Trend Analysis of the Nikkei Stock Average under Japan's Low Interest Rate Policy

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1. Introduction

This study focuses on the trend characteristics of the Nikkei Stock Average (Nikkei 225) using the Markov-Switching Autoregressive Moving Average GARCH (MS-ARMA-GARCH) model under the low interest rate environment in Japan. The Bank of Japan's monetary policy has been as follows: zero interest rate policy from February 1999 to August 2000, February 2001 to July 2006, and October 2010 to April 2013; quantitative easing policy from March 2001 to March 2006; inflation targeting policy from January 2013 to the present; and quantitative and qualitative monetary easing policy from April 2013 to the present. In this study, the period of low interest rates is from Friday, February 12, 1999, when the zero interest rate policy was implemented, to Saturday, April 8, 2023, when Bank of Japan Governor Kuroda stepped down.

If trends do exist, it should be possible to observe upward and downward trends – so-called bull and bear markets – using a time series model. One common trend analysis model is the Markov-switching model. When using the Markov-switching model for trend analysis, market trends are first separated into the two regimes: bull and bear. The mean of the change rate of stock prices has two states, negative or positive. If the positive value continues, this indicates an upward trend (bull market), and if the negative value continues, this indicates a downward trend (bear market). The assumption is that these two states follow the transition of a Markov process. In general, a model of changing volatility[1)] is used for time series analysis of asset prices. Studies using the Markov-switching model, a model of changing volatility, include Hamilton and Susmel (1994) and Cai (1994), which use the Markov-switching ARCH (Autoregressive conditional heteroscedasticity) model, and Gray (1996), Klaassen (2002), and Haas *et al*. (2004), which use the Markov-switching GARCH (Generalized ARCH) model (hereinafter, the MS-GARCH model)[2)] . Henneke *et al*. (2011) conducted Bayesian estimation with the Markov chain Monte Carlo (MCMC) method for the estimation of the MS-ARMA-GARCH model. In the Japanese stock market, Satoyoshi (2004) uses the Markov-switching model on the TOPIX, and Satoyoshi and Mitsui (2011b), Satoyoshi and Mitsui (2012), and Mitsui (2015) use the same on the Nikkei Stock Average.

In recent years, the stock market has seen violent fluctuations due to hedge funds and future-driven market prices by institutional investors. Accordingly, this paper analyzes bull and bear markets of the Nikkei Stock Average rather than stock price indexes such as the Nikkei Stock Average. An empirical analysis was conducted on

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the daily Nikkei Stock Averages from February 12, 1999, to April 7, 2023, using the MS-ARMA-GARCH model. The empirical analysis showed statistically significant bull and bear regimes in the Nikkei Stock Average. In other words, we were able to identify bull markets, with high expected returns and low volatility, and bear markets, with low expected returns and high volatility. It was firmly established that the MS-ARMA-GARH model is valid models for analyzing bull and bear markets in the Nikkei Stock Average.

The model was estimated using the maximum likelihood method. During the low interest rate period under study, 69% (or 4,087) of the days were in a bull market, with high expected returns and low volatility, while 31% (or 1,832) of the days were in a bear market, with low expected returns and high volatility. The average duration of the bull phase was 8.982 days, while that of the bear phase was approximately 4.026 days. It is clear that the MS-ARMA-GARCH model was effective in analyzing the bull and bear markets of the Nikkei Stock Average. It was also found that Nikkei Stock Average exhibited a long-term bull market trend after the financial crisis in 2008, the Great East Japan Earthquake in March 2011, and the big crash at the time of the Corona Shock in February 2020.

The remainder of the paper is organized as follows: Section 2 explains the MS-ARMA-GARCH model and the methods used to estimate the analytical model. Section 3 describes the Nikkei Stock Average data and discusses the estimation results. Section 4 concludes the discussion and identifies future issues.

2. Methodology

2.1 MS-ARMA-GARCH Model

We begin this section by briefly explaining the MS-ARMA-GARCH model, which is based on the MS-GARCH model developed in Gray (1996), Klaassen (2002), and Haas *et al.* (2004). Let R_t be the rate of return on the Nikkei Stock Average at time t. If P_t is the level of the Nikkei Stock Average at time t, the rate of return of the Nikkei Stock Average, R_t, can be expressed as

$$R_t = (\ln P_t - \ln P_{t-1}) \times 100. \tag{2.1}$$

In the MS-ARMA-GARCH model, the processes for determining R_t and volatility σ_t^2 can be shown as follows:

$$R_t = \mu(S_t) + \sum_{i=1}^{r} \phi_i(S_t) R_{t-i} + \epsilon(S_t) + \sum_{j=1}^{m} \psi_j(S_t)\epsilon_{t-j}(S_t), \tag{2.2}$$

$$\sigma_t^2(S_t) = \omega(S_t) + \sum_{i=1}^{q} \alpha_i(S_t)\epsilon_{t-i}^2(S_t) + \sum_{j=1}^{p} \beta_j(S_t)\sigma_{t-j}^2(S_t), \tag{2.3}$$

$$\epsilon_{t-j-1}(S_{t-j}) = E[\epsilon_{t-j-1}(S_{t-j-1})|S_{t-j}, I_{t-j-1}], \tag{2.4}$$

$$\sigma_{t-j-1}(S_{t-j}) = E[\epsilon_{t-j-1}(S_{t-j-1})|S_{t-j}, I_{t-j-1}]. \tag{2.5}$$

