Collusion and Participation Constraints

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1. Introduction

This paper studies optimal mechanisms with collusion in a model in which a government (the principal) procures two complementary products from two firms (the agents). Unlike the literature on optimal mechanisms with multiple agents under asymmetric information, we examine a setting in which each firm's cost comprises not only a variable cost but also a fixed cost, both of which depend on private information. We assume that there are two productivity types for each firm: one type has a high marginal cost and a low fixed cost, while the other has a low marginal cost and a high fixed cost.

We show that, if the difference in the amount of fixed costs with respect to each firm's type is sufficiently small, then the optimal contracts with collusion entail downward distortion. In contrast, we demonstrate that, if the difference in fixed costs with respect to productivity types is sufficiently large, then the countervailing incentives prevail and the optimal contracts with collusion entail upward distortion.

This paper is related to three strands of the literature on contract theory or mechanism design. The first is the literature on optimal organizations with multiple agents under asymmetric information. Dana (1993) analyzes optimal industry structures, in which the government procures two substitute products. The optimal industry structure, he concludes, depends on whether marginal costs of the two products are sufficiently positively correlated or not. Baron and Besanko (1992, 1999), Gilbert and Riordan (1995), and Severinov (2008) analyze optimal industry structures, in which marginal costs of the two products are independently determined. These papers show that the optimal industry structure depends on the degree of complementarity or substitutability. However, these papers do not consider fixed costs that depend on agents' types. In this paper, we extend the literature to a more general cost structure, in which not only a variable cost but also a fixed cost are considered.

The second strand of research related to this paper is concerned with the issues of countervailing incentives under asymmetric information (Laffont and Tirole,1993; Laffont and Martimort, 2002). Lewis and Sappington (1989) assume one-dimensional uncertainty regarding marginal costs and fixed costs and analyze countervailing incentives. Maggi and Rodriguez-Clare (1995) further examine the issues on countervailing incentives. Jullien (2000) explores the effects of type-dependent participation constraints on optimal contracts. However, these papers consider only a single agent. Kobayashi (2019) considers the relationship between multiple agents and type-dependent participation constraints and characterizes countervailing incentives in optimal contracts.

Third, our paper is also related to the literature on collusion under asymmetric information. Laffont and

Martimort (1997) examine collusion-proof contracts. Laffont and Martimort (1998) compare collusion-proof contracts under centralization with those under delegation. Further, Laffont and Martimort (2000) analyze the effects of correlation on collusion in optimal contracts.

In this paper, we examine optimal organizations in a model with a principal and multiple agents, in which the cost function of each firm comprises not only a variable cost but also a fixed cost, both of which depend on private information. We show that, when the difference in the amount of fixed costs with respect to productivity type is sufficiently large, countervailing incentives can result because the set of binding incentive compatibility and participation constraints depends on the value of this difference and that the optimal collusion-proof contract entails upward distortion.

The paper is organized as follows. In Section 2, we present a model and basic assumptions. In Section 3, we characterize optimal contracts without side mechanisms. In Section 4, we characterize optimal contracts with side mechanisms. Section 5 concludes.

2. The Model

We consider a two-product industry in which two firms produce products A and B. Output (quantity or quality) of product A or B is denoted q_A or q_B respectively. The two products are supposed to be complementary. A government procures these products and supplies a final product (a public good). Let $V(q_A,q_B)$ denote social benefit that consumers obtain from the final product, where

$$V(q_A,q_B)=S(q),\ \text{ with }\ q=\min\{q_A,q_B\}.$$

For all $q>0$, $S(q)$ is twice continuously differentiable, increasing and strictly concave.

