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Abstract

This paper considers how the choice of a *unit of account* affects the formation of an optimal currency area (OCA). We show that forming a currency union internalizes the exchange rate risk and leads to smoothing of consumption levels. However, changing the *unit of account* of the sovereign debt to a common currency may increase the debt burden if a country is less competitive. Therefore, the OCA is determined by this trade-off so that a less competitive debtor country is better off choosing a national currency and debt relief may be an effective way to maintain a currency union.

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Keywords: Optimal currency area, Unit of account, Sovereign debt

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1 Introduction

Following the 2011 sovereign debt crisis in the eurozone, considerable renewed attention has been given to optimal currency area (OCA) theory, with academics and policy-makers debating whether the eurozone must become an OCA to survive.

To determine the OCA, the benefits and costs of a common currency should be investigated. The classical OCA theory supposes that there exist various transaction costs and nominal rigidities in prices[1]. Under these circumstances, the theory stresses that the benefits are derived from a reduction in transaction costs, whereas the costs are derived from relinquishing an independent monetary policy to stabilize the economy. Therefore, as Mundell (1961) suggests, if there were no nominal rigidities, the benefits would always outweigh the costs so that a unique common currency would exist for the entire world.

However, as Goodhart (1998) and Eichengreen (2014) note, the recent turmoil in the eurozone revealed that the classical OCA theory overlooks crucial political economy factors, such as sovereign debt. If countries form a currency union, the burden of sovereign debt in each member country will be affected by the intraregional movements of the currency[2]. This is because, under the currency union, all the transactions in member countries are denominated by a common currency so that less (more) competitive countries attract less (more) currency, which, in turn, may make the burden of debt payment more (less) severe.

Then, the question is whether the less competitive debtor countries can become better

[1]Mongelli (2002) provides a detailed survey of the evolution of the OCA literature.

[2]Lane (2012) notes that the current account imbalance was a pre-crisis risk factor in the European sovereign debt crisis.

off by adopting a common currency given the discrepancy of competitiveness among member countries. To address this question, we construct a theoretical model in which the choice of a *unit of account* can affect not only the terms of trade but also the debt burden of agents, so that the formation of the OCA is also affected.

The outline of the model is as follows. There are two island countries, in which identical households and banks are located. A household, which is endowed with one unit of a good, deposits the endowment with a bank in the same country and, in return, receives a banknote. This banknote can be considered the national currency as it is generated by a bank in the country. In the next period, households exchange their banknotes to obtain the goods they want. Here, we introduce a preference shock that is common to all households. Under this setting, if a good in a country is preferred more (less), the currency of the country is demanded more (less). Therefore, fluctuations in the value of the currencies induce fluctuations in the value of the household's assets, so that consumption levels also fluctuate. However, if a common currency is introduced by the central bank, the common currency is chosen as a *unit of account* in a deposit contract, which, in turn, plays a role as a *medium of exchange* in goods transactions. Therefore, the shock does not affect the value of household assets, which leads to smoothing of the country's consumption levels.

However, when a country owes debt to another country, the result alters. Changing the *unit of account* of the debt from a national currency to a common currency may increase the debt burden if a country is less competitive. Under this arrangement, there arises a parameter space in which a national currency is preferred, and thus, the OCA can be depicted. This trade-off between the risk-sharing effect in consumption levels and the variation of the debt burden highlights the importance of the choice of the *unit of account* and its effect on the formation of the OCA.

Finally, we investigate whether the creditor country has an incentive to relieve some outstanding debt or not. Then, we show that in some cases, debt relief is effective for both a debtor country and a creditor country to maintain the common currency regime.

Related literature: After the 2011 sovereign debt crisis, several papers have considered the relationship between the sovereign debt problem and monetary policy in a currency union. Corsetti and Dedola (2016) consider the role of the central bank as a backstop for government funding and show that monetary authorities can rule out a self-fulfilling sovereign debt crisis by adopting an unconventional monetary policy. Aguiar et al. (2015) show that a high-debt country facing a potential for a rollover crisis may be better off if it belongs to a currency union with an intermediate mix of high- and low-debt members. Bolton and Huang (2018) consider the value of monetary sovereignty and find that the OCA is determined by the trade-off between monetary flexibility and the costs of strategic monetizations. These papers show that in a currency union, the burden of sovereign debt in each of the member countries is affected by a centralized monetary policy. In contrast, our paper shows that the burden of sovereign debt is affected by the choice of a *unit of account* even if monetary policy is constant.

