Network externalities in a quantity-setting vertically differentiated luxury goods market*

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1 Introduction

This study reconsiders how the strength of network externalities and the difference in product quality between high- and low-end firms influence their demand, price, profits, consumer surplus, and social welfare under vertical product differentiation in a luxury goods market, following Wu et al. (2024). We depart from the analysis of price competition considered in Wu et al. (2024) by exploring quantity competition as a duopoly of market competition composed of high-quality and low-quality firms. Following Wu et al. (2024), we incorporate vertically differentiated products with different network sizes; that is, two types of products: high-end luxury goods (high-quality products) and entry-level luxury goods (low-quality products).[1)] More concretely, to explain the phenomenon in which the availability or sales of entry-level luxury products affect the purchase choices of a particular consumer group, we model firm-specific network externalities, which are isolated from the impact of entry-level luxury product sales or presence in the fashion of Wu et al. (2024). On the other hand, the strength of the network externalities of low-quality products is influenced by the expected market share of both low- and high-quality products.

Recent studies of vertical product differentiation addressed the strength of network externalities. Baake and Boom (2001) modeled two firms that select their product's essential quality and then they may agree on providing an adapter with each other before engaging under price competition. Lambertini and Orsini (2005) derived the existence of a pure-strategy subgame-perfect equilibrium in qualities and prices in a duopoly model with vertical differentiation where quality improvements require a quadratic variable cost and network externalities operate. Cheng and Chan (2023) showed that, with significant network benefits obtained from quality enhancement, the

1) Corneo and Jeanne (1997) found that in equilibrium, the signaling value of conspicuous consumption depends on the number of consumers, and consumer behavior is characterized by either snobbism or conformism. As Wu et al. (2024) indicate, the former snob effect (snobbism) is especially apparent in the luxury goods market.

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effects of network externalities differ in response to the type of competition by examining how firms determine their vertical product quality and network size. Grilo et al. (2001) showed that consumer vanity reduces price competition, whereas weak conformity intensifies competition by taking not only positive network effects but also negative effects under the spatial model of product differentiation. Most recently, Wu et al. (2024) investigated the effect of network externalities and differences in product quality between high-quality and low-quality firms on the equilibrium market outcomes under price competition. They considered a situation in which consumers' luxury consumption triggers their vanity-driven utility, which decreases due to the snob effect as the market share of a high-quality firm and a low-quality firm expands.

We reconsider a model à la Wu et al. (2024) by changing the competition mode from price to quantity. In this study, we find three main results that hold only under quantity competition. First, we show that the demand and price of the low-quality firm's products and its profit can be positively associated with the strength of network externalities in a relatively large area on the quality of a low-quality firm's product and the strength of network externalities. Thus, the crowding-out effect of high-quality products on low-quality products should be weaker under quantity competition than under price competition. Second, we find that the profit-enhancing quality of a low-quality firm is higher under quantity competition than under price competition. This property is explained by the fact that a low-quality firm attempts to avoid excessive competition under price competition than under quantity competition by setting a lower level of quality.[2)] Third, under quantity competition, we find that both the profit-maximizing quality of a low-quality firm and its socially optimal quality for the government are positively associated with the strength of network externalities. Furthermore, under quantity competition, the profit-maximizing quality of a low-quality firm is higher than its socially optimal quality for an arbitrary level of network externality. The profit-maximizing firm tries to lower its product's quality to avoid excessively intense market competition under price competition than under quantity competition. Moreover, the relationship between the equilibrium quality of a low-quality firm and the profit-maximizing and socially optimal levels appears as some sort of complementarity, although part of the term in the consumers' utility function is represented as a product of the strength of network externalities and the quality of a low-quality firm's product.

The remainder of this paper is organized as follows. In Section 2, we present the basic model. Section 3 investigates the effect of the strength of network externalities and the quality differences between high- and low-quality firms on the equilibrium market outcomes under quantity competition. Finally, Section 4 concludes. Finally, in the Appendix, we provide the equilibrium market outcomes, except for the demand for the products of both a high-quality firm and a low-quality firm under quantity competition.[3)]

2）Furthermore, depending on the strength of the network externalities and the quality of a low-quality firm's products in the status quo, we show that the profit of a high-quality firm can increase, while that of a low-quality firm can decrease as the quality of a low-quality firm's products is enhanced. This result is also strikingly different from that obtained under price competition in Wu et al. (2024).

3）Moreover, Appendix provides the polynomial equations of the strength of network externalities and the quality of the product of a low-quality firm with the threshold levels of the strength of network externalities and its quality such that the signs of the changes in consumer surplus and social welfare along with a marginal change in its quality are altered.

2 The model

This study models a luxury goods market composed of firms h and l that produce vertically differentiated products with network externalities. Firm h produces only high-quality products and firm l produces only low-quality products. Following Benassi et al. (2016) and Wu et al. (2024), the product quality of firm i is u_i $(i = h, l)$.[4] We note that $u_h > u_l$, where $u_l \in (0, 1)$.[5] p_i denotes the product price of firm i, $(i = h, l)$. The product qualities of firms h and l are predetermined and considered parameters in this model. Consumers first form expectations about the network sizes of the products between firms h and l, after which they engage under price competition by considering these expectations. Then, consumers decide to purchase the products of the high-quality firm, low-quality firm, or purchase nothing, based on their initial expectations and the quantities determined by firms h and l. Following Katz and Shapiro (1985), Hoernig (2012), and Wu et al. (2024), we assume that consumers are rational and that their expectations are fulfilled in the Nash equilibrium in quantity-setting competition. Let D_i $(i = h, l)$ denote the firm's actual market share; then, $D_i = y_i$ owing to the rationality of consumers' expectations in equilibrium $(i = h, l)$.