Here, $\mu(S_t)$ is a constant term, and there is no autocorrelation in the returns. $E[\cdot|\cdot]$ is the conditional expected value. I_{t-j-1} is the information set $I_{t-j-1} = \{R_{t-1}, R_{t-2}, \cdots\}$ up until $t-j-1$. It is assumed that the constant term μ, ω and volatility σ_t^2 follow state variable S_t and switch simultaneously. To ensure nonnegativity of volatility, it is assumed that $\omega(S_t)$, $\alpha(S_t)$, $\beta(S_t)>0$. The order selection of ARMA and GARCH are ARMA(1,1)-GARCH(1,1); therefore, we will focus on the MS-ARMA(1,1)-GARCH(1,1) model.

$$R_t = \mu(S_t) + \phi(S_t)R_{t-1} + \epsilon(S_t) + \psi(S_t)\epsilon_{t-1}(S_t), \tag{2.6}$$

$$\epsilon_t(S_t) = \sigma_t(S_t)z_t,\ \ z_t \sim i.i.d., E[z_t] = 0,\ \ Var[z_t] = 1, \tag{2.7}$$

$$\sigma_t^2(S_t) = \omega(S_t) + \alpha(S_t)\epsilon_{t-1}^2(S_t) + \beta(S_t)\sigma_{t-1}^2(S_t), \tag{2.8}$$

$$\epsilon_{t-1}(S_t) = E[\epsilon_{t-1}(S_{t-1})|S_t, I_{t-1}], \tag{2.9}$$

$$\sigma_{t-1}(S_t) = E[\epsilon_{t-1}(S_{t-1})|S_t, I_{t-1}]. \tag{2.10}$$

In the above, $i.\ i.\ d.$ indicates independent and identically distributed values. z_t is an error term, $E[\cdot]$ is the expected value, and $Var[\cdot]$ is the variance. In the Markov-switching model, the state variable S_t that is not observed follows a Markov process and can be defined with the following transition probability:

$$p_{i|j} = Pr[S_{t+1} = i|S_t = j],\ \ i, j = 0, 1. \tag{2.11}$$

$Pr[S_{t+1}=i|S_t=j]$ is the probability that the state transitions from j to i. However, the probability that this term's state j transitions to next term's state i is dependent only on this term, as shown belows[3]:

$$Pr[S_{t+1} = i|S_t = j, S_{t-1}, S_{t-2,\ldots}] = p_{i|j} = Pr[S_{t+1} = i|S_t = j]. \tag{2.12}$$

Here, the following holds:

$$\sum_{i=0}^{1} p_{i|j} = 1,\ \ j = 0, 1. \tag{2.13}$$

Then the transition matrix $\mathbf{P}$ of S_t is

$$\mathbf{P} = \begin{pmatrix} p_{0|0} & p_{0|1} \\ p_{1|0} & p_{1|1} \end{pmatrix}. \tag{2.14}$$

Here, $0 \leq p_{0|0}, p_{1|1} \leq 1$.

This study considers the condition $S_t=0$ as indicative of a bull market and $S_t=1$ as indicative of a bear market[4]. Therefore, $p_{0|1}$ is the transition probability from bull to bear market, and $p_{1|0}$ is the transition probability from bear to bull market. Moreover, $p_{0|0}$ and $p_{1|1}$ represent the transition probabilities of a maintained bull market and maintained bear market, respectively. The restriction $\mu(0)>\mu(1)$ exists[5]. In our empirical analysis, the distribution of the error term is assumed to follow the standard normal distribution as shown below[6].

$$z_t \sim i.i.d.N\left(0, 1\right). \tag{2.15}$$

Here, the estimated parameters are $\Theta = \{\mu(0), \mu(1), \phi(0), \phi(1), \psi(0), \psi(1), \omega(0), \omega(1), \alpha(0), \alpha(1), \beta(0), \beta(1), p_{0|0}, p_{1|1}\}$. In this study, parameters are estimated with the maximum likelihood method for simplicity.

2.2 Estimation Method

Let $L(\Theta)$ denote the likelihood function. This likelihood function can be determined as follows:

$$\begin{aligned} L(\Theta) &= f\left(R_1, R_2, \cdots, R_T\right) = \prod_{t=1}^{T} f\left(R_t | I_{t-1}; \Theta\right) \\ &= \prod_{t=1}^{T} \sum_{j=0}^{1} \left(R_t | S_t = j, I_{t-1}\right) \Pr\left[S_t = j, | I_{t-1}\right]. \end{aligned} \tag{2.16}$$

Then the log-likelihood function $\ln L$ can be expressed as

$$\begin{aligned} \ln L &= \sum_{t=1}^{T} \ln \left\{ \sum_{j=0}^{1} \left(R_t | S_t = j, I_{t-1}; \Theta\right) \Pr\left[S_t = j, | I_{t-1}; \Theta\right] \right\} \\ &= \sum_{t=1}^{T} \ln \left\{ \mathbf{i}' \left(\boldsymbol{\eta}_t \odot \hat{\boldsymbol{\xi}}_{t|t-1} \right) \right\}, \end{aligned} \tag{2.17}$$

where

$$\mathbf{i} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ \boldsymbol{\eta}_t = \begin{pmatrix} f(R_t | S_t = 0, I_{t-1}; \Theta) \\ f(R_t | S_t = 1, I_{t-1}; \Theta) \end{pmatrix}, \ \hat{\boldsymbol{\xi}}_{t|t-1} = \begin{pmatrix} \Pr[S_t = 0, | I_{t-1}; \Theta] \\ \Pr[S_t = 1, | I_{t-1}; \Theta] \end{pmatrix}.$$

Here, the symbol $\odot$ denotes element-by-element multiplication. $\hat{\boldsymbol{\xi}}_{t|t-1}$ in equation (2.17) is obtained with the filtering method proposed by Hamilton (1989) (Hamilton Filter)[7]. We can express

$$\hat{\boldsymbol{\xi}}_{t|t-1} = \left(\mathbf{P} \otimes \mathbf{Q}\right) \hat{\boldsymbol{\xi}}_{t-1|t-1}, \tag{2.18}$$

$$\hat{\boldsymbol{\xi}}_{t|t} = \frac{\left(\boldsymbol{\eta}_t \odot \hat{\boldsymbol{\xi}}_{t|t-1} \right)}{\mathbf{i}' \left(\boldsymbol{\eta}_t \odot \hat{\boldsymbol{\xi}}_{t|t-1} \right)}. \tag{2.19}$$

By repeating the above equations (2.18) and (2.19) alternately, $\hat{\boldsymbol{\xi}}_{t|t-1}$ is calculated for $t=1, 2, \ldots, T$ and is substituted into equation (2.17)[8].