The cost function of each product is given by

$$C(q,\theta)=\theta q+F(\theta),$$

where θ is the marginal cost and $F(\theta)$ is the fixed cost. We assume that parameter θ takes either θ_1 or θ_2 with $\theta_1<\theta_2$. We also assume $F(\theta_1)\geqq F(\theta_2)\geqq 0$. This assumption implies that a high marginal cost is associated with a low fixed cost and vice versa. For instance, this inverse relationship can arise because a high fixed cost guarantees a low marginal cost and vice versa in constructing facilities such as highways and bridges.

We assume that the marginal cost θ is private information for each firm. For firm A, let θ_i denote its type, $i=1$, 2. Similarly, for firm B, let θ_j denote its type $j=1$, 2. Let p_{ij} denote a joint probability between θ_i and θ_j. For simplicity, we assume the probability distributions over θ_i and θ_j are independent. Let $p=\Pr(\theta_i=\theta_1)=\Pr(\theta_j=\theta_1)$, $0<p<1$. Hence, we have

$$\begin{aligned}&p_{11}=p^2,\\&p_{12}=p_{21}=p(1-p),\\\text{and}\quad&p_{22}=(1-p)^2.\end{aligned}$$

Each firm's payoff is defined as a monetary transfer from the government minus a realized production cost. Then, firm A's payoff is

$$u^A = \tau_A - \{\theta_A q_A + F(\theta_A)\},$$

where τ_A is a monetary transfer from the principal to firm A. Similarly, firm B's payoff is

$$u^B = \tau_B - \{\theta_B q_B + F(\theta_B)\}.$$

Following Baron and Myerson (1982), the principal's payoff is given as

$$S(q) - (\tau_A + \tau_B) + \alpha(u^A + u^B),$$
$$\alpha \in [0,1).$$

We consider two different mechanism structures. One is a mechanism in which the two agents are not allowed to cooperate. The other is a mechanism in which the two agents can collude.

When designing optimal contracts, the principal solves its payoff maximization problem subject to incentive compatibility constraints and participation constraints. An incentive compatibility constraint (ICC) guarantees that each firm prefers the contract that is designed for it. A participation constraint (PC) guarantees that each firm accepts the designated contract.

In addition to those constraints, we consider collusion-proof constraints, which are described below, when we examine optimal contracts with collusion.

Let $q_{ij} \equiv q(\theta_i, \theta_j)$ be the quantity when firm A's type is θ_i and firm B's type θ_j, $i, j = 1,2$. The government's expected payoff Π, *ex post* payoff u^A of firm A, and ex post payoff u^B of firm B are given by, respectively,

$$\Pi = \sum_{ij} p_{ij}[S(q_{ij}) - \tau_{ij}^A - \tau_{ij}^B],$$
$$u^A = \tau_{ij}^A - \theta_i q_{ij} - F(\theta_i),$$
$$\text{and} \quad u^B = \tau_{ij}^B - \theta_j q_{ij} - F(\theta_j), \quad i, j = 1,2,$$

where monetary transfers from the government to the firms are denoted τ_{ij}^A and τ_{ij}^B.

In this paper, we examine the possibilities of the firms' collusion. Following Laffont and Tirole (1997), we consider a side mechanism as follows. Assume that a third party offers a side contract to the firms. Let $\varphi(\cdot)$ denote two firms' reports to the third party. Let $x_i(\cdot)$ denote a monetary transfer to each agent by the third party. Then, a side mechanism is defined as follows: A side mechanism is a triple of $x_A(\cdot), x_B(\cdot)$, and $\varphi(\cdot)$ for $\forall(\theta_i, \theta_j)$, and denoted

$$\{x_A(\cdot), x_B(\cdot), \varphi(\cdot)\}.$$

Because there does not exist outside finance for the coalition of the two agents, a side mechanism must satisfy the budget balance constraint, which is $x_A(\theta_i, \theta_j) + x_B(\theta_i, \theta_j) = 0$, for $\forall(\theta_i, \theta_j)$.

Next, we describe the frametitletiming of the contracting game. The sequence of events proceeds as follows:

Stage 1: Nature determines each firm's type: θ_1 or θ_2. Each firm observes her own type only.

