Network Effects and Conjectures in Duopoly

Shinji Kobayashi

1. Introduction

The purpose of this paper is to examine the existence of equilibria and the effects of positive network externalities on competition in a duopolistic industry which provides network goods. In particular, as an equilibrium concept, we consider equilibria with consistent conjectural variations.

Over the past two decades, we have witnessed a rapid progress in information technology centered on the Internet. Along with the development of information technology, communication and financial industries have been dominated by a few large corporations. The emergence of large international oligopolistic companies is well known. One of the main characteristics of network goods is that as the number of consumers of the goods increases, the utility of each consumer increases.

So far, many researchers have studied the economics of competition in oligopoly industries, both theoretically and empirically. Furthermore, since the 1980s, the analysis of competition in industries that supply network goods has much progressed. There are pioneering contributions to the literature on network externalities in oligopolistic industries, for instance, Katz and Shapiro (1985) and Farrell and Saloner (1985). Katz and Shapiro (1985) analyze the private and social incentives for firms to accomplish product compatibility. They find that firms with small networks tend to favor product compatibility. Farrell and Saloner (1985) examine the possibility that the benefits of standardization impede the collective shift from an inferior standard to an efficient standard. They show that there can be inefficient inertia under incomplete information. Farrell and Saloner (1986) investigate the private and social incentives for the adoption of a new technology incompatible with the installed base. Katz and Shapiro (1986) examine the relationship between standardization and innovation. Farrell and Saloner (1992) analyze the relationship between the equilibrium adoption of conversion technology and its optimality. Katz and Shapiro (1992) study competition between old and new technologies in a market with network externalities. They show that entry may be profitable and a bias towards existing products, that is, excess inertia, cannot arise. Katz and Shapiro (1994) examine the relation between network effects and systems competition. Bental and Spiegel (1995) study market coverage in the presence of network externalities. They show that under free entry, market coverage is larger with a single industry standard. de Palma and Leruth (1996) study firms' compatibility choices and show that the firms prefer compatibility when they have an equal probability of becoming the largest under incompatibility. Cremer et al. (2000) show that the firm with larger installed bases chooses low compatibility. Baake and Boom (2001) examine firms' compatibility decisions in a vertical differentiation model with network externality and show that an adapter is provided in equilibrium. Useful surveys of the literature on network externalities can be found in Besen and Farrell (1994) and Economides (1996).

In the above studies on network effects, Nash equilibrium has been mainly examined as an equilibrium concept. In terms of the concept of conjectural variations, one special feature of Nash equilibrium concept is that each player has zero conjectural variations with respect to the reactions of other players. If each player has a non-zero conjectural variations, the outcome of competition among oligopolistic firms may differ from the outcome of Nash equilibrium. For a detailed study on conjectural variations in oligopoly, see Takenaka and Kobayashi (2020).

In this paper, we assume that each firm has non-zero conjectures about the strategies of its rival firms, and we perform an equilibrium analysis under the conditions that the firms' conjectural variations are consistent at the neighborhood of equilibrium. In addition, we present the results of comparative statics.

The main conclusions in this paper are as follows. The value of consistent conjectural variations is negative. It implies that the equilibrium outcome with consistent conjectural variations is more competitive than the Nash equilibrium outcome.

The paper is organized as follows. In Section 2, we present the model. In Section 3, we derive an equilibrium with consistent conjectural variations of the game and compare it with Nash equilibrium. Section 3 also presents some results from comparative statics analysis. In the final section, we state our conclusions.

2. The Model

In this section, we set up the model. Following Cremer et al. (2000), we consider an industry in which two firms produce differentiated network products. Let the two firms labeled firm 1 and 2. Let p_i denote the price of product i. Let $\beta_i \geq 0$ denote firm i's installed base, which is determined before the game starts, $i=1, 2$. For simplicity, we assume $\beta_2=0$.

New consumers enter the market and can purchase one of the two competing network products. We assume that consumers are distributed uniformly on the set $\Omega=[0, a]$, $a>0$. We assume the density function $f(x) = 1$ on Ω.

Suppose that each consumer purchases one of the two products. Each consumer obtains the direct benefit of a network good, ω. She also gets the network benefit of good i, α_i. Thus, a consumer of type $\omega \in [0,\ a]$ obtains a total surplus from purchasing good i:

$$U_i^{\omega} = \omega + \alpha_i - p_i.$$

Now consider the marginal consumer who are indifferent between purchasing good i or good j and not buying any of the two network products, $i, j=1, 2, i \neq j$.

Let $\widehat{\omega}$ denote the marginal consumer. Then, for the marginal consumer $\widehat{\omega}$, we must have

$$\widehat{\omega} + \alpha_i - p_i = \widehat{\omega} + \alpha_j - p_j = 0.$$

Hence, we get

$$\widehat{\omega} = p_i - \alpha_i = p_j - \alpha_j.$$

Note that $\widehat{\omega}$ is often referred to the quality-adjusted price.