For estimation of the parameters, maximum likelihood estimation is conducted using the statistical and time series analysis software *PcGive*[9].

3 Empirical Results

3.1 Data

Data for the Nikkei Stock Average were obtained from Bloomberg. The data period is from February 12, 1999, to April 7, 2023 (see Figure 1)[10]. The rate of return was calculated as the percentage change (%) in the closing price of each index (see Figure 2). The sample period is from February 15, 1999, to April 7, 2023, which produced a sample size of 5,920. The mean, standard deviation, skewness, excess kurtosis, maximum, minimum, and normality test statistics[11)] for the data are summarized in Table 1. The negative skewness value here indicates that the distribution of the Nikkei Stock Average is skewed to the left. Since excess kurtosis of the rate of return of the Nikkei Stock Average exceeds 0 and the normality test was significant, it is obvious that the distribution of the rate of return of the Nikkei Stock Average has thicker tails than the normal distribution. The histogram and density function of the rate of return are shown in Figure 3. In this figure, the density and normal distributions are superimposed. According to Table 1, $N\,(s\,=\,1.458)$ follows the normal distribution $N\,(0.011,\,1.458)$ with a mean of 0.011 and a variance of 1.458^2. Figure 4 depicts the autocorrelation of $|R_t|$. From Figure 4, we can see that the decay of autocorrelation of $|R_t|$ is very slow. This suggests that the series of $|R_t|$ has a long-term memory.

Table 1: Summary statistics for daily rates of return for the Nikkei Stock Average

February 15, 1999 – April 7, 2023, No. of Obs. 5,920

Mean	Std. Dev.	Skewness	Exc. Kurtosis	Max.	Min.	Normality Test
0.011	1.458	−0.349*	6.088*	13.235	−12.111	2890.2**
(0.019)		(0.032)	(0.064)			

(i) The numbers in parentheses indicate the standard errors. Let T be the sample size and $\hat{\sigma}$ be the standard deviation; then the standard errors of the mean, skewness, and kurtosis are, respectively, $\hat{\sigma}/\sqrt{T}$, $\sqrt{6/T}$, $\sqrt{24/T}$.
(ii) * indicates that the result is significant at the significance level of 5%.
(iii) ** indicates that the result is significant at the significance level of 1%.

Figure 1: Daily closing prices on the Nikkei Stock Average (2/12/1999 – 4/7/2023)

Figure 2: Daily closing prices on the Nikkei Stock Average (2/12/1999 – 4/7/2023)

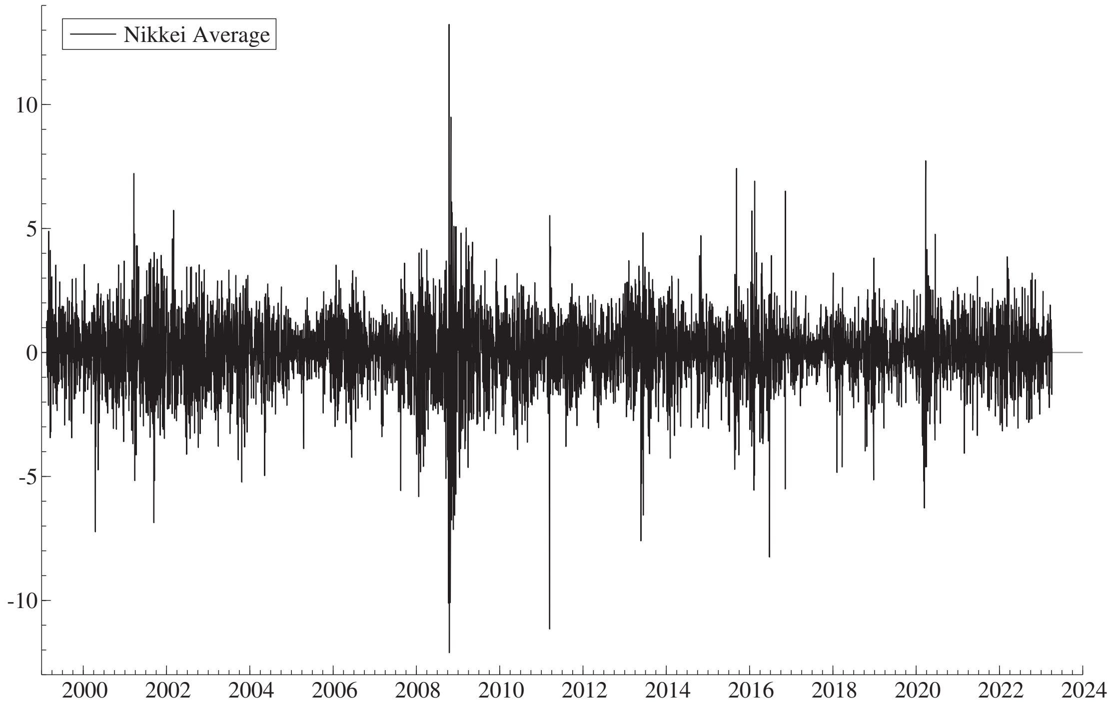


Figure 3: Histogram of the Nikkei Stock Average returns and its normal approximation