The benefits and costs of a common currency as a *medium of exchange* are often considered in an environment in which search-matching frictions in transactions exist (Matsuyama et al. 1993, Trejos and Wright 1996, Ravikumar and Wallace 2002, Kiyotaki and Moore 2003). In contrast, the benefits and costs of a common currency as a *unit of account* have been considered only rarely. Freeman and Tabellini (1998) find that even when privately issued IOUs can be circulated in an economy, a common currency is chosen not only as a *medium of exchange* but also as a *unit of account*. Doepke and Schneider (2017) show that countries choose a common currency as a *unit of account* if the intensity of cross-border trade increases and the value of a national currency is too volatile. In contrast to these papers, our paper shows that the burden of sovereign debt affects the choice of a *unit of account* and thus the formation of an OCA. Moreover, our paper shows the effectiveness of debt relief to maintain a currency union. Therefore, we believe that our results provide a new perspective on the benefits and costs of a common currency and contribute to developing a new theoretical framework for currency unions.

The remainder of this paper is organized as follows. Section 2 provides the model. Section 3 describes the case in which banknotes are the only medium of exchange. Section 4 shows that the introduction of a common currency improves welfare. Section 5 shows that, under the existence of sovereign debt, choosing a national currency may be optimal for a less competitive debtor country. Section 6 shows the conditions required for debt relief to be effective. Section 7 concludes.

2 The model

Time consists of two periods, 0 and 1. There are two island countries, indexed by $i \in \{1, 2\}$. In each country, there are two agents, households and banks. In the subsequent analysis, a household and a bank located in country i are denoted as household i and bank i.

In country i, there is a continuum of households, the population of which is normalized to one. At the beginning of period 0, a representative household i is endowed with one unit of good i, e_i. Households do not engage in any production activity in this economy. In period 1, a representative household i receives utility by consuming both e_1 and e_2 . The value of utility from consumption for a representative household i in period 1, U_i, is denoted by:

$$U_i(\omega_s) = \alpha(\omega_s) \ln C_i^1(\omega_s) + (1 - \alpha(\omega_s)) \ln C_i^2(\omega_s),$$

where $C_i^j(\omega_s)$ is consumption of e_j ($j \in \{1, 2\}$) by a representative household i when a state ω_s ($s \in \{a, b\}$) occurs. In terms of α and ω_s, we assume that all households face the same preference shock at the beginning of period 1, such that ω_a (ω_b) occurs with probability q $(1 - q)$ and:

$$\alpha(\omega_s) = \begin{cases} \alpha & \text{when } \ \omega_s = \omega_a, \\ 1 - \alpha & \text{when } \ \omega_s = \omega_b, \end{cases}$$

where $0 < \alpha < 1$. For example, suppose that $\alpha = 2/3$, all households in both countries put twice (half) as much weight on consuming e_1 as on consuming e_2 when ω_a (ω_b) is realized. In terms of this preference shock, we assume that households cannot arrange mutual insurance against the shock among them in period 0 owing to limited commitment[3].

The good with which the representative household i is endowed is stored until period 1. Here, we assume that households do not have a storage technology, so they rely on another

[3]This assumption of incomplete capital markets is appropriate for the eurozone. Sørensen and Yosha (1998) show that before the introduction of the euro, capital markets in the eurozone were not integrated because neither factor income flows nor cross-border flows of physical goods contributed significantly to international risk sharing.

agent, the “bank”, which has a large “freezer” in which the goods can be perfectly stored until period 1. In each country, there is a continuum of banks, the population of which is normalized to one. Banks in country i are perfectly competitive and, due to spatial separation, a representative household i deposits its endowment only with bank i. A representative bank i is endowed with one unit of asset a, which is common to banks 1 and 2, at the beginning of period 0 and consumes assets in period 1. Banks are risk neutral and the value of utility from consumption for a representative bank i in period 1, V_i, is simply given by:

$$V_i(\omega_s) = A_i(\omega_s),$$

where A_i is bank $i^{'}$s consumption of a bank asset when ω_s is realized.

3 Banknotes as a sole medium of exchange

Next, we consider the case in which banknotes are the only medium of exchange in the economy. In period 0, a representative household i deposits its endowment, e_i, to a representative bank i and receives bank i's banknote, which specifies that a holder of the banknote is certain to receive a unit of e_i in period 1. In this sense, the *unit of account* of this deposit contract is the amount of e_i. In period 1, after the realization of a preference shock, all households exchange their banknotes to obtain the goods they want[4]. In these transactions in this economy, banknotes are used as a *medium of exchange*.

[4]Banknotes are bearer notes so they can be exchanged between households.

First, the maximization problem of a representative household i is defined as:

$$\begin{aligned} \max_{C_i^j(\omega_s)} \quad & U_i(\omega_s) \\ \text{s.t.} \quad & \sum_j P_j(\omega_s) C_i^j(\omega_s) = P_i(\omega_s), \end{aligned} \tag{1}$$

where $P_i(\omega_s)$ is the goods-market price of good i and it also denotes the price of banknote i in period 1 when ω_s is realized.

Next, the maximization problem of a representative bank i is defined as:

$$\begin{aligned} \max_{A_i(\omega_s)} \quad & V_i(\omega_s) \\ \text{s.t.} \quad & P^a(\omega_s) A_i(\omega_s) = P^a(\omega_s), \end{aligned} \tag{2}$$

where $P^a(\omega_s)$ denotes the asset-market price of an asset in period 1 when ω_s is realized.