Consumers are heterogeneous in their product quality preferences for firms h and l, denoted as $\theta \in (0, 1)$. θ is equally distributed in a unit line, similar to the assumptions employed in Wu et al. (2024). Each consumer can choose to purchase one product produced by either firm h or firm l, or not to purchase both products. Consumers regard the possession of a rare luxury good that others do not own as a representation of their prestige and preference. The strength of the network externalities when consuming a product is denoted by $\alpha \in (0, 1)$, which is symmetric among all consumers. The utility functions of consumer purchasing the high-quality and low-quality product are, respectively,

$$\begin{cases} U(\theta, u_h, p_h) = \theta u_h p_h + \alpha u_h (1 - y_h), \\ U(\theta, u_l, p_l) = \theta u_l p_l + \alpha u_l (1 - y_h - y_l), \end{cases} \tag{1}$$

where y_i is consumers' expectations of firm i's equilibrium market share $(i = h, l)$. Following studies such as those by Grilo et al. (2001) and Wu et al. (2024), consumer utility is composed of the following two factors:

1. Intrinsic utility $(\theta u_i - p_i)$: This utility is directly obtained from its consumption, which consists of a product price p_i, quality u_i, and consumer's preference for quality θ, $(i = h.l)$.
2. Interactive utility $(\alpha u_h (1 - y_h)$ or $\alpha u_l (1 - y_h - y_l))$: This utility is obtained from the social contexts of its consumption, which consists of the network effects cultivating the vanity brought from its consumption.

Note that in the model à la Wu et al. (2024), the network effects diminish as more people acquire the same products with each other.

4） We employ an approach similar to vertical product differentiation. Wang and Wang (2021) explored the influence of the delegation of downstream firms that produce different quality of products on the profits, consumer surplus and social welfare in a vertically related market, and Wang and Wang (2022) investigated a model in which the public firm competes with a private firm taking consumer surplus into account in a vertically differentiated market.

5） As mentioned below, we assume that $u_h = 1$.

This approach to consumer utilities has the following three characteristics, as described in Wu et al. (2024).

1. We incorporate $(1 - y_h)$ as the interactive utility on the basis of the luxury consumption by reflecting the phenomenon such that the vanity obtained from high-quality luxury goods yields some sort of positive utility. Note that it decreases in the number of people who purchase the same luxury products.
2. As in Tolotti and Yepez (2001), αu_i is regarded as firm-specific social recognition or brand power which are linked with firm i's product.
3. The interactive utility of consumers of high-quality products is solely affected by the market share of high-quality products, whereas the network of the low-quality product extends to the entire industry.[6)]

The marginal consumer who is indifferent to the consumption of either good is

$$\theta_h = \frac{p_h - p_l - \alpha[u_h(1-y_h) - u_l(1-y_h-y_l)]}{u_h - u_l}.$$

We denote by θ_l the consumer who is indifferent to the purchase of a low-quality good and who refrains from buying. As in Wauthy (1996), from (1), we define this type of marginal consumer as $\theta_l = \dfrac{p_l}{u_l} - \alpha(1 - y_h - y_l)$. Then, we derive the inverse demand functions for the products of firms h and l:

$$\begin{cases} D_h = 1 - \dfrac{p_h - p_l - \alpha[u_h(1-y_h) - u_l(1-y_h-y_l)]}{u_h - u_l} \iff p_h = -D_l u_l + u_h[1 - D_h + \alpha(1-y_h)], \\ D_l = \dfrac{p_l u_h - p_h u_l + \alpha u_h u_l y_l}{u_l(u_l - u_h)} \iff p_l = u_l[1 - D_h - D_l + \alpha(1-y_h-y_l)]. \end{cases} \tag{2}$$

Additionally, following Motta (1993), Wauthy (1996), Aoki (2003), and Wu et al. (2024), we assume that firm i's unit cost is $u_i^2/2$, which is convex and positively associated with product quality ($i = h, l$). Firm i's profit, consumer surplus, and social welfare are, respectively, as follows($i = h, l$):

$$\begin{aligned} \pi_i &= \left(p_i - \frac{u_i^2}{2}\right) D_i, \\ CS &= \int_{\theta_h}^{1} U(\theta, u_h, p_h) d\theta + \int_{\theta_l}^{\theta_h} U(\theta, u_l, p_l) d\theta, \\ SW &= CS + \pi_h + \pi_l. \end{aligned}$$

In the next section, we investigate how the strength of network externalities α and the difference in product quality between firms h and $l$that is, $u_h - u_l = 1 - u_l$mediate the demand for their products, profits, consumer surplus, and social welfare under quantity competition.

6) As indicated in Wu et al. (2024), the feature of this model reflects the fact that when consumers of low-quality products encounter consumers of high-quality products, the vanity they receive from their purchases vanishes. See Wu et al. (2024) for examples that reflect the asymmetric influence between high-quality and low-quality luxury products.

3 Quantity competition

In the market stage, given the inverse demand 2 faced by firms h and l, their profit maximization for a given y_h and y_l lead to the following reaction function of firms h and l, respectively.

$$\frac{\partial \pi_h}{\partial D_h} = -D_l u_l + \frac{1}{2}u_h[2(1-2D_h) - u_h + 2\alpha(1-y_h)] = 0$$
$$\Rightarrow D_h = R_h(D_l; y_h, y_l) = \frac{1}{4}\left[2 - u_h + 2\alpha(1-y_h) - \frac{2D_l u_l}{u_h}\right], \qquad (3)$$
$$\frac{\partial \pi_l}{\partial D_i} = -\frac{u_l^2}{2} + u_l[1 - D_h - 2D_l + \alpha(1-y_h-y_l)] = 0$$
$$\Rightarrow D_l = R_l(D_h; y_h, y_l) = \frac{1}{4}\{2[1 - D_h + \alpha(1-y_h-y_l)] - u_l\}. \qquad (4)$$

By imposing the assumption of consumer rational expectations $y_h = D_h$ and $y_l = D_l$ on (3) and (4), we obtain the *re* reaction functions:[7)]

$$\begin{cases} D_h = R_h^{re}(D_l) = -\dfrac{u_h^2 + 2D_l u_l - 2u_h(1+\alpha)}{2u_h(2+\alpha)}, \\ D_l = R_l^{re}(D_h) = -\dfrac{u_l - 2(1+\alpha) + 2D_h(1+\alpha)}{2(2+\alpha)}, \end{cases}$$

which yield [8)]

$$\begin{cases} D_h^{qq}(u_h, u_l; \alpha) = \dfrac{(1+\alpha)[2u_h(2+\alpha) - u_l(2+u_l)]}{2[u_h(2+\alpha)^2 - u_l(1+\alpha)]}, \\ D_l^{qq}(u_h, u_l; \alpha) = \dfrac{u_l^2(1+\alpha) + u_h[2(1+\alpha) - u_l(2+\alpha)]}{2[u_h(2+\alpha)^2 - u_l(1+\alpha)]}. \end{cases} \qquad (5)$$

Henceforth, similar to the approach employed by Wu et al. (2024), we set the following assumption to focus on the difference in product quality between firms h and l, that is, $u_h - u_l$:

Assumption 1. $u_h = 1$.