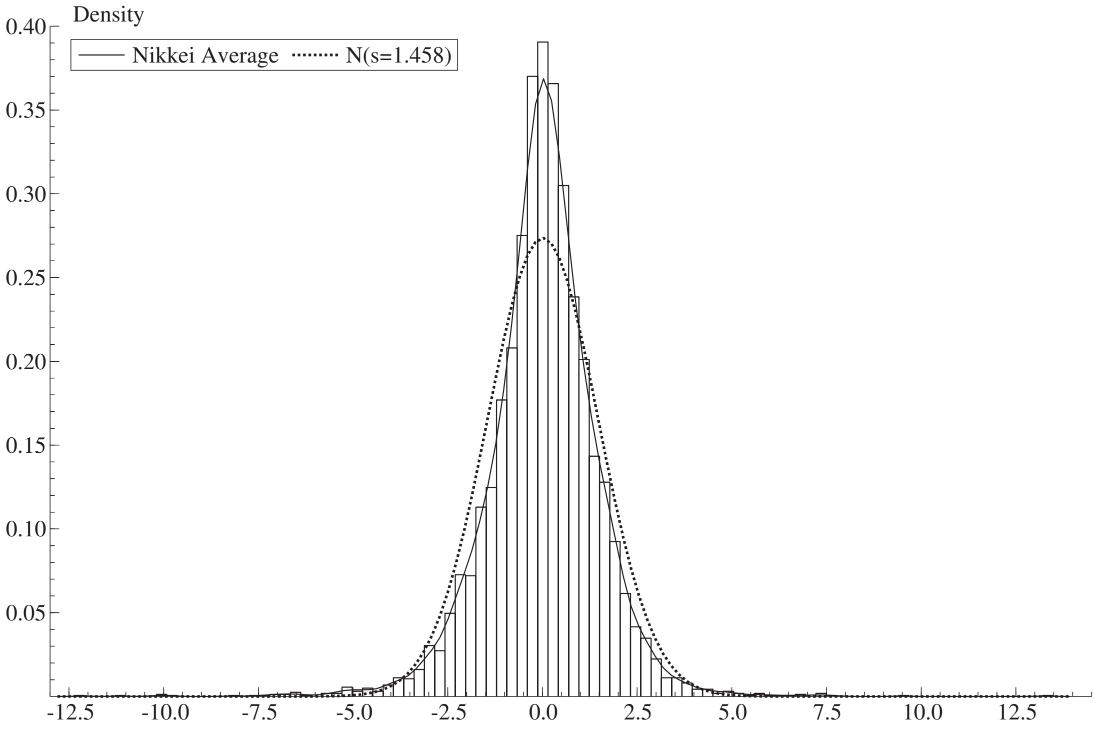
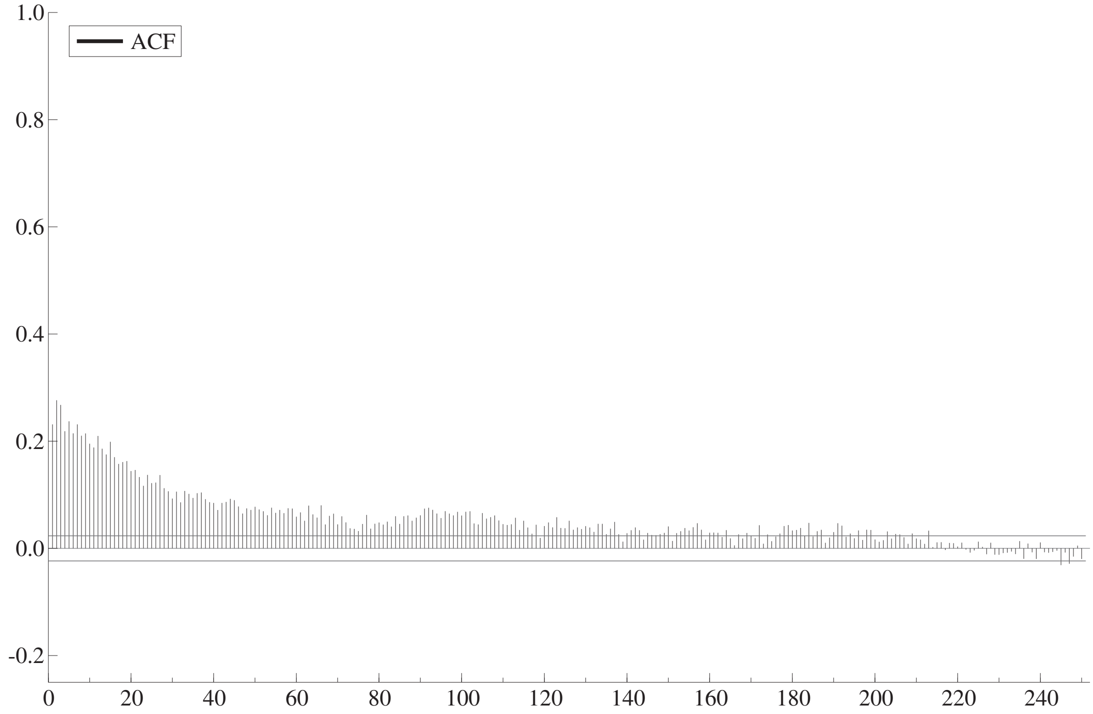