Let q_i denote the quantity of product i. Because the marginal consumer has valuation $\omega = \widehat{\omega}$ and the assumption of the uniform distribution on the valuation of consumers, we have

$$q_1 + q_2 = a - \widehat{\omega}.$$

Let μ denote the degree of network effects, θ the degree of compatibility between the two products, and β a mass of consumers (installed base of firm 1). Parameter μ represents the strength of network externality, that is, the effects of network externality regarding to the rival good.

We assume that network benefits are given as follows:

$\alpha_1 = \mu(\beta + q_1 + \theta q_2)$

and

$\alpha_2 = \mu(\theta\beta + q_2 + \theta q_1)$.

We assume that each firm's profit from its installed base is fixed, and thus it does not affect the two firms' strategies.

Thus, we obtain the following inverse demands:

$p_1 = a + \mu\beta - (1-\mu)q_1 - (1-\theta\mu)\, q_2$

and

$p_2 = a + \theta\mu\beta - (1-\mu)q_2 - (1-\theta\mu)\, q_1$.

Finally, let us assume that the cost functions are given by

$$c_i(q_i) = c\frac{q_i^2}{2}, \quad c > 0, \ i = 1, \ 2.$$

For the sake of brevity, hereafter, we assume that $a = 1$.

3. Equilibrium and Consistent Conjectures

In this section, we examine an equilibrium with consistent conjectural variations. Profit of firm i is given by

$\pi(q_i) = p_i\, q_i - c\frac{q_i^2}{2}$, $\mathrm{i} = 1, 2$.

The two firms undertake quantity competition. Because we consider consistent conjectural variations in the duopoly game, the equilibrium outcome under price competition is the same as that under quantity competition (see the remark below).

The literature on network externalities focuses on Nash equilibrium. Nash conjectures imply that conjectural variations are zero. In contrast to the literature on network externalities and oligopoly, this paper considers consistent conjectural variations (CCVs).

Let γ_i^j denote firm i's conjecture about firm j's response when firm i changes its output. The conditions of consistency are given as follows: Given the two reaction functions, $q_1 = R_1(q_2)$ and $q_2 = R_2(q_1)$, the conditions of consistency are

$$\frac{dR_1(q_2)}{dq_2} = \gamma_1^2 \quad \text{and} \quad \frac{dR_2(q_1)}{dq_2} = \gamma_2^1.$$

Assumption I: $\frac{1+\mu\beta}{1-\mu\theta} > \frac{1+\theta\mu\beta}{2(1-\mu)+c}$.

Assumption II: $\frac{1+\mu\beta}{1-\mu\theta} > \frac{1+\theta\mu\beta}{2(1-\mu)+(1-\mu\theta)\gamma_1^2+c}$ and $2(1-\mu) + (1-\mu\theta)\gamma_1^2 + c > 0$.

Proposition 1.

There exists a unique equilibrium (q_1^*, q_2^*) with consistent conjectures (γ_1^*, γ_2^*) such that (γ_1^*, γ_2^*) by (1) and (q_1^*, q_2^*) given by (2) provided that Assumptions I and II hold.

Proof:

From the first-order condition of profit maximization of firm i, $i=1, 2$, we have the following reaction functions:

$$q_1 = \frac{(1+\mu\beta)}{2(1-\mu)+(1-\mu\theta)\gamma_1^2+c} - \frac{(1-\mu\theta)}{2(1-\mu)+(1-\mu\theta)\gamma_1^2+c}\, q_2 \equiv R_1\,(\,q_2\,)$$

and

$$q_2 = \frac{(1+\theta\mu\beta)}{2(1-\mu)+(1-\mu\theta)\gamma_2^1+c} - \frac{(1-\mu\theta)}{2(1-\mu)+(1-\mu\theta)\gamma_2^1}\, q_1 \equiv R_2\,(\,q_1\,)\,.$$

Next we show that consistent conjectural variations at the equilibrium are negative.

Given $q_1 = R_1\,(\,q_2\,)$ and $q_2 = R_2\,(\,q_1\,)$, the conditions of consistency are

$$\frac{dR_1\,(\,q_2\,)}{dq_2} = \gamma_1^2 \quad \text{and} \quad \frac{dR_2\,(\,q_1\,)}{dq_2} = \gamma_2^1\,.$$

Hence, we obtain

$$\gamma_i^j = \gamma^* = \frac{-[2(1-\mu+c]+\sqrt{[2(1-\mu+c]^2-4(1-\mu\theta)}}{2(1-\theta\mu)} < 0\ , \quad i, j=1, 2, i\neq j\,. \tag{1}$$

Thus, under Assumptions I and II, the equilibrium quantities are

$$q_1^* = \frac{(A+B\gamma^*)(1+\mu\beta)}{(A+B\gamma^*)^2-B^2} - \frac{B\,(1+\theta\mu\beta)}{(A+B\gamma^*)^2-B^2}$$

and (2)

$$q_2^* = \frac{(A+B\gamma^*)(1+\theta\mu\beta)}{(A+B\gamma^*)^2-B^2} - \frac{B\,(1+\mu\beta)}{(A+B\gamma^*)^2-B^2}\,,$$

where

$$\mathrm{A} \equiv 2(1-\mu)+c\,, \quad B \equiv 1-\mu\beta\,.$$

Q.E.D.