First, we summarize the same qualitative properties of the change in equilibrium market outcomes against the marginal change in α between price and quantity competition (Wu et al. (2024)) as follows:

Result 1. *Under quantity competition, the following results hold, similar to those obtained for price competition:*
(1-1) $\dfrac{\partial D_h^{qq}(1, u_l; \alpha)}{\partial \alpha} > 0$, *(1-2)* $\dfrac{\partial [D_h^{qq}(1, u_l; \alpha) + D_l^{qq}(1, u_l; \alpha)]}{\partial \alpha} > 0$, *(2)* $\dfrac{\partial p_h^{qq}(1, u_l; \alpha)}{\partial \alpha} > 0$, *(3)* $\dfrac{\partial \pi_h^{qq}(1, u_l; \alpha)}{\partial \alpha} > 0$,

7) "*re*" denotes the reaction functions of firms h and l under rational consumer expectations.
8) Note that superscript "*qq*" denotes the equilibrium market outcomes under quantity competition.

(4-1) $\dfrac{\partial CS^{qq}(1,u_l;\alpha)}{\partial \alpha} > 0$, *and (4-2)* $\dfrac{\partial SW^{qq}(1,u_l;\alpha)}{\partial \alpha} > 0$.

As Wu et al. (2024) showed, under quantity competition, Result 1 indicates that the market position of firm h becomes more advantageous, and both consumer surplus and social welfare increase as the strength of network externalities increases.

However, under quantity competition, we obtain different results from those under price competition shown in Wu et al. (2024) on the comparative static analyses of the equilibrium market outcomes of firm l with respect to the strength of network externalities, α, as in the following proposition.

Proposition 1. *Under quantity competition, we have*

$$(1)\ \frac{\partial D_l^{qq}(1,u_l;\alpha)}{\partial \alpha}\begin{cases}\leq 0, & \text{if} \quad \alpha \geq \bar{\alpha}(u_l),\\ > 0, & \text{otherwise}.\end{cases} \qquad (2)\ \frac{\partial p_l^{qq}(1,u_l;\alpha)}{\partial \alpha}\begin{cases}\leq 0, & \text{if} \quad \alpha \geq \bar{\alpha}(u_l),\\ > 0, & \text{otherwise}.\end{cases}$$

$$(3)\ \frac{\partial \pi_l^{qq}(1,u_l;\alpha)}{\partial \alpha}\begin{cases}\leq 0, & \text{if} \quad \alpha \geq \bar{\alpha}(u_l),\\ > 0, & \text{otherwise}.\end{cases}$$

We note that $\bar{\alpha}(u_l) := \dfrac{3-2u_l-\sqrt{9-u_l^2(3-u_l)}}{u_l-3}$.

The trajectories of the threshold level on the changes in the demand and price of firm l's product and its profit, along with a marginal change in the strength of network externalities α, are illustrated in Figures 1, 2, and 3. Wu et al. (2024) explained that the crowding-out effect of high-quality products on low-quality products intensifies as the strength of network externalities increases, since (1) $\dfrac{\partial D_l^{pp}(1,u_l;\alpha)}{\partial \alpha} < 0$, (2) $\dfrac{\partial p_l^{pp}(1,u_l;\alpha)}{\partial \alpha} < 0$, and (3) $\dfrac{\partial \pi_l^{pp}(1,u_l;\alpha)}{\partial \alpha} < 0$ hold for any $u_l \in (0, 1)$ and any $\alpha \in (0, 1)$.[9] However, Proposition 1 indicates that this global crowding effect does not hold under quantity competition. More precisely, when α is sufficiently low ($\alpha < \overline{\alpha}(u_l)$), we find that (1) $\dfrac{\partial D_l^{qq}(1,u_l;\alpha)}{\partial \alpha} > 0$, (2) $\dfrac{\partial p_l^{qq}(1,u_l;\alpha)}{\partial \alpha} > 0$, and (3) $\dfrac{\partial \pi_l^{qq}(1,u_l;\alpha)}{\partial \alpha} > 0$ for any $u_l \in (0, 1)$. This fact can be explained as follows. Compared with price competition, market competitiveness is weak under quantity competition owing to the strategic substitutability between D_h and D_l. Thus, because the effect of consumers deriving higher marginal utility from consuming high-quality products than low-quality ones is weaker under quantity competition than under price competition, the crowding-out effect of high-quality products on low-quality products does not work under quantity competition unless the level of α becomes sufficiently high.[10]

9）Note that "*pp*" denotes the equilibrium market outcomes under price competition.

10）Even under Bertrand market competition, Nakamura (2024) obtained a similar result such that this crowding out effect of a high-quality firm becomes weaker by the managerial delegation within firms h and l in the fashion of Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985).