Stage 2: The principal offers a grand mechanism to the firms.

Stage 3: The firms accept or refuse it.

Stage 4: The third party offers a side mechanism to the firms.

Stage 5: Each firm accepts or refuses the side contract .

Stage 6: When accepting the side mechanism, each firm reports her information to the third party.

Stage 7: The transfers in the side contract take place.

Stage 8: The firms produce the products. The principal provides monetary transfers to the firms.

3. Optimal Contracts Without Side Mechanisms

In this section, we examine frametitleoptimal mechanisms when the agents do not collude. The results in this section serve as benchmark cases and are compared with the optimal mechanisms in Section 4.

The government's (the principal's) problem (P-1), can be stated as follows:

$$Maximize\ \Pi=\sum_{ij} p_{ij}[S(q_{ij})-\tau_{ij}^{A}-\tau_{ij}^{B}] \tag{P-1}$$

$$\begin{aligned}
\text{subject to } & \sum_{j=1}^{2} p_{1j}[\tau_{1j}^{A}-\theta_1 q_{1j}-F(\theta_1)] \geq \sum_{j=1}^{2} p_{1j}[\tau_{2j}^{A}-\theta_1 q_{2j}-F(\theta_1)],\\
& \sum_{i=1}^{2} p_{i1}[\tau_{i1}^{B}-\theta_1 q_{i1}-F(\theta_1)] \geq \sum_{i=1}^{2} p_{i1}[\tau_{i2}^{B}-\theta_1 q_{i2}-F(\theta_1)],\\
& \sum_{j=1}^{2} p_{2j}[\tau_{2j}^{A}-\theta_2 q_{2j}-F(\theta_2)] \geq \sum_{j=1}^{2} p_{2j}[\tau_{1j}^{A}-\theta_2 q_{1j}-F(\theta_2)],\\
& \sum_{i=1}^{2} p_{i2}[\tau_{i2}^{B}-\theta_2 q_{i2}-F(\theta_2)] \geq \sum_{i=1}^{2} p_{i2}[\tau_{i1}^{B}-\theta_2 q_{i1}-F(\theta_2)],\\
& \tau_{1j}^{A}-\theta_1 q_{1j}-F(\theta_1) \geq 0,\\
& \tau_{2j}^{A}-\theta_2 q_{2j}-F(\theta_2) \geq 0,\\
& \tau_{i1}^{B}-\theta_1 q_{i1}-F(\theta_1) \geq 0,\\
\text{and } & \tau_{i2}^{B}-\theta_2 q_{i2}-F(\theta_2) \geq 0, \quad i=1,2 \text{ and } j=1,2.
\end{aligned}$$

We distinguish five regimes depending on the magnitude of the difference $F(\theta_1)-F(\theta_2)$, which affects which constraints in (P-1) are binding. Due to the symmetry between the two agents in our model, we can follow the analysis of Kobayashi (2018) to find the optimal solutions to the contracting problem (P-1). For a complete analysis of the case of a single agent, the reader is referred to the work of Kobayashi (2018). In addition, Kobayashi (2019) presents for the case of multiple agents. In this paper, for simplicity, we focus on the cases in which firms earn positive informational rents.

Because the agents are symmetric, it is natural to consider symmetric mechanisms. Let us define $\tau_{11}\equiv\tau(\theta_1,\theta_1), \tau_{12}\equiv\tau(\theta_1,\theta_2), \tau_{21}\equiv\tau(\theta_2,\theta_1)$, and $\tau_{22}\equiv\tau(\theta_2,\theta_2)$.

First, we analyze the case when a difference in fixed costs with respect to the firm's productivity types, $F(\theta_1)-F(\theta_2)$, is sufficiently small. Next, we consider the case in which the difference in fixed costs with respect to the firms' types is sufficiently large.

