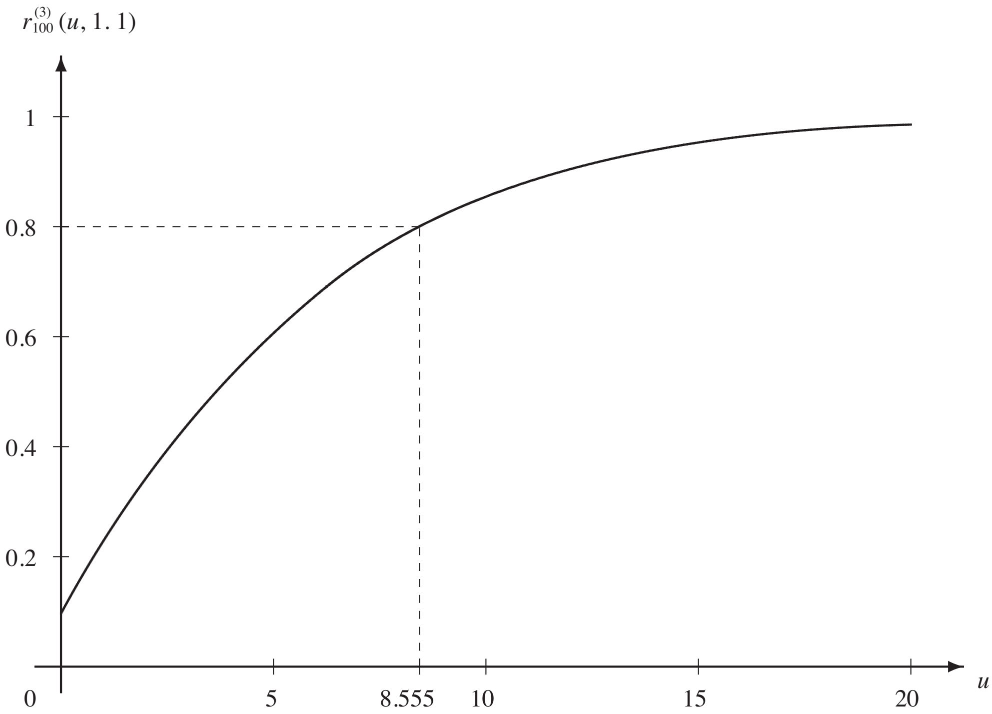


Figure 4.



5　REMARKS 2

For R|Ex|Er model we have obtained the general formula that derives the non-ruin probability in finite time. Although we assume that the claim inter-arrival time has the exponential distribution and the claim size has the Erlang distribution, it is an interesting problem to consider the other distributions for the inter-arrival time and the claim size. The premium rate is constant in this paper, however, it may be variable. Furthermore, it is an important problem to get the expected ruin time for our model.

6　RUIN PROBABILITY AND RUIN TIME

In this section we discuss the ruin probability and the ruin time.

6.1　RUIN PROBABILITY

We could find the ruin probabilities in finite time for Model-1 and Model-2 in Theorem 1 and Theorem 2, respectively.

THEOREM 3　*For Model-1, we obtain the ruin probability at n-th claim as follows:*

$$p_n(u,c) = \frac{1}{\mu}\left(\frac{\lambda\mu}{\lambda + c\mu}\right)^n e^{-u\mu} \sum_{i=0}^{n-1} K_{n,n-i} \frac{u^i}{i!} \left(\frac{c}{\lambda + c\mu}\right)^{n-i-1} \quad (n \geq 1). \tag{30}$$

Also the ruin probability in finite time for Model-2 is obtained by

THEOREM 4

$$p_n^{(k)}(u,c) = \left(\frac{\lambda}{c}\right)^n \alpha^{-n} A^{k(n-1)} e^{-u\beta} \sum_{i=0}^{kn-1} K_{n,kn-i}^{(k)} \frac{(u\alpha)^i}{i!} \quad (n \geq 1). \tag{31}$$

6.2　RUIN TIME

Since the ruin occurs at the n-th claim time, we define the ruin time distribution function:

$$F_n(c,u,t) = P(Z_1 < u, Z_2, u - S_1, \cdots, Z_{n-1} < u - S_{n-2}, Z_n \geq S_{n-1}, T_n < t).$$

Thus we have the following theorem.

THEOREM 5

(32)
$$F_n(c,u,t) = \int_0^t \int_{t-w}^t \int_0^{u+c\xi_1} f(x_1) \int_0^{u+c(2\xi_1+w)-x_1} f(x_2) \cdots$$
$$\cdot \int_0^{u+c((n-1)\xi_1+(n-2)w)-\sum_{j=1}^{n-2} x_j} f(x_{n-1}) \int_{u+c(n\xi_1+(n-1)w)-\sum_{j=1}^{n-1} x_j}^{\infty} f(x_n)dx_n$$
$$\cdot dx_{n-1} \cdots dx_1 h(\xi_1)d\xi_1 h(w)dw$$

where

$f(x)$ is the p.d.f. for claims
and $h(y)$ is the p.d.f. for the inter-arrival times.

Especially, in the case of For R|Ex|Ex we could think of $F_n\ (c, u, t)$ as a conditional Erlang distribution.

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