Figure 1: The effect of α on $D_l^{qq}(1, u_l; \alpha)$

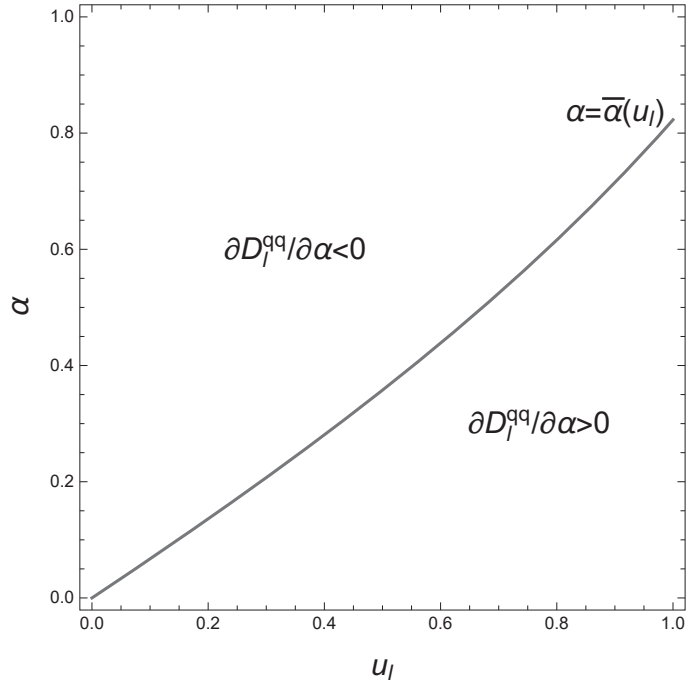


Figure 2: The effect of α on $p_l^{qq}(1, u_l; \alpha)$

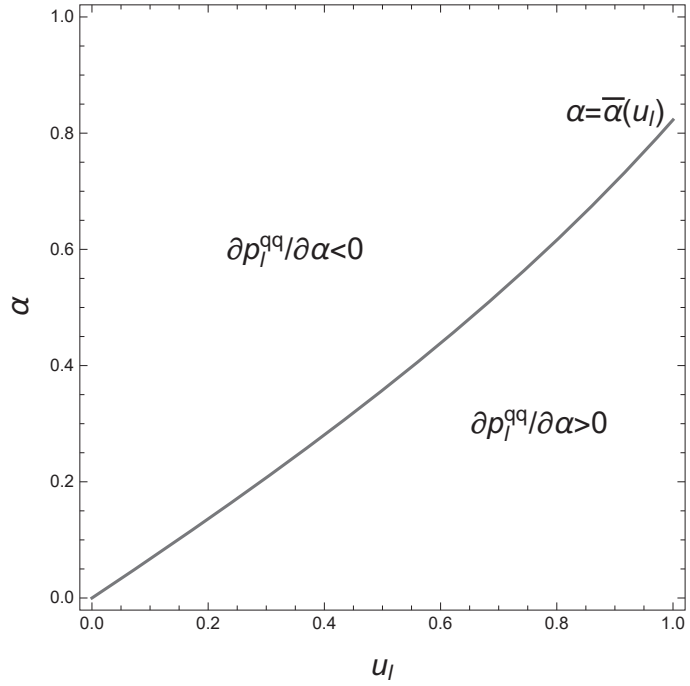
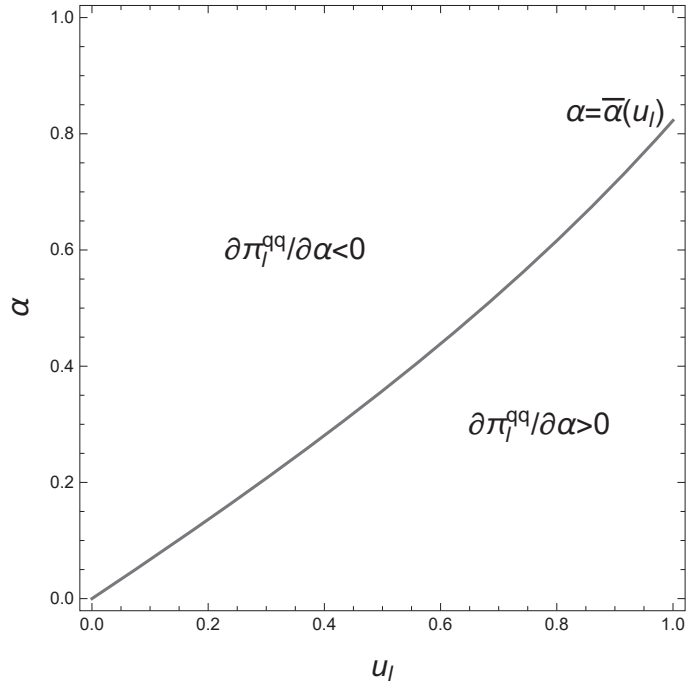


Figure 3: The effect of α on $\pi_l^{qq}(1, u_l; \alpha)$



Next, we investigate the change in the difference between the quality of products for firms h and l on the equilibrium market outcomes by considering the comparative statistics of the equilibrium market outcomes with respect to the product quality for firm l, u_l. Then, we have Proposition 2 on the comparative statistics of demand for the products of firms h and l with respect to u_l.

Proposition 2. *Under quantity competition, we have*

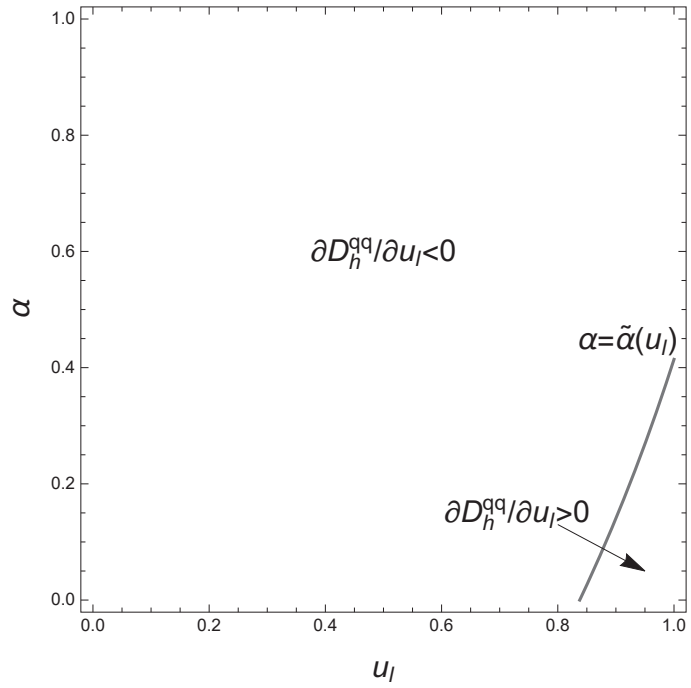
$$\frac{\partial D_h^{qq}(u_h, u_l; \alpha)}{\partial u_l} \begin{cases} \geq 0, & \text{if} \quad \alpha \leq \tilde{\alpha}(u_l), \quad \text{when} \quad u_l \geq 4 - \sqrt{10} \simeq 0.837722, \\ < 0, & \text{otherwise}. \end{cases}$$

$$\frac{\partial D_l^{qq}(u_h, u_l; \alpha)}{\partial u_l} < 0.$$

We note that $\tilde{\alpha}(u_l) := \dfrac{9u_l(8u_l) - \sqrt{9 + u_l^2[6u_l(8u_l)]}}{4u_l 6}$.