The following two propositions summarize the main results. Proposition 1 shows the second best output is smaller than the first best output if the difference in fixed costs with respect to the firms' types is sufficiently small. Proposition 2 demonstrates the second-best output is larger than the first-best output because of countervailing incentives if the difference in fixed costs with respect to the firms' types is sufficiently large.

Proposition 1 *When $F(\theta_1)-F(\theta_2)$ is sufficiently small, the optimal contracts are characterized as follows:*

$$S_q(q_{11}^*)=2\theta_1,$$
$$S_q(q_{12}^*)=S_q(q_{21}^*)=\theta_1+\theta_2+\frac{p}{1-p}(1-\alpha)(\theta_2-\theta_1),$$
$$\textit{and } S_q(q_{22}^*)=2\theta_2+\frac{2p}{1-p}(1-\alpha)(\theta_2-\theta_1),$$

where $S_q(\cdot)$ denotes the partial derivative with respect to q and q_{ij}^* an equilibrium output.

Proof: First, define $\tau_{12}\equiv\tau(\theta_1,\theta_2)$, $\tau_{21}\equiv\tau(\theta_2,\theta_1)$ and $q_m\equiv q(\theta_1,\theta_2)=q(\theta_2,\theta_1)$. Suppose that the difference $F(\theta_1)-F(\theta_2)$ is sufficiently small. Then, the following constraints are binding:

$$\tau_{11}-\theta_1 q_{11}\geq\tau_{21}-\theta_1 q_m,$$
$$\tau_{12}-\theta_1 q_m\geq\tau_{22}-\theta_1 q_{22},$$

and

$$p[\tau_{21}-\theta_2 q_m-F(\theta_2)]+(1-p)[\tau_{22}-\theta_2 q_{22}-F(\theta_2)]\geq 0.$$

From these binding conditions, monetary transfers to the firms are given by

$$\tau_{22}=p[\theta_1 q_{22}+(\theta_2-\theta_1)q_m]+(1-p)\theta_2 q_{22}+F(\theta_2),$$
$$\tau_{12}=\tau_{22}-\theta_1 q_{22}+\theta_1 q_m,$$

and

$$\tau_{11}=\tau_m-\theta_1 q_m+\theta_1 q_{11}.$$

It can be shown that these transfers satisfy the remaining ICCs and PCs. Substituting these transfers into (P-1) and taking the first order conditions with respect to q_{ij} yield

$$S_q(q_{11}^*)=2\theta_1,$$
$$S_q(q_{12}^*)=S_q(q_{21}^*)=\theta_1+\theta_2+\frac{p}{1-p}(1-\alpha)(\theta_2-\theta_1),$$
$$\text{and } S_q(q_{22}^*)=2\theta_2+\frac{2p}{1-p}(1-\alpha)(\theta_2-\theta_1).$$

This completes the proof.

We note that in this regime, the difference $F(\theta_1)-F(\theta_2)$ must satisfy $F(\theta_1)-F(\theta_2)<(\theta_2-\theta_1)q_{22}^*$. In addition, we note that q_{11}^{FB} is the first best output when both types have the low marginal cost θ_1. When one of two productivity types has the low marginal cost θ_1, output q_{12}^* and q_{21}^* are distorted and smaller than q_{11}^*. When both productivity types have the high marginal cost θ_2, output q_{22}^* is distorted and smaller than q_{12}^* and q_{21}^*.

We get $\tau_{11}=\theta_1 q_{11}^*+(\theta_2-\theta_1)q_{21}^*+F(\theta_2)$ and $\tau_{12}=\theta_1 q_{12}^*+(\theta_2-\theta_1)q_{22}^*+F(\theta_2)$. The firms' payoffs are

$$u_{11}=(\theta_2-\theta_1)\left(\frac{q_{21}^*+q_{12}^*}{2}\right)+F(\theta_2)-F(\theta_1)>0,$$
$$u_{12}=u_{21}=(\theta_2-\theta_1)q_{22}^*+F(\theta_2)-F(\theta_1)>0.$$

This result is a generalization of Proposition 1 of Laffont and Martimort (1997), in which they assumed that $F(\theta_1)=F(\theta_2)=0$.