Then, given the set of parameters, (α, q), an equilibrium in this economy consists of a vector of parameters $(C_i^j(\omega_s), A_i(\omega_s), P_i(\omega_s), P^a(\omega_s))$, such that: 1) a representative household i solves (1) for given levels of α and $P_i(\omega_s)$; 2) a representative bank i solves (2) under given levels of α and $P^a(\omega_s)$; 3) the exchange rate of banknotes is determined competitively; and 4) markets in e_i and a clear so that

$$\sum_i C_i^j(\omega_s) = 1 \tag{3}$$

$$\sum_i A_i(\omega_s) = 2 \tag{4}$$

are satisfied.

The following proposition characterizes the optimal allocation in the economy with only banknotes.

Proposition 1 (The economy with only banknotes)**.** For all $j \in \{1, 2\}$, $C_1^j(\omega_s) = \alpha(\omega_s), C_2^j(\omega_s) = 1 - \alpha(\omega_s), P_1(\omega_s)/P_2(\omega_s) = \alpha(\omega_s)/(1 - \alpha(\omega_s))$, and $A_i(\omega_s) = 1$.

(Proof) See Appendix 1.

Proposition 1 shows that whereas a representative bank i consumes its own assets in both states, the consumption level of a representative household i varies depending on the realization of the states. This is because the value of banknote i becomes high (low) when e_i is preferred more (less) in period 1. Therefore, when banknote i is measured by a *unit of* e_i, a preference shock induces fluctuation in the relative price of banknotes, so that the consumption levels of households also fluctuate.

4 Introduction of a common currency

Next, we introduce the third agent, called "the central bank", which is located between the two island countries. The central bank has the ability to credit its tickets to both banks at the end of period 0 if the banks' assets are considered eligible as collateral. We call these tickets "currency". In this sense, the supply of currency can be interpreted as an open market operation, as the central bank exchanges currency for banks' assets if the assets comply with the eligibility criteria[5].

In this environment, if a bank chooses to exchange its asset with currency, the *unit of account* in a deposit contract can be not only goods but also currency. That is, in a deposit contract, a representative household i is repaid a fraction γ_i in the form of e_i and a fraction $1 - \gamma_i$ in the form of currency, m_i, in period 1. Because γ_i is an endogenous variable for a representative household i, the supply of currency is generated endogenously in this model.

[5]For Eurosystem monetary policy operations, the credit assessment framework lays down the procedures, rules, and techniques to ensure that eligible assets comply with the credit quality requirements.

Under this modification, the maximization problem of a representative household i in period 1 is given by:

$$\begin{aligned} \max_{C_i^j(\omega_s)} \quad & U_i(\omega_s) \\ \text{s.t.} \quad & \sum_j P_j(\omega_s) C_i^j(\omega_s) = \gamma_i P_i(\omega_s) + (1-\gamma_i) m_i. \end{aligned} \tag{5}$$

The budget constraint of (5) differs from that of (1) because a fraction $1-\gamma_i$ of a deposit is repaid in the form of currency. Then, in period 0, a representative household i chooses γ_i to maximize its expected utility.

Next, the maximization problem of a representative bank i in period 0 is given by:

$$\begin{aligned} \max_{\mu_i} \quad & E[V_i(\omega_s)] \\ \text{s.t.} \quad & M_i \leq E[P^a(\omega_s)] \\ & a_i \equiv M_i + B_i - (1-\mu_i) m_i \geq 0, \end{aligned} \tag{6}$$

where M_i denotes the amount of currency supplied by the central bank, B_i denotes the amount of currency that it borrows from other banks in period 0, μ_i denotes the fraction of a deposit that it repays in the form of e_i, and a_i denotes the balance of the amount of currency that it holds. The first constraint implies that the central bank supplies currency only up to the expected value of collateralized bank assets[6]. The second constraint implies that the amount of currency that a representative bank i can commit to paying a depositor is up to the sum of the currency supplied by the central bank and the currency borrowed from other banks.

[6] In the Eurosystem, assets are subject to specific valuation haircuts, the rates of which differ in accordance with the quality, risk, and the issuer of the assets. In this paper, for simplicity, it is assumed that a bank asset is subject to a haircut of 0%.

The maximization problem of a representative bank i in period 1 is given by:

$$\begin{aligned} \max_{A_i(\omega_s)} \quad & V_i(\omega_s) \\ \text{s.t.} \quad & P^a(\omega_s)A_i(\omega_s) = a_i - R_i + (1-\mu_i)P_i(\omega_s), \end{aligned} \tag{7}$$

where R_i denotes the amount of currency that it repays to other banks in period 1[7]. Consider the third term on the right-hand side of the constraint. According to the deposit contract, a fraction $1-\mu_i$ of the deposit is repaid by currency so that a representative bank i can sell $1-\mu_i$ units of e_i at the competitive price $P_i(\omega_s)$.