The trajectory of the threshold level of the change in demand for firm h's product, along with a marginal change in the quality of firm l's product, u_l, is illustrated in Figure 4. The situation in which α is sufficiently low (near zero) and u_l is sufficiently high (near one) is likely to be equal to quantity competition with homogeneous goods. From the viewpoint of the model setting of consumers' purchasing behaviors, the demand for a low-quality firm that is more disadvantageous than a high-quality firm decreases in u_l owing to the strategic substitutability between D_h and D_l. Conversely, the demand for the products of a high-quality firm, which is more advantageous than that of a

Figure 4: The effect of u_l on $D_h^{qq}(1,u_l;\alpha)$



low-quality firm, increases in u_l when α is sufficiently low (near zero) and u_l is sufficiently high (near one).

Under quantity competition, we obtain the following result on the changes in the prices of both firms h and l and the profit of firm h along with a marginal change in u_l, which are similarly explained by the intuition behind those of the demand for their products.

Corollary 1. *Under quantity competition, we have*

$$\frac{\partial p_h^{qq}(1,u_l;\alpha)}{\partial u_l}\begin{cases}\geq 0, & \text{if} \quad \alpha \leq \tilde{\alpha}(u_l), \quad \text{when} \quad u_l \geq 4-\sqrt{10} \simeq 0.837722,\\ <0, & \text{otherwise}.\end{cases}$$

$$\frac{\partial p_l^{qq}(1,u_l;\alpha)}{\partial u_l} > 0,$$

$$\frac{\partial \pi_h^{qq}(1,u_l;\alpha)}{\partial u_l}\begin{cases}\geq 0, & \text{if} \quad \alpha \leq \tilde{\alpha}(u_l), \quad \text{when} \quad u_l \geq 4-\sqrt{10} \simeq 0.837722,\\ <0, & \text{otherwise}.\end{cases}$$

Figures 5 and 6 illustrate the trajectories of the threshold levels of the changes in the price of firm h's product and its profit, along with a marginal change in the quality of firm l's product, u_l.

By differentiating firm l's equilibrium profit with respect to u_l, we find that the impact of u_l on $\pi_l\,(1,\,u_l;\,\alpha)$\$ is contingent on the interplay of its product demand and price dynamics, as in the following proposition.

Figure 5: The effect of u_l on $p_h^{qq}(1, u_l; \alpha)$

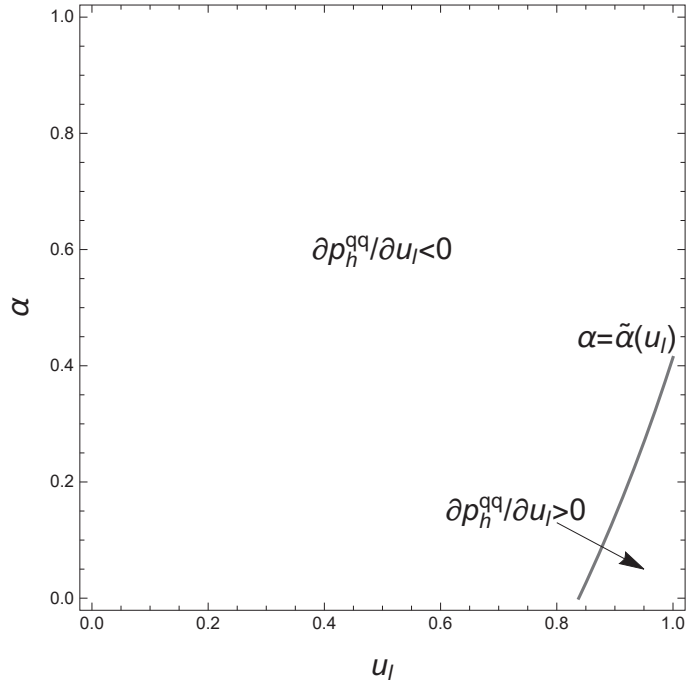
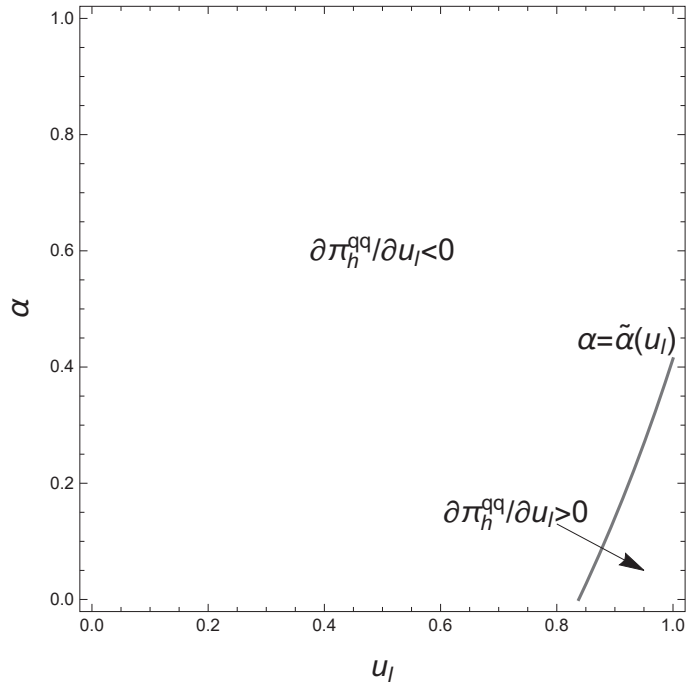


Figure 6: The effect of u_l on $\pi_h^{qq}(1, u_l; \alpha)$



Proposition 3. *Under quantity competition, we have*

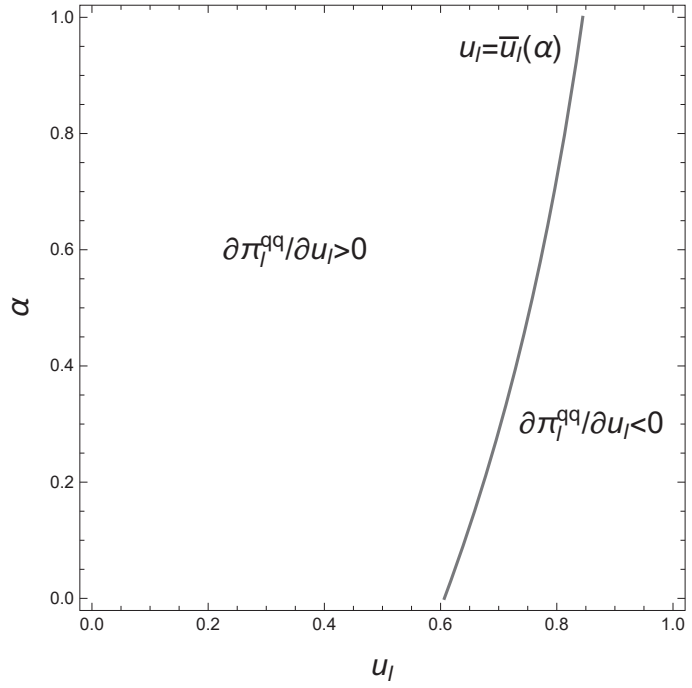
$$\frac{\partial \pi_l^q(1,u_l;\alpha)}{\partial u_l}\begin{cases}\leq 0, & \text{if} \quad u_l \geq \bar{u}_l(\alpha),\\ > 0, & \text{otherwise}.\end{cases}$$