Figure 4: ACF for the Nikkei Stock Average $|R_t|$



3.2 Estimation Results

Table 2 shows the estimation results of the Nikkei Stock Average. Let us consider the MS-ARMA(1,1)-GARCH(1,1) model. The estimated values of $\mu(0)$ and $\mu(1)$ are 0.709 and -0.279, respectively, and are statistically significant results. $\mu(0)$, which indicates a bull market, is a positive value and $\mu(1)$, which indicates a bear market, is a negative value. This confirms that when the state variable $S_t=0$, the Nikkei Stock Average is in a bull market, and when $S_t=1$, the Nikkei Stock Average is in a bear market. The estimated values of $\omega(0)$ and $\omega(1)$ are 0.018 and 0.279, respectively, and are statistically significant results. Because $\omega(0)<\omega(1)$, we can see that a bear market has a higher volatility value than a bull market. The estimated value of the parameter showing a clustering of volatility shock is $\alpha(0)+\beta(0)=0.982$, $\alpha(1)+\beta(1)=0.992$, demonstrating that the clustering of shock is higher in both bear and bull markets. The estimated values of transition probabilities $p_{0|0}$ and $p_{1|1}$ are 0.852 and 0.795, respectively, and are statistically significant results. $p_{0|0}$ is extremely close to 1, implying that if there is a switch to a bull market, that state will be maintained for a relatively long period. Because $p_{0|0}>p_{1|1}$, we can see that a bear market does not last as long as a bull market. Also, the mean μ and volatility σ switch simultaneously following the state variable S_t, so when it switches to a low volatility state it remains in that state for a long period, whereas when it switches to a high volatility state it does not stay there for long. $Q(20)$ and $Q^2(20)$ represent the normalization residuals $(\hat{\epsilon}\hat{\sigma}^{-1})$ up to 20, and the Q-statistic of the squared Ljung-Box test. It follows the asymptotic degree of freedom 20 with χ^2 distribution. Statistically significant estimation values were not produced with the MS-ARMA(1,1)-GARCH(1,1) model. Regarding $Q(20)$ and $Q^2(20)$, a null hypothesis cannot be rejected with a

Table 2: Estimation results for the MS-ARMA(1,1)-GARCH(1,1) model

$$R_t = \mu(S_t) + \phi(S_t)R_{t-1} + \epsilon(S_t) + \psi(S_t)\epsilon_{t-1}(S_t),$$
$$\epsilon_t(S_t) = \sigma_t(S_t)z_t,\ z_t \sim i.i.d.N(0,1)$$
$$\sigma_t^2(S_t) = \omega(S_t) + \alpha(S_t)\epsilon_{t-1}^2(S_t) + \beta(S_t)\sigma_{t-1}^2(S_t).$$
$$\mathbf{P} = \begin{pmatrix} p_{0|0} & p_{0|1} \\ p_{1|0} & p_{1|1} \end{pmatrix}.$$

	$\mu(0)$	$\mu(1)$	$\phi(0)$	$\phi(1)$	$\psi(0)$	$\psi(1)$
Estimates	0.709*	− 0.279*	0.809*	0.680*	− 0.843*	− 0.615*
Standard Errors	(0.127)	(0.111)	(0.044)	(0.052)	(0.033)	(0.057)

	$\omega(0)$	$\omega(1)$	$\alpha(0)$	$\alpha(1)$	$\beta(0)$	$\beta(1)$	$p_{0\|0}$	$p_{1\|1}$
Estimates	0.018	0.279*	0.016*	0.138*	0.966*	0.854*	0.852*	0.795*
Standard Errors	(0.043)	(0.049)	(0.005)	(0.021)	(0.008)	(0.022)	(0.035)	(0.066)

	$\ln L$	AIC	$SBIC$	$Q(20)$	$Q^2(20)$
Estimates	− 9790.33	3.313	3.329	15.149	39.741

* denotes statistical significance at the 5% level.

10% significance level. Therefore, we can see that the MS-ARMA(1,1)-GARCH(1,1) model captures the autocorrelation of Nikkei Stock Average volatility.

3.3 Discussion

Figure 5 shows the periods of the Nikkei Stock Average when $S_t = 0$ (bull market). As noted earlier, 4,087 (or 69.05%) of the total days in the sample were in a bull market state. The average duration of the bull market state was just under 9 days. Tables 3 and 4 show the summary statistics for the number of days in bull and bear markets, respectively. Overall, 1,832 (or 30.95%) of the total days in the sample were in a bear market state. The average duration of the bear market state was slightly more than 4 days. These results indicate that it takes time for stock prices to rise but that they fall rather quickly. Table 5 shows the top 10 long-running bull trends, and Table 6 shows the top 10 long-running bear trends. The black lines in Figures 5 and 6 indicate the smoothed probabilities of bull and bear markets. The shaded areas in Figure 5 show bull markets; in Figure 6, they show bear markets.

The smoothed probabilities can be computed using the backward iteration suggested by Kim (1993):

$$\boldsymbol{\xi}_{t|T} = \boldsymbol{\xi}_{t|t} \odot \{\boldsymbol{P}'[\boldsymbol{\xi}_{t+1|T}(\boldsymbol{P} \oslash \boldsymbol{\xi}_{t|t})]\}, \tag{3.1}$$

$$\boldsymbol{\xi}_{t|t} = \frac{diag(\boldsymbol{\eta}_t)}{\boldsymbol{\xi}'_{t|t-1}\boldsymbol{\eta}_t},\ \boldsymbol{\xi}_{t+1|t} = \boldsymbol{P}\boldsymbol{\xi}_{t|t}, \tag{3.2}$$

where $\oslash$ is used for element-by-element division and $diag(\boldsymbol{\eta}_t)$ creates a diagonal matrix with $\boldsymbol{\eta}_t$ on the diagonal.