Given the set of parameters, (α, q), an equilibrium is characterized as in the previous case, except that we should add three equations concerning the demand and supply of currency, as follows:

$$\gamma_i = \mu_i, \tag{8}$$

$$\sum_i m_i = \sum_i M_i \equiv M, \tag{9}$$

$$\sum_i B_i = \sum_i R_i = 0. \tag{10}$$

(8) shows that, in a deposit contract, the demand for currency by a representative household i equals the supply of currency by a representative bank i. (9) shows that the total demand for currency by banks equals the total supply of currency generated by the central bank. (10) shows the condition for the interbank market to be cleared.

Solving this equilibrium, we can derive the following proposition.

Proposition 2 (Introduction of a common currency)**.** When a common currency is introduced, it is optimal for both a representative household i and bank i to choose $\mu_i = \gamma_i =$

[7] B_i and R_i become negative when a representative bank i lends to other banks in period 0.

0 $\forall i \in \{1,2\}$ so that a common currency becomes the unique *unit of account* in the deposit contract. Under this, $C_1^j(\omega_s) = \rho, C_2^j(\omega_s) = 1-\rho, A_1(\omega_s) = 1-2\rho+P_1(\omega_s)/P^a(\omega_s), A_2(\omega_s) = 2\rho-1+P_2(\omega_s)/P^a(\omega_s)$ $\forall j \in \{1,2\}$ and $s \in \{a,b\}$, where $\rho \equiv q\alpha+(1-q)(1-\alpha)$. In addition, $P_1(\omega_a) = P_2(\omega_b) = \alpha M$, $P_1(\omega_b) = P_2(\omega_a) = (1-\alpha)M, P^a(\omega_s) = M/2$.

(Proof) See Appendix 2.

Proposition 2 indicates that, in an equilibrium with a common currency, all households choose a common currency as a *unit of account* in the deposit contract. This is because it is optimal for households to share the risk induced by the preference shock *among* countries by committing to using a common currency as a *unit of account* in a deposit contract. As a result of this risk-sharing effect, a common currency is accepted not only as a *unit of account* in period 0, but is also used as a *medium of exchange* in period 1. In addition, $E[V_i] = 1$ holds for $i \in \{1,2\}$ so that the relative price shock faced by the household sector is fully absorbed in the risk-neutral banks. In this sense, as Mundell (1961) notes, introducing a common currency enhances the efficiency of the economy because of the better performance of a common currency not only as a *unit of account* but also as a *medium of exchange*[8].

Although the economy with a common currency can achieve a more efficient allocation than can the economy with only banknotes, the consumption level will vary between households 1 and 2. This is because the expected value of banknotes varies. That is, when e_i is more (less) likely to be preferred in period 1, bank i should promise household i that it will provide more (less) currency in period 1 in return for receiving a unit of e_i in period 0.

[8]This result is robust because we can obtain the same result even if we consider different environments, such as varying the population of the islands or introducing a home bias in consuming goods.

5 Sovereign debt and the optimal currency area

Next, we introduce sovereign outstanding debt into the model and consider how it affects the choice of the *unit of account* and the formation of the OCA.

From January 1, 1999, the euro began to be substituted for the currencies of the member countries of the EU. This substitution required the adoption of irrevocable conversion rates, which were determined according to the principle described in a joint communiqué issued on May 2, 1998. Using this rate, all goods and financial transactions that had been denominated in each national currency were transformed into the common currency, the euro.

Then, we assume that country 1 owes d $(0 < d < 1)$ units of debt denominated by e_1 to country 2 and this must be repaid by the end of period 1. First, we consider the case in which the representative household 1 chooses a national currency as a *unit of account*. Because d units of e_1 are repaid in period 1, the budget constraint of household 1 becomes:

$$\sum_j P_j(\omega_s)C_1^j(\omega_s) = (1-d)P_1(\omega_s). \tag{11}$$

On the other hand, if it chooses a common currency, the debt is newly denominated in the common currency according to the predetermined irrevocable conversion rate, r. Thus, when $\gamma_i = 0$, its budget constraint becomes:

$$\sum_j P_j(\omega_s)C_1^j(\omega_s) = m_1 - r \cdot d. \tag{12}$$

Then, we focus on the two maximization problems for representative household 1[9]. One is (1), except that the budget constraint is (11), and the other is (5) with $\gamma_i = 0$, except that

[9]Note that, from Proposition 2, households in country 2 always have an incentive to choose a common currency as a *unit of account*. In addition, without loss of generality, we assume that $M = 2$ in the later explanation.

the budget constraint is (12). As the equilibrium cannot be solved analytically, we derive the utility of both cases and consider numerical examples when $r \in \{0.7, 1.2\}$ and $d \in \{0.2, 0.7\}$. Proposition 3 shows the results.

Proposition 3 (Optimal currency area)**.** Suppose that representative household 1 owes d units of debt denominated by e_1 to country 2. Then, there arises a parameter space (α, q) in which representative household 1 prefers to choose a national currency as a *unit of account*. In addition, the region expands as d becomes larger and r becomes higher.