Note that $\bar{u}_l(\alpha) := \dfrac{21+3\alpha[10+\alpha(5+\alpha)]-\sqrt{3}\sqrt{\{5+\alpha[6+\alpha(3+\alpha)]\}\{23+\alpha[34+\alpha(17+3\alpha)]\}}}{2(1+\alpha)(2+\alpha)}$. *Then, the profit enhancing quality of firm l is higher under quantity competition than under price competition.*

Figure 7 illustrates the trajectories of the threshold levels of the changes in firm l's profit, along with a marginal change in the quality of firm l's product, u_l. As Wu et al. (2024) demonstrated under price competition, $\dfrac{\partial D_l^{pp}(1,u_l;\alpha)}{\partial u_l} < 0$ and $\dfrac{\partial p_l^{pp}(1,u_l;\alpha)}{\partial u_l} > 0$ are satisfied. Thus, the qualitative properties of the change in the price and demand for the product of firm l with a marginal change in u_l under quantity competition are the same as those under price competition. Thus, the result that the profit-enhancing quality of firm l is higher under quantity competition than under price competition is explained by the fact that firm l tends to avoid intense market competition rather than price competition by setting a lower level of u_l.

Additionally, by summing the statements in Corollary 1 and Proposition 3, we obtain the following proposition.

Figure 7: The effect of u_l on $\pi_l^{qq}(1,u_l;\alpha)$



Proposition 4. *Under quantity competition, if the quality of the product of firm l, u_l, approaches that of firm h, the profit of firm l decreases, whereas the profit of firm h increases when the strength of the network externalities, α, is sufficiently low, that is, $\alpha < \tilde{\alpha}(u_l)$.*

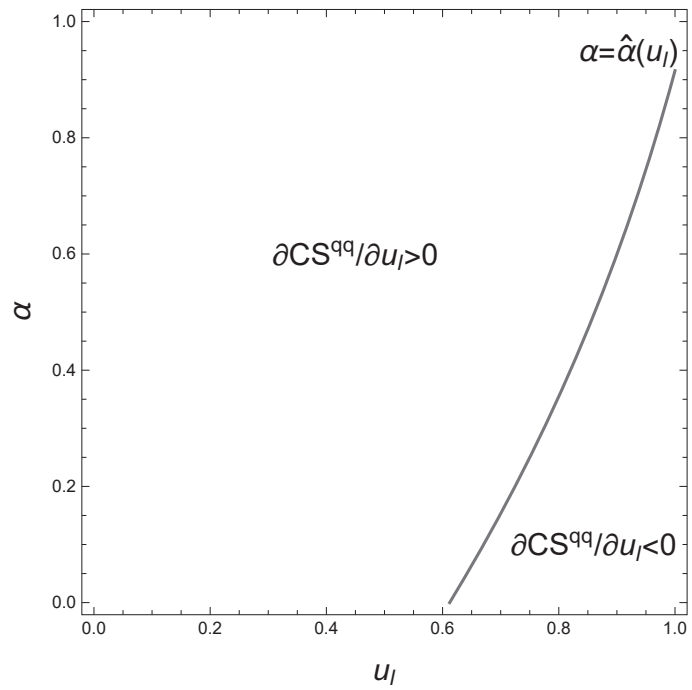
Proposition 4 states that firms h and l do not always enter a lose-lose situation as the quality of firm l's products approaches that of firm h. This is in sharp contrast to the results obtained for price competition in Wu et al. (2024).

Figures 8 and 9 depict the trajectories of the threshold level of changes in consumer surplus and social welfare, along with a marginal change in the quality of firm l's products, u_l.[11)]

In Figures 8 and 9, we find that $CS^{qq}(1, u_l; \alpha)$ and $SW^{qq}(1, u_l; \alpha)$ initially increase and then decrease as u_l increases, irrespective of the strategies of firms h and l in market competition. Thus, they are explained based on the same intuition as under price competition, as indicated in Wu et al. (2024).

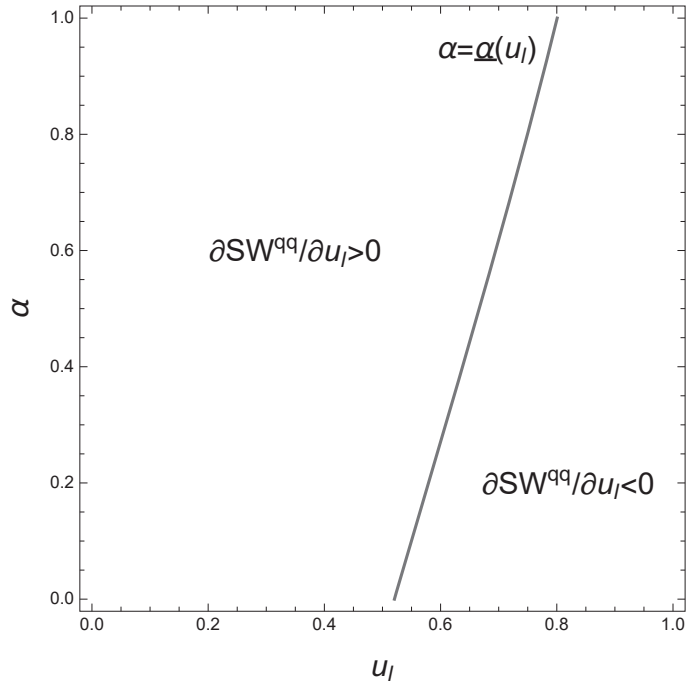
When u_l is relatively low in the status quo, an improvement in the quality of low-quality products initially facilitates consumer willingness to pay higher prices for enhanced quality. In contrast, when the quality of the products of a low-quality firm is relatively high in the status quo and gradually approaches the quality of products of a high-quality firm, market competition intensifies; consequently, social welfare deteriorates owing to higher prices because less quality-sensitive consumers exit the market.