Next, we show that because consistent conjectural variations are negative, the outcome of the equilibrium with CCVs is more competitive than that of Nash equilibrium. Note that we have

$$q_1^* - q_2^* > 0\,.$$

Because the installed base of firm 1 is larger than that of firm 2, the equilibrium quantity of firm 1 exceeds that of firm 2.

To find a Nash equilibrium, we use the following reaction functions,

$$q_1 = \frac{1+\mu\beta}{2(1-\mu)+c} - \frac{1-\theta\mu}{2(1-\mu)+c}\, q_2$$

and

$$q_2 = \frac{1+\theta\mu\beta}{2(1-\mu)+c} - \frac{1-\theta\mu}{2(1-\mu)+c}\, q_1 .$$

Then, we get a Nash equilibrium $(q_1^N,\ q_2^N)$ such that

$$q_1^N = \frac{A(1+\mu\beta)}{A^2-B^2} - \frac{B(1+\theta\mu\beta)}{A^2-B^2}$$

and

$$q_2^N = \frac{A(1+\theta\mu\beta)}{A^2-B^2} - \frac{B(1+\mu\beta)}{A^2-B^2} .$$

We note that $q_1^N - q_2^N > 0$.

By comparing the equilibrium outcome with consistent conjectural variations, $(q_1^*,\ q_2^*)$, with the outcome of Nash equilibrium, $(q_1^N,\ q_2^N)$, we obtain

$$q_1^* + q_2^* > q_1^N + q_2^N .$$

Remark: We have considered Cournot competition in our game. Because we have obtained consistent conjectural variations in quantities, we can show that there are the corresponding consistent conjectural variations in prices and that the equilibrium outcomes are the same. For general results on this point, see for example, Takenaka and Kobayashi (2020).

Next, by examining the effects of changes in parameters on the equilibrium outcomes and the consistent conjectural variations, we obtain the following results, that is, Propositions 2 and 3.

Proposition 2.

$$\frac{\partial(q_1^* - q_2^*)}{\partial\theta} > 0 .$$

Thus, the difference in equilibrium quantities increases when the degree of compatibility increases.

Proposition 3.

$$\frac{\partial\gamma^*}{\partial\theta} > 0 \ \ and\ \ \frac{\partial\gamma^*}{\partial c} > 0 .$$

This proposition says that the value of consistent conjectural variations increases when the degree of compatibility increases. It also shows that the value of consistent conjectural variations increases when the production costs increase.

4. Conclusion

This paper has analyzed network effects and competition in a differentiated duopoly.

We have considered network externalities in the duopolistic competition. We have shown that the unique equilibrium with consistent conjectural variations exists and that consistent conjectural variations at the equilibrium are negative. It implies that the outcome at the unique equilibrium with consistent conjectural variations is more competitive than the Nash equilibrium outcome.

Future work should examine firms' compatibility decisions. For instance, we may extend the original game to a two-stage in which in the first stage, firms decide compatibility choices and in the second stage, they undertake Cournot or Bertrand competition.

References

Baake, P. and A. Boom (2001) "Vertical Product Differentiation, Network Externalities, and Compatibility Decisions," *International Journal of Industrial Organization*, 19, 267-284.

Bental, B. and M. Spiegel (1995) "Network Competition, Product Quality, and Market Coverage in the Presence of Network Externalities," *Journal of Industrial Economics*, 18, 197-208.

Besen, S. and J. Farrell (1994) "Choosing How to Compete: Strategies and Tactics in Standardization," *Journal of Economic Perspectives*, 8, 117-131.

Crémer, J., P. Rey, and J. Tirole (2000) "Connectivity in the Commercial Internet," *Journal of Industrial Economics*, 48, 433-472.

de Palma, A. and L. Leruth (1996) "Variable Willingness to pay for Network Externalities with Strategic Standardization Decisions," *European Journal of Political Economy*, 12, 235-251.

Economides, N. (1996) "The Economics of Networks," *International Journal of Industrial Organization*, 14, 673-700.

Farrell, J. and Saloner (1985) "Standardization, Compatibility, and Innovation," *Rand Journal of Economics*, 16, 70-83.

Farrell, J. and Saloner (1986) "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation," *American Economic Review*, 76, 940-955.

Farrell, J. and Saloner (1992) "Converters, Compatibility and the Control of Interfaces," *Journal of Industrial Economics*, 40, 9-35.

Katz, M. and C. Shapiro (1985) "Network Externalities, Competition, and Compatibility," *American Economic Review*, 75, 424-440.

Katz, M. and C. Shapiro (1986) "Technology Adoption in the Presence of Network Externalities," *Journal of Political Economy*, 94, 822-841.

Katz, M. and C. Shapiro (1992) "Product Introduction with Network Externalities," *Journal of Industrial Economics*, 40, 55-83.

Katz, M. and C. Shapiro (1994) "Systems Competition and Network Effects," *Journal of Economic Perspectives*, 8, 93-115.

Takenaka, K. and S. Kobayashi (2020) *Oligopoly and Conjectural Variations* (in Japanese), Keio University Press, Tokyo.