(Proof) See Appendix 3.

[insert Figure 1 around here]

In Figure 1, representative household 1 chooses a common currency when (α, q) exists in the white region, whereas it chooses a national currency in the blue region. Figure 1(1) shows that representative household 1 prefers a national currency when 1) q is large and α is low, or 2) q is low and α is large. The former (latter) case means that the state a (b) is more likely to be realized and in that state, e_1 is less preferred by all households. Therefore, in both cases, the expected value of banknote 1 becomes low. Thus, representative household 1 faces a trade-off. That is, to achieve the risk-sharing effect on its consumption level, it would prefer to choose a common currency. However, this may increase the debt burden because, as (12) shows, the amount of debt repayment is constant irrespective of the state realized. Therefore, if the undesirable state for country 1 is more likely to occur, the debt burden becomes more severe so that representative household 1 may be better off choosing a national currency as a *unit of account*. Moreover, other figures in Figure 1 show that representative household 1 is more likely to choose a national currency when d and r becomes larger.

Proposition 3 highlights the cost of a common currency. That is, adopting a common currency as a *unit of account* generates the variation of the debt burden. Therefore, a less competitive debtor country may be better off choosing a national currency. Moreover, as the blue region expands from the upper-left or lower-right region to the center of the figure, representative household 1 is more likely to choose a national currency when the values of α and q are located around 1/2. This means that in these regions, the household feels that the risk-sharing effect of a common currency is less attractive.

6 Optimal currency area with debt relief

Finally, we consider whether country 2 has an incentive to relieve its debt claim on country 1. That is, if debt relief occurs, a representative household 2 reduces the amount of debt to $\acute{d}$ $(0 \leq \acute{d} < d)$, with an additional operating cost, s, per unit of renounced debt[10].

Under this modification, if representative household 2 accepts the debt relief under a common currency regime, its budget constraint becomes

$$\sum_j P_j(\omega_s) C_2^j(\omega_s) = m_2 + r \cdot \acute{d} - s(d - \acute{d}). \tag{13}$$

On the other hand, when it chooses a national currency, its budget constraint becomes

$$\sum_j P_j(\omega_s) C_2^j(\omega_s) = P_2(\omega_s) + dP_1(\omega_s). \tag{14}$$

[10]This cost includes various operating costs in relieving a debt such as negotiating a debt relief agreement among creditors or renewing a financial contract.

Then, we compare the outcome of two maximization problems for representative household 2 when (α, q) locates in a blue-region in Figure 1. One is (1), except that the budget constraint is (14), and the other is (5) with $\gamma_i = 0$, except that the budget constraint is (13). Proposition 4 shows the result.

Proposition 4 (Optimal currency area with debt relief)**.** Suppose that country 2 can reduce the amount of debt to $\acute{d}$. Then, $\acute{d} = \max\left\{2\left(\rho - \alpha^q(1-\alpha)^{(1-q)}(1-d)\right)/r, 0\right\}$ and there arises a parameter space (α, q) in which country 1 remains in a currency union. The region expands as s becomes smaller.

(Proof) See Appendix 4.

[insert Figure 2 around here]

Figure 2 shows the numerical results when $r \in \{0.7, 1.2\}$, $d \in \{0.2, 0.7\}$ and $s = 0.2$. This figure shows that when (α, q) exists in the light-blue region, country 2 renounces $d - d'$ units of debt so that country 1 remains in a common currency regime. This result suggests that, in that region, a creditor country has an incentive to renounce the debt for a debtor country to stay in a common currency regime. This incentive becomes stronger as r and d become larger. Moreover, the region expands as s becomes smaller[11]. However, as the blue region still remains in Figure 2, if more debt should be renounced for country 1 to stay in a currency union, country 2 cannot afford to accept it so that a common currency regime will collapse.

[11] In a hypothetical situation in which there is no cost in renouncing a debt (i.e., $s = 0$), country 1 always chooses a common currency regime if (α, q) exists in the blue region in Figure 1.

Proposition 4 suggests two policy implications about sovereign debt management. First, a creditor country may be willing to accept a debt relief plan *ex post* to maintain a currency union. On the other hand, to reduce the blue region in Figure 2, it is necessary for a debtor country to reduce an outstanding debt *ex ante* before it enters a currency union. This gives a theoretical rationale for the Maastricht Treaty, which lays down the preconditions for the level of sovereign debt for a smoothly functioning currency union.

7 Conclusion

This paper provides a theoretical model in which the choice of a *unit of account* may affect not only the terms of trade but the burden of debt repayment, which therefore also influences the formation of an OCA. In particular, a less competitive debtor country will be better off when it chooses a national currency as a *unit of account* so that debt relief may be useful to maintain a common currency regime. These results indicate the importance of the *unit of account* because it affects the total surplus accrued from various transactions in the country. We believe that these viewpoints will contribute to the development of a new theoretical framework for currency unions, which are particularly relevant to the euro and the eurozone.