Figure 8: The effect of u_l on $CS_h^{qq}(1, u_l; \alpha)$



11）Analytically deriving the threshold levels of α as a function of u_l such that the sign of $\frac{\partial CS^{qq}(u_h, u_l; \alpha)}{\partial u_l}$ and $\frac{\partial SW^{qq}(u_h, u_l; \alpha)}{\partial u_l}$, which are defined as $\hat{\alpha}(u_l)$ and $\underline{\alpha}(u_l)$, respectively, change. Thus, in the Appendix, we describe how $\hat{\alpha}(u_l)$ and $\underline{\alpha}(u_l)$ are solutions to the polynomial equations of u_l and α.

Figure 9: The effect of u_l on $SW_h^{qq}(1, u_l; \alpha)$



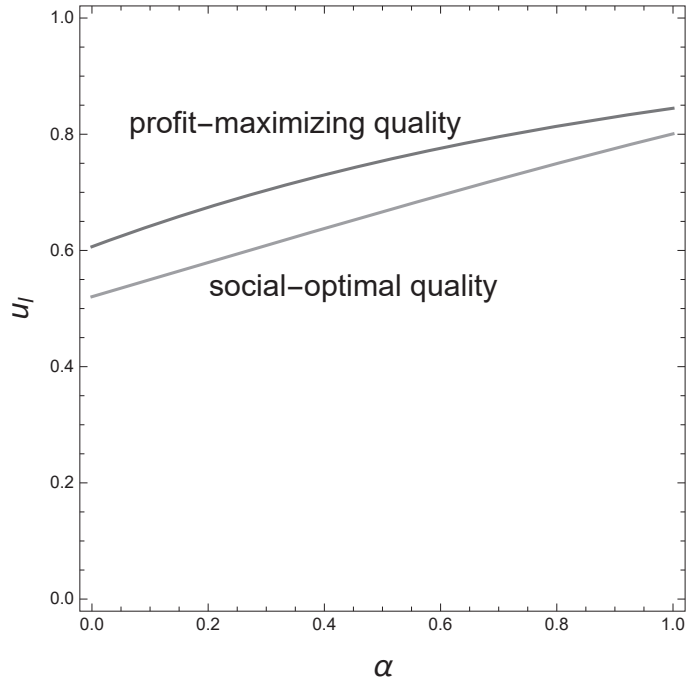
Finally, following Wu et al. (2024), we compare the equilibrium quality of the low-quality products chosen for firm l from the viewpoint of maximizing its profit and the product quality of the low-quality firm selected by the government to optimize social welfare, as illustrated in Figure 10.

Proposition 5. *Under quantity competition, both the profit-maximizing quality and the socially optimal quality of firm l are positively associated with the strength of network externalities, α. Furthermore, under quantity competition, the profit-maximizing quality of firm l is higher than its socially optimal quality for any* $\alpha \in (0, 1)$.

Proposition 5 is in sharp contrast to the result under price competition in Wu et al. (2024). Under quantity competition, as stated in Corollary 1, $\frac{\partial \pi_h^{qq}(u_h, u_l; \alpha)}{\partial u_l} < 0$ holds for almost all areas in the (u_l, α) plane. This works more strongly under quantity competition than under price competition; thus, the government refrains from setting a higher level of u_l to enhance social welfare. Consequently, we find that the profit-maximizing quality of firm l is higher than its socially optimal quality for an arbitrary network externality strength α.

Additionally, as indicated in Wu et al. (2024), under price competition, the equilibrium quality decisions for firm l and the government exhibit a decreasing trend with an increasing degree of network externalities. Therefore, the relationship of the equilibrium quality of firm l between the profit-maximizing level and the socially optimal level occurs as a complementarity; nevertheless, some terms in the utility function are represented as a product of the degree of network externalities with the quality of firm l.

Figure 10: The optimal u_l profit-maximizing v.s social-optimal



4 Conclusion

This study revisits the effect of snob network externalities on the market performance of vertically differentiated luxury products, as investigated in Wu et al. (2024). We depart from price competition in the fashion of Wu et al. (2024) by considering quantity competition involving two types of firms. We obtain several qualitative results that only hold under quantity competition in a market of luxury goods with vertical product differentiation, which consists of a high-quality firm and a low-quality firm.

First, under quantity competition, we find that the relationship between the demand and price of a product and the profit of a low-quality firm with the strength of network externalities depends on both the quality of the low-quality firm's product and the strength of network externalities in the status quo. In particular, under quantity competition, the demand for low-quality firms' products can be positively associated with the strength of network externalities, implying that the profit of a low-quality firm can also be positively associated with the strength of its network externalities. Thus, under quantity competition, the crowding-out effect of a high-quality product on a low-quality product relaxes compared to price competition when the degree of network externalities increases. Second, strikingly different from the results obtained under price competition, we find that the demand, price, and profit of the high-quality firm's product depend on both the quality of the low-quality firm's product and the strength of the network externalities in the status quo. Thus, the profit-enhancing quality for a low-quality firm is higher under quantity competition than under price competition. Third, we show that under quantity competition, both the profit-maximizing quality and socially optimal quality of a low-quality firm are positively associated with the strength of network externalities. Furthermore, under quantity competition, the profit-maximizing quality of a

low-quality firm is higher than the socially optimal quality for the government. In particular, the latter result implies that the implications for the government of the promotion and/or regulation of the quality of products of a low-quality firm are strikingly different under price and quantity competition. Future studies could explore the effect of luxury taxes and subsidies in a vertically differentiated luxury goods market in the context of both price competition and quantity competition.