Table 3: Summary statistics for regime 0 (bull market)

Total: 4087 days (69.05%) with average duration of 8.98 days. No. of Obs. 455

Mean	Std. Dev.	Skewness	Exc. Kurtosis	Max.	Min.	Median
8.982	9.000	2.375	7.544	61.000	1.000	6.000

Table 4: Summary statistics for regime 1 (bear market)

Total: 1832 days (30.95%) with average duration of 4.03 days. No. of Obs. 455

Mean	Std. Dev.	Skewness	Exc. Kurtosis	Max.	Min.	Median
4.026	3.977	2.218	6.251	26.000	1.000	3.000

Table 5: Top 10 (bull trend)

Period	days
05/09/2014 – 08/04/2014	61
08/03/2020 – 10/29/2020	61
12/10/2004 – 02/22/2005	48
06/02/2017 – 08/08/2017	47
05/24/2011 – 07/27/2011	46
04/03/2020 – 06/10/2020	45
09/30/2015 – 12/03/2015	44
03/16/2011 – 05/20/2011	43
06/10/2005 – 08/03/2005	38
03/27/2018 – 05/22/2018	38

Table 6: Top 10 (bear trend)

Period	days
09/30/2008 – 11/06/2008	26
04/16/2010 – 05/27/2010	26
02/25/2020 – 03/27/2020	23
09/08/2005 – 10/07/2005	20
06/04/2002 – 06/27/2002	18
11/29/2005 – 12/22/2005	18
01/04/2008 – 01/29/2008	17
05/23/2013 – 06/14/2013	17
02/22/2011 – 03/15/2011	16
12/04/2018 – 12/26/2018	16
10/30/2020 – 11/24/2020	16

4 Conclusions

In this study, an MS-ARMA(1,1)-GARCH(1,1) model was used to conduct a trend analysis of the Nikkei Stock Average under the low interest rate environment in Japan. During the low interest rate period under study, 69% of the days were in a bull market with high expected returns and low volatility, while 31% were in a bear market with low expected returns and high volatility. The average duration of the bull phase was approximately 9 days, while that of the bear phase was approximately 4 days. Results of the study make clear that the MS-ARMA-GARCH model is effective in analyzing the bull and bear markets of the Nikkei Stock Average. It was also found that the trend of the Nikkei Stock Average has been a long-term bull market after the financial crisis in 2008, the Great East Japan Earthquake in March 2011, and the big crash at the time of the Corona Shock in February 2020 [12)].

Future issues to be addressed include conducting an analysis using the MS-ARMA-EGARCH model. While this study modeled volatility changes with the GARCH model, Henry (2009) proposed the MS-EGARCH model [13)], which incorporates the EGARCH (Exponential GARCH) model formulated by Nelson (1991). In addition, Maheu *et al.* (2012) offers a four-state Markov-switching model that identifies four trends – bull market, bear market, bear market rally, and bear market correction[14)]. It is important to analyze trends in detail. Because this study uses daily

Figure 5: The smoothed probability of bull regime for the Nikkei Stock Average

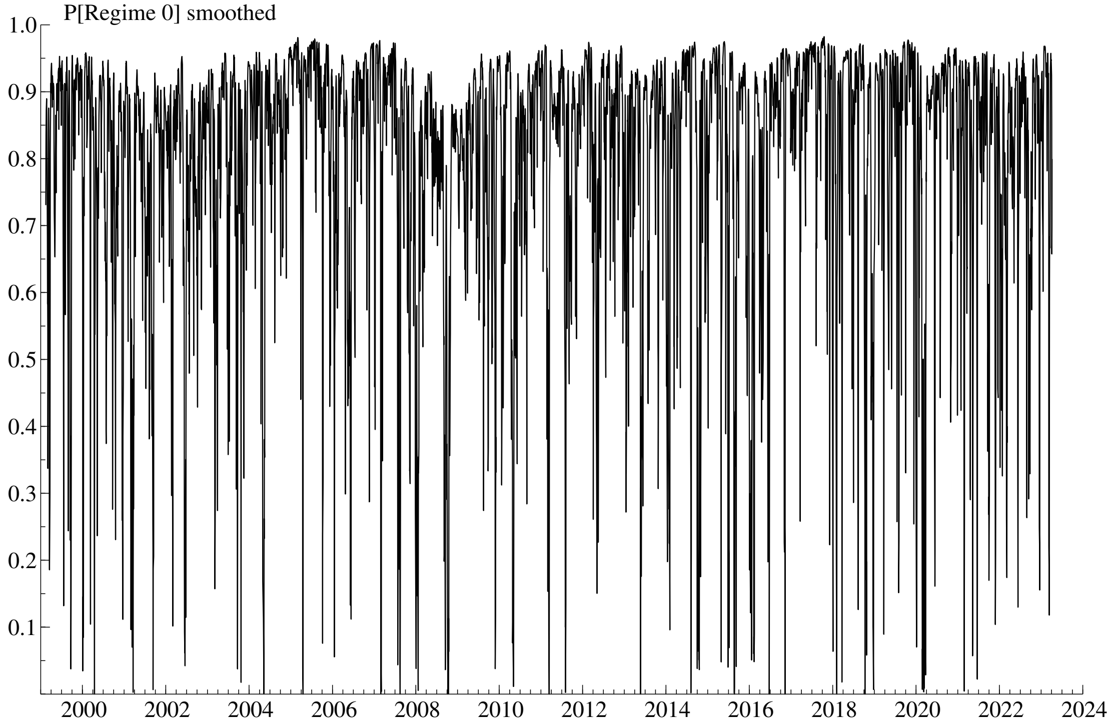
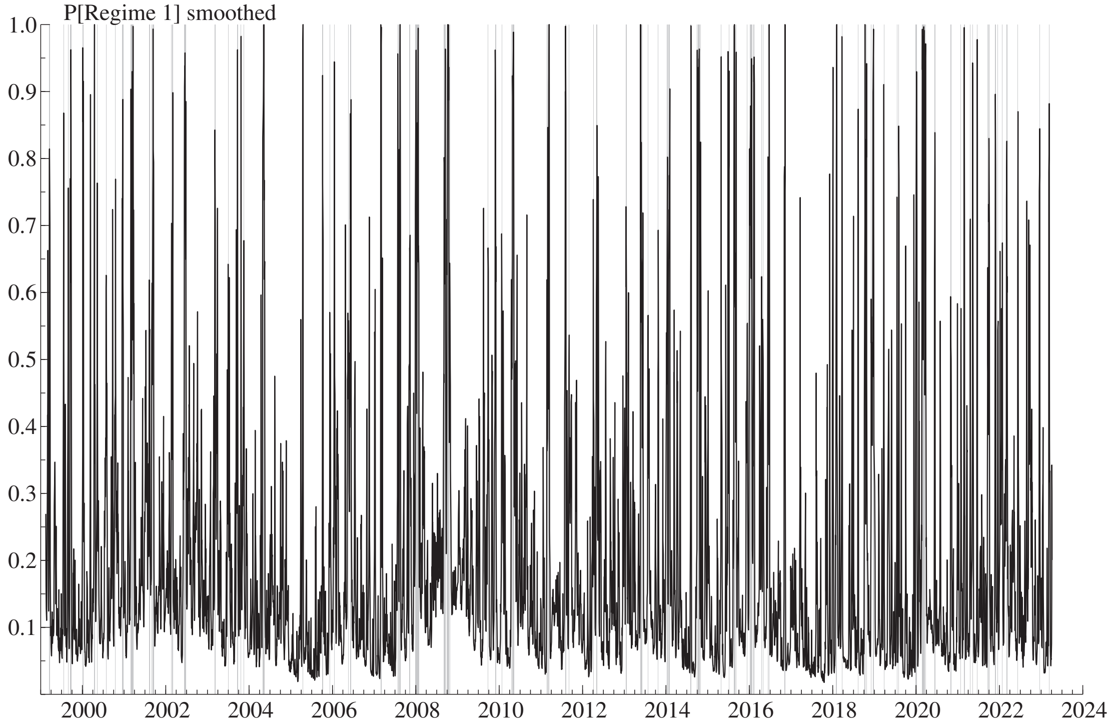