Finally, to focus in a simple way on how a *unit of account* affects the formation of an OCA, we have not considered a production sector in our model. As Mundell (1961) denotes, labor and wage mobility are important elements in considering an OCA. Incorporating these elements into our model in future will provide richer implications for the literature on OCAs.

Appendix 1

By solving the maximization problem of a representative household i, (1), $C_i^j(\omega_s)(j \in \{1,2\})$ can be written as follows:

$$\begin{cases} C_i^1(\omega_s) &= \dfrac{\alpha(\omega_s)P_i(\omega_s)}{P_1(\omega_s)} \\ C_i^2(\omega_s) &= \dfrac{(1-\alpha(\omega_s))P_i(\omega_s)}{P_2(\omega_s)}. \end{cases}$$

By inserting these equations into (3), we can derive $C_1^j(\omega_s) = \alpha(\omega_s), C_2^j(\omega_s) = 1-\alpha(\omega_s)$ and $P_1(\omega_s)/P_2(\omega_s) = \alpha(\omega_s)/(1-\alpha(\omega_s))$.

In addition, as a representative bank i does not deal with other banks in this case, it is obvious that $A_i(\omega_s) = 1$. (q.e.d.)

Appendix 2

First, consider the maximization problem of a representative household i in period 1, (5). As in Appendix 1, $C_i^j(\omega_s)$ is written as follows:

$$\begin{cases} C_i^1(\omega_s) = \dfrac{\alpha(\omega_s)(\gamma_i P_i(\omega_s) + (1-\gamma_i)m_i)}{P_1(\omega_s)} \\ C_i^2(\omega_s) = \dfrac{(1-\alpha(\omega_s))(\gamma_i P_i(\omega_s) + (1-\gamma_i)m_i)}{P_2(\omega_s)}. \end{cases} \tag{15}$$

Then, inserting (15) into (3), we can derive the equilibrium price levels as:

$$\begin{cases} P_1(\omega_s) = \dfrac{\alpha(\omega_s)\bar{m}}{1-\alpha(\omega_s)\gamma_1 - (1-\alpha(\omega_s))\gamma_2}, \\ P_2(\omega_s) = \dfrac{(1-\alpha(\omega_s))\bar{m}}{1-\alpha(\omega_s)\gamma_1 - (1-\alpha(\omega_s))\gamma_2}, \end{cases} \tag{16}$$

where $\bar{m} \equiv (1-\gamma_1)m_1 + (1-\gamma_2)m_2$.

Next, consider the maximization problem of a representative household i in period 0. Given $(\alpha, q, P_i(\omega_s))$, it chooses γ_i to satisfy the following first order condition:

$$\frac{\partial E[U_i]}{\partial \gamma_i} = E\left[\sum_j \frac{\alpha_j(\omega_a)}{C_i^j(\omega_a)} \frac{\partial C_i^j(\omega_a)}{\partial \gamma_i}\right] = 0. \tag{17}$$

From (15), we have:

$$\frac{\partial C_i^j(\omega_s)}{\partial \gamma_i} = \frac{\alpha_j(\omega_s)(P_j(\omega_s) - m_i)}{P_j(\omega_s)}. \tag{18}$$

From the maximization problem of a representative bank i, (6) and (7), we have:

$$m_i = E[P_i(\omega_s)]. \tag{19}$$

Then, inserting (16), (18), and (19) into (17), we can derive:

$$\begin{aligned}\frac{\partial E[U_i]}{\partial \gamma_i} &= \sum_s \left(\frac{P_i(\omega_s) - m_i}{\gamma_i P_i(\omega_s) + (1-\gamma_i) m_i} \right) \\ &= q(1-q)(P_i(\omega_a) - P_i(\omega_b)) \left(\frac{1}{\gamma_i P_i(\omega_a) + (1-\gamma_i) m_i} - \frac{1}{\gamma_i P_i(\omega_b) + (1-\gamma_i) m_i} \right).\end{aligned}$$

The last term in parentheses in the second equation becomes zero only when $\gamma_i = 0$. Therefore, $\gamma_i = 0$ is optimal for all households.

Inserting $\gamma_i = 0$ and (9) into (15) and (16), we can derive:

$$\begin{cases} C_1^j(\omega_s) = q\alpha + (1-q)(1-\alpha) \equiv \rho \\ C_2^j(\omega_s) = q(1-\alpha) + (1-q)\alpha = 1-\rho \end{cases} \tag{20}$$

$$\begin{cases} P_1(\omega_a) = P_2(\omega_b) = \alpha M \\ P_1(\omega_b) = P_2(\omega_a) = (1-\alpha) M. \end{cases} \tag{21}$$

Inserting (8), (19), and $\gamma_i = 0$ into the second constraint of (6) implies that:

$$E[P_i(\omega_s)] - E[P^a(\omega_s)] \leq B_i. \tag{22}$$

Therefore, when $E[P_i(\omega_s)] - E[P^a(\omega_s)] > 0$, B_i becomes positive so that a representative bank i borrows from other banks.