Next, we examine the case when the difference in fixed costs with respect to the firm's productivity types, $F(\theta_1)-F(\theta_2)$, is sufficiently large. In that case, countervailing incentives result. Note that in that regime, $(\theta_2-\theta_1)q_{11}^C<F(\theta_1)-F(\theta_2)$ must hold.

Proposition 2 *When $F(\theta_1)-F(\theta_2)$ is sufficiently large, the optimal contracts are characterized as follows:*

$$S_q(q_{11}^C)=2\theta_1-\frac{2(1-p)}{p}(1-\alpha)(\theta_2-\theta_1),$$
$$S_q(q_{12}^C)=S_q(q_{21}^C)=\theta_1+\theta_2-\frac{(1-p)}{p}(1-\alpha)(\theta_2-\theta_1),$$
$$\text{and } S_q(q_{22}^C)=2\theta_2.$$

Proof: Suppose that $F(\theta_1)-F(\theta_2)$ is sufficiently large. Then, the following constraints are binding.

$$p[\tau_{21}-\theta_2 q_m-F(\theta_2)]+(1-p)[\tau_{22}-\theta_2 q_{22}-F(\theta_2)]$$
$$\geqq p[\tau_{11}-\theta_2 q_{11}-F(\theta_2)]+(1-p)[\tau_{12}-\theta_2 q_m-F(\theta_2)],$$

$$p[\tau_{11}-\theta_1 q_{11}-F(\theta_1)]+(1-p)[\tau_{12}-\theta_1 q_m-F(\theta_1)]\geqq 0,$$
$$\tau_{12}-\theta_2 q_m=\tau_{11}-\theta_2 q_{11},$$

and

$$\tau_{22}-\theta_2 q_{22}=\tau_{12}-\theta_2 q_m.$$

From the binding conditions, monetary transfers τ_{11}, τ_{12}, τ_{21}, and τ_{22} are

$$\tau_{11}=-pq_{11}(\theta_2-\theta_1)+\theta_2 q_{11}-(1-p)(\theta_2-\theta_1)+F(\theta_1),$$
$$\tau_{21}=\tau_{11}-\theta_2 q_{11}+\theta_2 q_m,$$

and

$$\tau_{22}=\tau_{12}-\theta_2 q_m-\theta_2 q_{22}.$$

Substituting these transfers into (P-1) and taking the first order conditions with respect to q_{ij} yield

$$S_q(q_{11}^C)=2\theta_1-\frac{2(1-p)}{p}(1-\alpha)(\theta_2-\theta_1),$$

$$S_q(q_m^C)=\theta_1+\theta_2-\frac{1-p}{p}(1-\alpha)(\theta_2-\theta_1),$$

$$\text{and}\quad S_q(q_{22}^C)=2\theta_2.$$

This completes the proof.

The monetary transfers are

$$\tau_{12}=\tau_{21}=\theta_2 q_{21}^C+(\theta_1-\theta_2)q_{11}^C+F(\theta_1)$$

$$\text{and}\quad \tau_{22}=\theta_2 q_{22}^{FB}+(\theta_1-\theta_2)q_{12}^C+F(\theta_1).$$

The firms' payoffs are

$$u_{12}=u_{21}=(\theta_1-\theta_2)q_{11}^C+F(\theta_1)-F(\theta_2)>0,$$

$$\text{and}\ u_{22}=(\theta_1-\theta_2)\left(\frac{q_{21}^C+q_{12}^C}{2}\right)+F(\theta_1)-F(\theta_2)>0.$$

Proposition 2 shows that when $F(\theta_1)-F(\theta_2)$ is sufficiently large, countervailing incentives result. Proposition 1 shows that the agent with lower marginal costs obtains positive informational rents whereas Proposition 2 proves that the agent with high marginal costs earns positive informational rents. Propositions 1 and 2 extend a result of Laffont and Martimort (1997) to the case with fixed costs that depend on each agent's private information.