Figure 6: The smoothed probability of bear regime for the Nikkei Stock Average



data, the bear market state turned out to be quite long in the empirical analysis. Using weekly or monthly data will likely eliminate this problem. Finally, Mitsui and Totsuka (2022) and Totsuka and Mitsui (2022), for example, conduct empirical studies using Bayesian estimation methods based on Hamiltonian Monte Carlo.

Notes

1）Volatility is defined based on the variance or standard deviation of the return on asset, and is used as the index of the risk of risky assets in finance theory.

2）Engle (1982) proposed the ARCH model that formulates the volatility at each time as the linear function of the square of the past unexpected shock. In addition, Bollerslev (1986) added the past volatility values to the explanatory variables, and extended the GARCH model to a more general model.

3）This can also be expressed as: $p_{ji} = Pr[S_{t+1} = i | S_t = j]$.

4）In equations (2.4)–(2.5), when $S_t = 0$, indicating a bull market, the following equations are obtained:

$$R_t = \mu(0) + \phi(0)R_{t-1} + \epsilon(0) + \psi(0)\epsilon_{t-1}(0),$$
$$\sigma_t^2(0) = \omega(0) + \alpha(0)\epsilon_{t-1}^2(0) + \beta(0)\sigma_{t-1}^2(0).$$

Also, when $S_t = 1$, indicating a bear market, the following equation is obtained:

$$R_t = \mu(1) + \phi(1)R_{t-1} + \epsilon(1) + \psi(1)\epsilon_{t-1}(1),$$
$$\sigma_t^2(1) = \omega(1) + \alpha(1)\epsilon_{t-1}^2(1) + \beta(1)\sigma_{t-1}^2(1).$$

5）$\mu(0) > 0$, $\mu(1) < 0$ does not always hold depending on the asset data.

6）It is known that the distribution of stock returns follows a distribution with a thicker tail compared to a normal distribution. Therefore, it is necessary to analyze the error term using fat-tailed distributions, such as the t-distribution, skewed t distribution, generalized hyperbolic skew t distribution, GED (Generalized Error Distribution), and skewed GED (SGED).

7）For details, refer to Kim and Nelson (1999).

8）For details, refer to Satoyoshi and Mitsui (2011a).

9）For further information on using *PcGive* for Markov-switching estimation, refer to Doornik and Hendry (2013).

10）The figures were created by *PcGive* (statistical and time series analysis software).

11）In this paper, we use the method of Jarque and Bera (1987), which employs skewness and kurtosis in testing the normality of the profitability distribution. The Jarque - Bera test statistic JB is

$$JB = \frac{\hat{skew}^2 T}{6} + \frac{(\hat{kurt} - 3)^2 T}{24} \sim \chi^2(2)$$

where $\hat{skew}$ and $\hat{kurt}$ are skewness and kurtosis calculated from the data, respectively, and T is the number of samples. For a normal distribution, JB is $JB = 0$, and the value of JB increases as the distribution deviates from the normal distribution. For details, see Jarque and Bera (1987).

12）For verification of the BOJ's ETF purchases, see Harada and Okimoto (2019).

13）Satoyoshi and Mitsui (2011a), Mitsui (2012,2013), and Satoyoshi and Mitsui (2016) conducted bull and bear market analysis on the Nikkei Stock Average using the MS-EGARCH model.

14）Satoyoshi and Mitsui (2013) conducted trend analysis on the Nikkei Stock Average using a four-state Markov-switching model, as with Maheu *et al*. (2012). Mitsui (2014b) also conducted empirical analysis on Nikkei 225 Futures.