Now, we focus on the case in which ω_a is realized in period 1. Then, the budget constraint of a representative bank i becomes:

$$P^a(\omega_a)A_i(\omega_a) = -R_i + P_i(\omega_a). \tag{23}$$

As (4) and (10), we can derive $P^a(\omega_a) = M/2$. In addition, as $R_i = -B_i$ and (19), we can derive:

$$\begin{cases} A_1(\omega_a) = -2\rho + 1 + 2\alpha \\ A_2(\omega_a) = -1 + 2\rho + 2(1-\alpha). \end{cases}$$

Using the same procedure, we can derive $P^a(\omega_b) = M/2$ and

$$\begin{cases} A_1(\omega_b) = -2\rho + 1 + 2(1-\alpha) \\ A_2(\omega_b) = -1 + 2\rho + 2\alpha. \end{cases}$$

(q.e.d.)

Appendix 3

First, consider a maximization problem (1) subject to the budget constraint (11). As in Appendix 1, the equilibrium exchange rate becomes:

$$\frac{P_1(\omega_s)}{P_2(\omega_s)} = \frac{\alpha(\omega_s)}{1-\alpha(\omega_s)}. \tag{24}$$

Using (24), $C_1^j(\omega_s)$ becomes:

$$C_1^j(\omega_s) = \alpha(\omega_s)(1-d). \tag{25}$$

Let $E[U_1^N]$ be the expected payoff of representative household 1 when it chooses a national currency. Then, from (25), $E[U_1^N]$ is given by:

$$E[U_1^N] = \ln \alpha^q (1-\alpha)^{1-q}(1-d).$$

Next, consider a maximization problem (5) subject to the budget constraint (12) and $\gamma_i = 0$. As in the previous case, $C_1^j(\omega_s)$ is given by:

$$C_1^j(\omega_s) = \frac{m_1 - r \cdot d}{m_1 + m_2}.$$

Here, from (9) and it is assumed that $M = 2$, (19) can be rewritten by:

$$m_1 = \rho\bar{m}, \quad m_2 = (1-\rho)\bar{m}.$$

Then, letting $E[U_1^C]$ be the expected utility when it chooses a common currency, we can write:

$$E[U_1^C] = \ln\frac{2\rho - r \cdot d}{2}.$$

Therefore, representative household 1 prefers a national currency if:

$$\alpha^q(1-\alpha)^{1-q}(1-d) > \frac{2\rho - r \cdot d}{2} \tag{26}$$

is satisfied.(q.e.d.)

Appendix 4

First, from (26), representative household 1 chooses a common currency if d is reduced to $\acute{d}$, which satisfies

$$\acute{d} \geq \max\left\{\frac{2}{r}\big(\rho - \alpha^q(1-\alpha)^{(1-q)}(1-d)\big), 0\right\}.$$

Let $E[U_2^N]$ and $E[U_2^D]$ be the expected payoff of representative household 2 when it chooses a national currency and a common currency with debt relief, respectively. Then, we can derive

$$E[U_2^N] = \ln \big(\alpha d + (1-\alpha)\big)^q \big(\alpha + (1-\alpha)d\big)^{(1-q)}$$

$$E[U_2^D] = \ln \frac{2 - 2\rho + r \cdot \acute{d} - s(d - \acute{d})}{2}.$$

Therefore, a representative household 2 accepts the debt relief if and only if

$$\frac{2 - 2\rho + r \cdot \acute{d} - s(d - \acute{d})}{2} > \big(\alpha d + (1-\alpha)\big)^q \big(\alpha + (1-\alpha)d\big)^{(1-q)}$$

is satisfied. (q.e.d.)