4. Optimal Contracts With Side Mechanisms

In this section, we consider the possibility of collusion between the agents. For simplicity, we assume $\alpha=0$ in the principal's objective function in this section. First, we examine collusion-proofness of contracts and frametitlethen characterize collusion-proof optimal mechanisms.

In our overall contract game, the principal offers a grand mechanism to the agents. Given the grand mechanism, the third party offers a side mechanism to the agents. A side mechanism is a triple of $x_A(\cdot)$, $x_B(\cdot)$, and $\varphi(\cdot)$ for $\forall(\theta_i,\theta_j)$, and denoted

$$\{x_A(\cdot),x_B(\cdot),\varphi(\cdot)\}.$$

A side mechanism must satisfy the budget balance constraint, which is $x_A(\theta_i,\theta_j)+x_B(\theta_i,\theta_j)=0$, for $\forall(\theta_i,\theta_j)$. The third party's maximization problem can be stated as follows.

$$\max\ \tau_A(\varphi(\theta_i,\theta_j))+\tau_B(\varphi(\theta_i,\theta_j))-(\theta_i+\theta_j)q(\varphi(\theta_i,\theta_j))-F(\varphi(\theta_i,\theta_j))-F(\varphi(\theta_i,\theta_j))$$

subject to

$$x_A(\theta_i,\theta_j)+x_B(\theta_i,\theta_j)=0,$$

$$\tau_A(\varphi(\theta_i,\theta_j))-\theta_A q(\varphi(\theta_i,\theta_j))-F(\varphi(\theta_i,\theta_j))+x_A(\theta_i,\theta_j)$$
$$\geqq \tau_A(\varphi(\tilde{\theta}_i,\theta_j))-\theta_A q(\varphi(\tilde{\theta}_i,\theta_j))-F(\varphi(\tilde{\theta}_i,\theta_j))+x_A(\tilde{\theta}_i,\theta_j),\text{ for }\tilde{\theta}_i\neq\theta_i,$$

$$\tau_B(\varphi(\theta_i,\theta_j))-\theta_B q(\varphi(\theta_i,\theta_j))-F(\varphi(\theta_i,\theta_j))+x_B(\theta_i,\theta_j)$$
$$\geqq \tau_B(\varphi(\tilde{\theta}_j,\theta_j))-\theta_b q(\varphi(\tilde{\theta}_j,\theta_j))-F(\varphi(\tilde{\theta}_j,\theta_j))+x_B(\tilde{\theta}_j,\theta_j),\text{for }\tilde{\theta}_j\neq\theta_j,$$

$$\tau_A(\varphi(\theta_i,\theta_j))-\theta_A q(\varphi(\theta_i,\theta_j))-F(\varphi(\theta_1,\theta_2))+x_B(\theta_i,\theta_j)\geqq\hat{u}^A(\theta_i,\theta_j),$$

and

$$\tau_B(\varphi(\theta_i,\theta_j))-\theta_B q(\varphi(\theta_i,\theta_j))-F(\varphi(\theta_i,\theta_j))+x_B(\theta_i,\theta_j)\geqq\hat{u}^B(\theta_i,\theta_j).$$

Here, $\hat{u}^A(\theta_i,\theta_j)$ and $\hat{u}^B(\theta_i,\theta_j)$ are each agent's payoff when the grand mechanism is played without collusion between the agents.