References

［1］ Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics,* 31, pp. 307-327.

［2］ Cai, J. (1994), "A Markov Model of Switching-Regime ARCH," *Journal of Business & Economic Statistics*, 12, pp. 309-316.

［3］ Doornik, J. A. and D. F. Hendry (2013), *Econometric Modelling PcGive 14 Volume IV*, Timberlake Consultants Ltd.

［4］ Engle, R. F. (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica*, 50, pp. 987-1007.

［5］ Gray, S. F. (1996), "Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process," *Journal of Financial Economics*, 42, pp. 27-62.

［6］ Haas, M., S. Mittnik and M. S. Paolella (2004), "Mixed Normal Conditional Heteroskedasticity," *Journal of Financial Econometrics*, 2, pp. 211-250.

［7］ Hamilton, J. D. (1989), "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica*, 57, pp. 357-384.

［8］ Hamilton, J. D. and R. Susmel (1994), "Autoregressive Conditional Heteroskedasticity and Changes in Regime," *Journal of Econometrics*, 64, pp. 307-333.

［9］ Harada, K and T. Okimoto (2019), "The BOJ's ETF Purchases and Its Effects on Nikkei 225 Stocks," RIETI Discussion Paper Series 19-E-014. <https://www.rieti.go.jp/jp/publications/dp/19e014.pdf>.

［10］Henneke, J. S., S. T. Rachev, F. J. Fabozzi and M. Nikolov (2011), "MCMC-based Estimation of Markov-Switching ARMA-GARCH Models," *Journal of Applied Economics*, 43, pp. 259-271.

［11］Henry, O. T. (2009), "Regime Switching in the Relationship between Equity Returns and Short-Term Interest Rates in the UK," *The Journal of Banking & Finance*, 33, pp. 405-414.

［12］Jarque, C. M. and A. K. Bera (1987), "Test for Normality of Observations and Regression Residuals," *International Statistical Review*, 55, pp. 163-172.

［13］Kim, C. -J. (1993), "Unobserved Component Time Series Models with Markov-Switching Heteroskedasticity: Changes in Regime and the Link between Inflation Rates and Inflation Uncertainty," *Journal of Business and Economic Statistics*, 11, pp. 341-349.

［14］Kim, C. -J. and C. R. Nelson (1999), *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, The MIT Press.

［15］Klaassen, F. (2002), "Improving GARCH Volatility Forecasts with Regime-Switching GARCH," *Empirical Economics*, 27, pp. 363-394.

［16］Maheu, J. M., T. H. McCurdy and Y. Song (2012), "Components of Bull and Bear Markets: Bull Corrections and

Bear Rallies," *Journal of Business & Economic Statistics*, 30, pp. 391-403.

[17] Mitsui, H. (2012), "Application of Markov-Switching EGARCH Models to the Nikkei Stock Average Variation," *KEIZAI SHUSHI* (The Nihon University Economic Review), 82, pp. 79-84.

[18] Mitsui, H. (2013), "A Note on the Stock Market Trend Analysis Using Markov-Switching EGARCH Models," *KEIZAI SHUSHI* (The Nihon University Economic Review), 83, pp. 111-117.

[19] Mitsui, H. (2014a), "Bull and Bear Market Analysis of the Nikkei 225 Futures and TOPIX Futures," *KEIZAI SHUSHI* (The Nihon University Economic Review), 84, pp. 19-39.

[20] Mitsui, H. (2014b), "Trend Analysis of Nikkei 225 Futures Price," *KEIZAI SHUSHI* (The Nihon University Economic Review), 84, pp.101-107.

[21] Mitsui, H. (2015), "Nikkei 225 Long-term Trend Analysis," *KEIZAI SHUSHI* (The Nihon University Economic Review), 85, pp. 63-69.

[22] Mitsui, H. and H. Totsuka (2022), "Bayesian Estimation of Stochastic Volatility Model Using Hamiltonian Monte Carlo Method and its Application to Nikkei 225 Data," *Bulletin of Research Institute of Economic Science College of Economic Nihon University*, 52, pp. 17-25.

[23] Nelson, D. B. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, 59, pp. 347-370.

[24] Satoyoshi, K. (2004), "An Empirical Study of Volatility in the Japanese Stock Market Using the Markov Switching GARCH Model," *Journal of the Japan Statistical Society*, 34, pp. 1-19. (in Japanese).

[25] Satoyoshi, K. and H. Mitsui (2011a), "An Empirical Study of Option Pricing under Bull and Bear Market Conditions," *Journal of Business Research*, 33, pp. 63-87. (in Japanese).

[26] Satoyoshi, K. and H. Mitsui (2011b), "Empirical Study of Nikkei 225 Options with the Markov Switching GARCH Model," *Asia-Pacific Financial Markets*, 18, pp. 55-68.

[27] Satoyoshi, K. and H. Mitsui (2012), "Option Valuation under Bulls and Bears Market Conditions," *Working Papers Series* 12-01, Nihon University. <http://www.eco.nihonu.ac.jp/center/economic/publication/pdf/12-01.pdf>.

[28] Satoyoshi, K. and H. Mitsui (2016), "Trend Analysis of Nikkei 225 and Option Valuation: Markov Switching EGARCH Models," *SHOUKEN KEIZAI KENKYU* (The JSRI Journal of Financial and Securities Markets), 96, pp. 59-82. (in Japanese).

[29] Totsuka, H. and H. Mitsui (2022), "Bayesian Estimation of a Stable Distribution Using the Hamiltonian Monte Carlo Method with Application to Stock Indices," *Bulletin of Research Institute of Economic Science College of Economic Nihon University*, 52, pp. 3-15.