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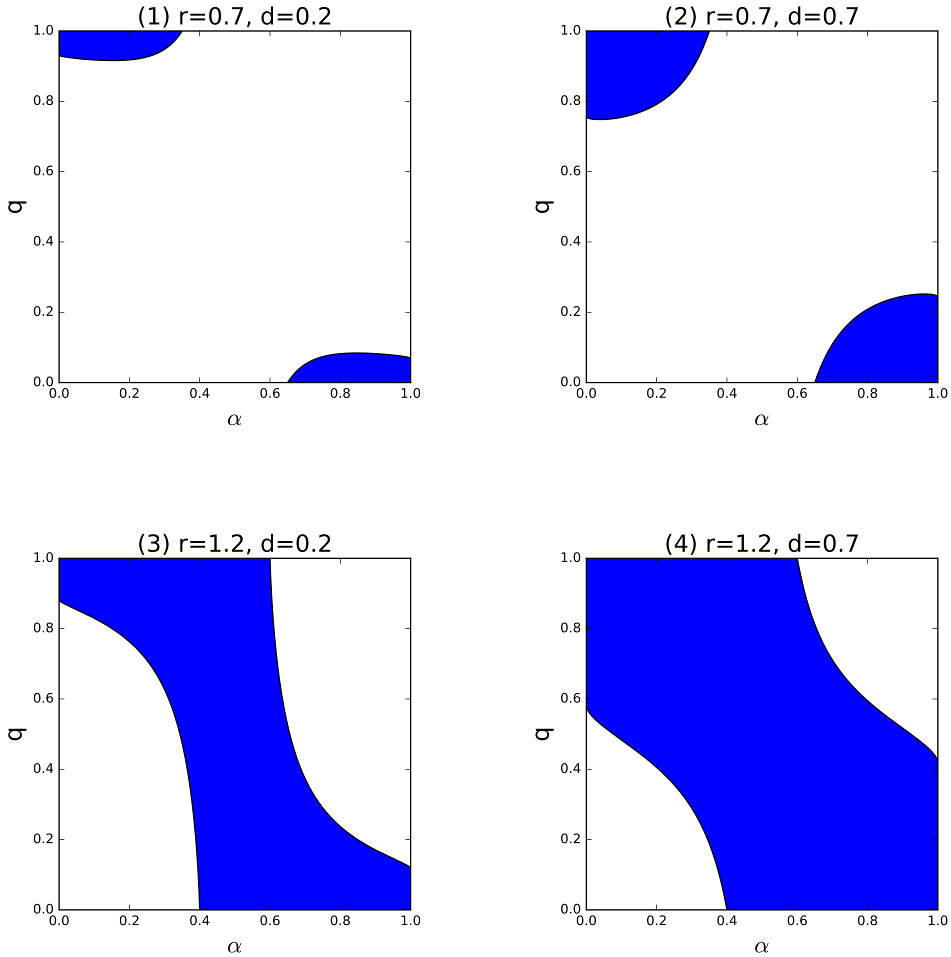

Figure 1: Optimal currency area

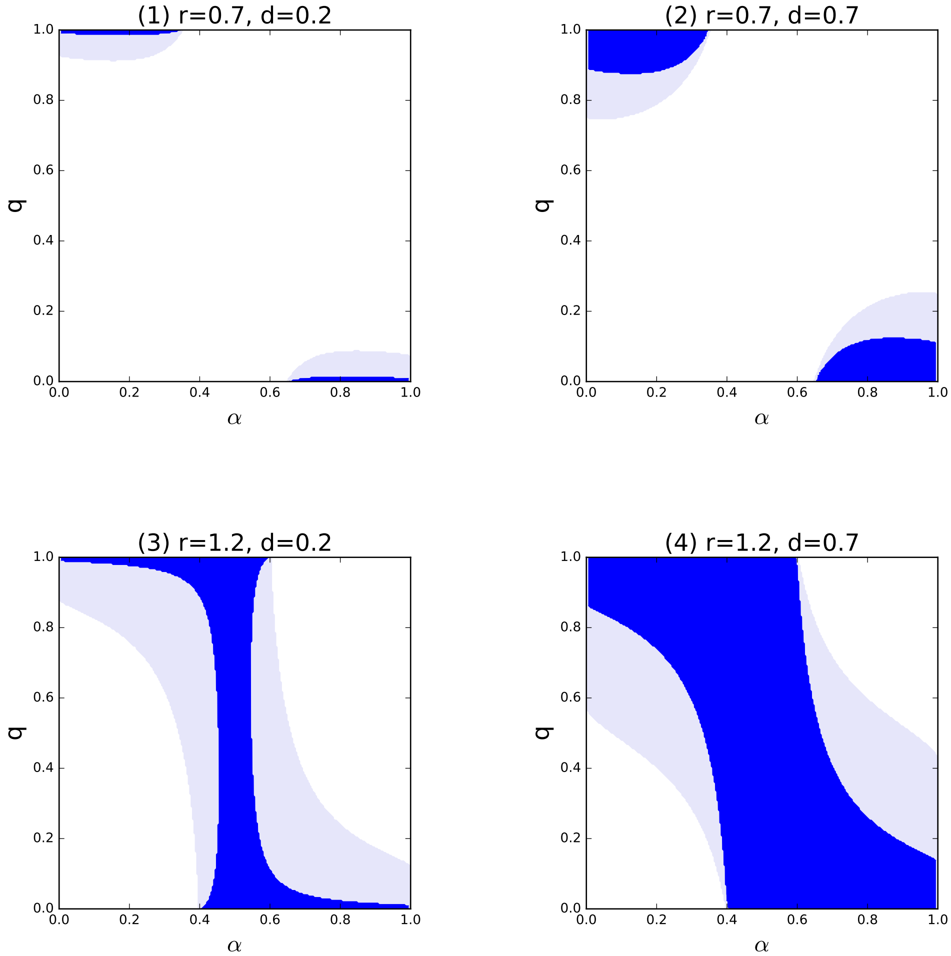

Figure 2: Optimal currency area with a debt relief

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