Following Laffont and Martimort (1997), we can show that collusion-proof constraints are characterized as follows. A ground mechanism is collusion-proof if and only if

$$2\tau_{11}-2\theta_1 q_{11}\geqq\tau_{12}+\tau_{21}-2\theta_1 q_{12}$$
$$\geqq 2\tau_{22}-2\theta_1 q_{22},$$

$$\tau_{12}+\tau_{21}-(\theta_1+\theta_2)q_{12}\geqq\tau_{22}-(\theta_1+\theta_2)q_{22}$$
$$\geqq 2\tau_{11}-(\theta_1+\theta_2)q_{11},$$

and

$$2\tau_{22}-2\theta_2 q_{22}\geqq\tau_{12}+\tau_{21}-2\theta_2 q_{12}$$
$$\geqq 2\tau_{11}-2\theta_2 q_{11}.$$

We are ready to examine collusion-proof contracts. First, we consider unlimited communication between the two agents. With unlimited communication and a side mechanism, we obtain the same optimal mechanism as the one in Proposition 1 when the difference $F(\theta_1)-F(\theta_2)$ is sufficiently small. Similarly, with unlimited communication and a side mechanism, we obtain the same optimal mechanism as the one in Proposition 2 when the difference $F(\theta_1)-F(\theta_2)$ is sufficiently large.

Next, we consider limited communication between the agents. For anonymous contracts, we define $\tau_{12}=\tau_{21}=\tau_m$. First, we consider a standard case, in which $F(\theta_1)-F(\theta_2)$ is sufficiently small, and downward distortion occurs.

The next proposition shows the collusion-proof contracts in the case that the difference $F(\theta_1)-F(\theta_2)$ is sufficiently small.

Proposition 3 *When $F(\theta_1)-F(\theta_2)$ is sufficiently small, the optimal contracts are characterized as follows:*

$$S_q(q_{11}^{SC})=2\theta_1,$$
$$S_q(q_m^{SC})=\theta_1+\theta_2+\frac{1+p}{2(1-p)}(\theta_2-\theta_1),$$
$$\textit{and } S_q(q_{22}^{SC})=2\theta_2+\frac{p}{1-p}(\theta_2-\theta_1).$$

Proof: From the binding conditions, the monetary transfers are

$$\tau_{11}=2\theta_1 q_{11}+(\theta_2-\theta_1)\left(\frac{q_{21}+q_{12}}{2}+q_{22}\right)+2F(\theta_2),$$
$$\tau_m=(\theta_1+\theta_2)\left(\frac{q_{21}+q_{12}}{2}\right)+(\theta_2-\theta_1)q_{22}+2F(\theta_2),$$
$$\text{and } \tau_{22}=2\theta_2 q_{22}+2F(\theta_2).$$

These transfers satisfy the remaining ICCs and PCs. Substituting these transfers into (P-2) and taking the first order conditions with respect to q_{ij} yield

$$S_q(q_{11}^{SC})=2\theta_1,$$
$$S_q(q_{12}^{SC})=S_q(q_{21}^{SC})=\theta_1+\theta_2+\frac{1+p}{2(1-p)}(\theta_2-\theta_1),$$
$$\text{and } S_q(q_{22}^{SC})=2\theta_2+\frac{p}{1-p}(\theta_2-\theta_1).$$

This completes the proof.

In this case, we must have that $F(\theta_1)-F(\theta_2)<(\theta_2-\theta_1)q_{22}^{SC}$ holds. Thus we have

$$q_{22}^{SC}<q_{22}^{FB},\quad q_{22}^{SC}<q_{12}^{SC}=q_{21}^{SC}=q_m^{SC}<q_{11}^{SC}=q_{11}^{FB}.$$

Output q_{11} is at the first-best level q_{11}^{FB}, and q_m^{SC} is distorted and smaller than q_{11}^{SC}. Output q_{22}^{SC} is distorted and smaller than q_m^{SC} .