5 Appendix

5.1 Equilibrium prices of firms h and l, consumer surplus, and social welfare

Here, we provide the equilibrium prices and profits of the products for firms h and l, consumer surplus, and social welfare, excluding the equilibrium demand for their products.

$$\begin{cases} p_h^{qq}(u_h,u_l;\alpha)=\dfrac{2u_h^2(1+\alpha)(2+\alpha)-u_l^3(1+\alpha)-u_hu_l\{2(1+\alpha)-u_l[3+\alpha(3+\alpha)]\}}{2[u_h(2+\alpha)^2-u_l(1+\alpha)]}, \\ p_l^{qq}(u_h,u_l;\alpha)=\dfrac{u_hu_l(1+\alpha)(2+u_l(2+\alpha)]}{2[u_h(2+\alpha)^2-u_l(1+\alpha)]}. \end{cases}$$

$$\begin{cases} \pi_h^{qq}(u_h,u_l;\alpha)=\dfrac{u_h(1+\alpha)^2[u_l(2+u_l)-2u_h(2+\alpha)]^2}{4[u_l(1+\alpha)-u_h(2+\alpha)^2]^2}, \\ \pi_l^{qq}(u_h,u_l;\alpha)=\dfrac{u_l\{u_l^2(1+\alpha)+u_h[2(1+\alpha)-u_l(2+\alpha)]\}^2}{4[u_l(1+\alpha)-u_h(2+\alpha)^2]^2}. \end{cases}$$

$$\begin{cases} CS^{qq}(u_h,u_l;\alpha)=\dfrac{\left\{\begin{array}{c}4u_h^3(2+3\alpha+\alpha^2)^2+u_h^2u_l[4(1+\alpha)^2+u_l^2(2+\alpha)^2-8u_l(1+\alpha)(2+\alpha)^2] \\ -u_hu_l^2(1+\alpha)[4(1+\alpha)-u_l^2(1+\alpha)-4u_l(5+5\alpha+\alpha^2)]-u_l^4(4+u_l)(1+\alpha)^2\end{array}\right\}}{8[u_l(1+\alpha)-u_h(2+\alpha)^2]^2}, \\ SW^{qq}(u_h,u_l;\alpha)=\dfrac{\left\{\begin{array}{c}12u_h^3(1+\alpha)^2(2+\alpha)^2-(4-u_l)u_l^4(1+\alpha)^2+u_h^2u_l[3u_l^2(2+\alpha)^2-16u_l(1+\alpha)(2+\alpha)^2 \\ -4(1+\alpha)^2(5+4\alpha)]+u_hu_l^2(1+\alpha)\{4(1+\alpha)-u_l^2(5+\alpha)+4u_l[9+\alpha(9+\alpha)]\}\end{array}\right\}}{8[u_l(1+\alpha)-u_h(2+\alpha)^2]^2}. \end{cases}$$

5.2 Polynomial derivations of $\hat{\alpha}(u_l)$ and $\underline{\alpha}(u_l)$

- $\hat{\alpha}(u_l)$ is a solution of $2u_l^4(1+\alpha)(3+2\alpha)+3u_l^2(2+\alpha)^2[14+\alpha(18+5\alpha)]-u_l^3\{62+\alpha[112+\alpha(67+13\alpha)]\}-u_l(1+\alpha)\{179+\alpha[352+\alpha(249+76\alpha+8\alpha^2)]\}+3(1+\alpha)^2(2+\alpha)^2(5+4\alpha)=0$.

- $\underline{\alpha}(u_l)$, is a solution of $2u_l^4(1+\alpha)(1+2\alpha)-u_l^3\{30+\alpha[80+\alpha(61+15\alpha)]\}+3u_l^2(2+\alpha)^2[14+\alpha(18+7\alpha)]-u_l\left(241+\alpha\{613+\alpha[643+\alpha(347+92\alpha+8\alpha^2)]\}\right)+3(1+\alpha)(2+\alpha)^2[7+\alpha(7+4\alpha)]=0$.

References

Aoki, R. (2003) "Effect of credible quality investment with Bertrand and Cournot competition" *Economic Theory* **21**(2), 653-672.

Baake, P. and A. Boom (2001) "Vertical product differentiation, network externalities, and compatibility decisions" *International Journal Industrial Organization*, **19**(1-2), 267-284.

Benassi, C., C. Massimiliano, and M. Maurizio (2016) "Price equilibrium and willingness to pay in a vertically differentiated mixed duopoly" *Journal of Economic Behavior and Organization* **125**, 86-96.

Cheng, Y., L. and Y. Y. Chan (2023) "Quality competition and market-share leadership in network industries" *BE Journal of Theoretical Economics* **23**(1), 259-289.

Corneo, G. and O. Jeanne (1997) "Conspicuous consumption, snobbism and conformism" *Journal of Public Economics* **66**(1), 55-71.

Fershtman, C. and K. Judd (1987) "Equilibrium Incentives in Oligopoly" *American Economic Review* **77**, 927-940.

Grilo, I., O. Shy, and J. F. Thisse (2001) "Price competition when consumer behavior is characterized by conformity or vanity" *Journal of Public Economics* **80**(3), 385-408.

Hoernig, S. (2012) "Strategic delegation under price competition and network effects" *Economics Letters*, **117**(2), 487489.

Katz, M. L., and L. Shapiro (1985) "Network externalities, competition, and compatibility" *American Economic Review* **75**, 424-440.

Lambertini, L. and R. Orsini (2005) "The existence of equilibrium in a differentiated duopoly with network externalities" *Japanese Economic Review*, **56**, 55-66.

Motta, M. (1993) "Endogenous quality choice: price vs. quantity competition" *Journal of Industrial Economics*, **41**(2), 113-131.

Nakamura, Y. (2024) "Network externalities in a vertically differentiated luxury goods market: A managerial delegation case" *Mimeograph*.

Sklivas, S. D. (1987) "The Strategic Choice of Management Incentives" *RAND Journal of Economics* **18**, 452-458.

Tolotti, M. and J. Yepez (2020) "Hotelling-Bertrand duopoly competition under firm-specific network effects" *Journal of Economic Behavior and Organization* **176**, 105-128.

Vickers, J. (1985) "Delegation and the Theory of the Firm" *Economic Journal* **95**, 138-147.

Wang, X. and L. F. S. Wang (2021) "Vertical product differentiation, managerial delegation and social welfare in a vertically-related market" *Mathematical Social Sciences* **113**, 149-159.

Wang, X. and L. F. S. Wang (2022) "Corporate social responsibility, vertical product differentiation, and privatization policy" *Japanese Economic Review* **73**, 403-425.

Wauthy, X. (1996) "Quality Choice in Models of Vertical Differentiation" *Journal of Industrial Economics*, **44**(3), 345-353.

Wu, D., J. Sun, L. F. S. Wang, and H. Liu (2024) "Network externalities in a vertically differentiated luxury goods market" *Mathematical Social Sciences* **128**, 100-109.