The firms' payoffs are

$$u_{11}=(\theta_2-\theta_1)(q_{21}^{SC}+q_{12}^{SC})+2F(\theta_2)-2F(\theta_1),$$
$$u_{12}=(\theta_2-\theta_1)q_{22}^{SC}+F(\theta_2)-F(\theta_1),$$

In this case, we must have $F(\theta_1)-F(\theta_2)<(\theta_2-\theta_1)q_{11}^{SC}$. The firms' payoffs are

$$u_{22}=(\theta_2-\theta_1)\left(\frac{q_{21}^{SC}+qS_{12}^{C}}{4}+q_{11}^{C}\right)+\frac{3[F(\theta_1)-F(\theta_2)]}{2}>0.$$

Proposition 3 shows that the agent with low marginal costs obtains positive informational rents.

Next, we consider the case in which countervailing incentives arise. The next proposition shows the collusion-proof contracts in the case that the difference $F(\theta_1)-F(\theta_2)$ is sufficiently large.

Proposition 4 *When $F(\theta_1)-F(\theta_2)$ is sufficiently large, the optimal contracts are characterized as follows:*

$$S_q(q_{11}^{SCC})=2\theta_1-\frac{2(1-p)}{p}(\theta_2-\theta_1),$$
$$S_q(q_{12}^{SCC})=S_q(q_{21}^{SC})=\theta_1+\theta_2-\frac{1-p}{p}(\theta_2-\theta_1),$$
and $S_q(q_{22}^{SCC})=2\theta_2.$

Proof: From the binding conditions, the monetary transfers are

$$\tau_{11}=2\theta_1 q_{11}+(\theta_2-\theta_1)\left(\frac{q_{21}+q_{12}}{2}+q_{22}\right)+2F(\theta_2),$$
$$\tau_m=(\theta_1+\theta_2)\left(\frac{q_{21}+q_{12}}{2}\right)+(\theta_2-\theta_1)q_{22}+2F(\theta_2),$$
and $\tau_{22}=2\theta_2 q_{22}+2F(\theta_2).$

These transfers satisfy the remaining ICCs and PCs. Substituting these transfers into (P-1) and taking the first-order conditions with respect to q_{ij} yield

$$S_q(q_{11}^{SCC})=2\theta_1-\frac{2(1-p)}{p}(\theta_2-\theta_1)$$
$$S_q(q_{12}^{SCC})=S_q(q_{21}^{SCC})=\theta_1+\theta_2-\frac{1-p}{p}(\theta_2-\theta_1)$$
and $S_q(q_{22}^{SCC})=2\theta_2.$

This completes the proof.

In contrast to Proposition 3, Proposition 4 shows that the agent with high marginal costs earns positive informational rents. We note that

$$q_{11}^{FB}<q_{11}^{SCC},\ \ q_{12}^{SCC}=q_{21}^{SCC}=q_m^{SCC},\ \text{ and } q_{22}^{SCC}=q_{22}^{FB}.$$

Propositions 3 and 4 show that participation constraints and coalition incentive constraints do not conflict. Thus, the optimal mechanism implements the second-best outcome.

5. Conclusion

We have examined optimal mechanisms in an adverse selection model of one principal and multiple agents in which the principal procures complementary products from the agents. We have characterized optimal contracts with and without side contracts. We have shown that with side contracts, when the difference in fixed costs with respect to productivity types, $F(\theta_1)-F(\theta_2)$, is sufficiently small, the optimal contracts exhibit downward distortion. We have also examined the case in which the value of $F(\theta_1)-F(\theta_2)$ is sufficiently large. In this case, we have shown that the optimal contracts entail countervailing incentives and upward distortion arises at equilibrium.

This paper has shown that the difference in fixed costs with respect to productivity types, $F(\theta_1)-F(\theta_2)$, affects which ICCs and PCs are binding and thus the firms' informational rents. Thus, it is critically important to understand that optimal mechanisms with and without collusion depend on the difference in fixed costs with respect to the agents' types when designing regulatory policies under asymmetric information and addressing the issue of collusion among agents.

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