# On the Structure of the Spatial Distribution of Recent Urban Population in Japan

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# F I G U R E S

**A B S T R A C T**————————————————————————

The object of this paper is to analyze the structure of the spatial distribution of the recent urban population in Japan by applying the rank-size rule to the population of the cities in each region of this country.

According to the studies which have been done, the rank-size rule is regarded as an adequate model to describe the structure of the urban population in each region of Japan. On the other hand, the fact that under certain conditions the rank-size rule can be simultaneously applied to the urban population in a country was found possible theoretically.

Therefore, in this paper, the rank-size rule was used as a model to describe the urban population of the regions in Japan for the period of time from 1960 to 1975. We also found that the rank-size rule could be applied successfully to the population of the cities in each region in Japan for the period of time observed.

From the results obtained, we can conclude that the structure of the spatial distribution of the recent urban population in Japan is homogeneous.

## I. Introduction: Description of Structure of Regional Urban Population

A notable characteristic of the spatial distribution of the population of Japan is a concentration of population in the urban regions. Table 1 shows the transition of the structure of the population by urban and rural residences in Japan.

The transition of the proportion of urban population to the total population $p_u$ is described by the logistic curve expressed by:

$$p_u = \frac{89.87603}{1 + 4.191\ e^{-0.28275\ (t - 1920)\ /\ 5}} \tag{1}$$

where t is the number of a year (Figure 1).

According to the figures in Table 1, in 1975, 75 percent of the total population of Japan was in the urban regions. Therefore, when

Table 1. Population by Urban and Rural Residences

| Year | Population (in thousand) | | Proportion (%) | |
|------|--------|--------|--------|--------|
| | Urban | Rural | Urban | Rural |
| 1920 | 10,097 | 45,866 | 18.0 | 82.0 |
| 1925 | 12,897 | 46,840 | 21.6 | 78.4 |
| 1930 | 15,444 | 49,006 | 24.0 | 76.0 |
| 1935 | 22,666 | 46,588 | 32.7 | 67.3 |
| 1940 | 27,578 | 45,537 | 37.7 | 62.3 |
| 1945 | 20,022 | 51,976 | 27.8 | 72.2 |
| 1950 | 31,366 | 52,749 | 37.3 | 62.7 |
| 1955 | 50,532 | 39,544 | 56.1 | 43.9 |
| 1960 | 59,678 | 34,622 | 63.3 | 36.7 |
| 1965 | 67,356 | 31,853 | 67.9 | 32.1 |
| 1970 | 75,429 | 29,237 | 72.1 | 27.9 |
| 1975 | 84,964 | 26,973 | 75.9 | 24.1 |

Source: Sorifu, Tokeikyoku (1977). "Wagakuni no Jinko, Showa 50 nen Kokusei Chosa Kaisetsu, Series No. 1 (Population of Japan, Interpretation of the Census Report in 1975, Series No. 1)," Nihon Tokei Kyokai, p. 21.

Figure 1.  Application of Logistic Curve
           to Transition of Structure of Urban
           Population to Total Population in Japan



$$P_u = \frac{89.87603}{1 + 4.1491 \, e^{-0.28275 \, (t - 1920) / 5}}$$
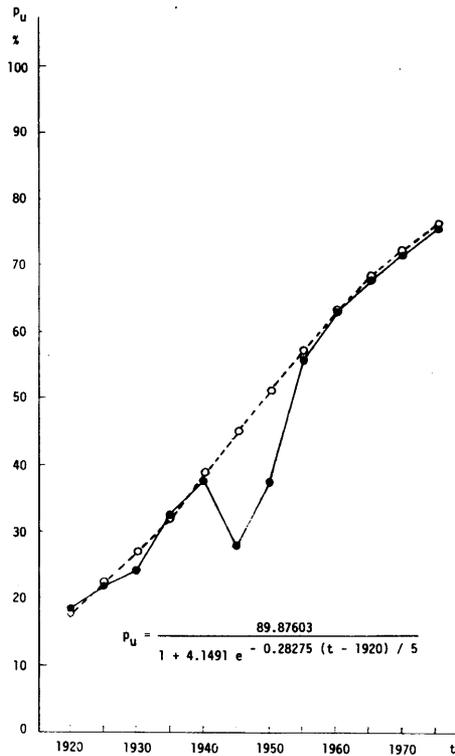
we try to grasp the structure of the population of Japan, especially the structure of the spatial distribution of this country, the characteristics of the spatial distribution of the urban regions should be observed.

In this paper, the characteristics of the regional system of the urban population in Japan are clarified.

The structure of urban population has been described by several methods.  Gibrat (1931) applied the lognormal distribution to the population of great cities in Europe in 1850 and 1926.  Christaller (1933) found a systematic structure of the cities from the observation of the urban population in Southern Germany.  According to his study, (1) the population of the cities can be divided into many classes by size, (2) the number of cities which belong to the mth class is $3^{m-1}$, and (3) the population of the cities which belong to the mth class is $K/3^{m-1}$, where K is the population of the "primate city." Zipf (1949) enlarged on the rank-size rule which had been originally

- 2 -

developed by Auerbach (1913) and Isard (1956). The rank-size rule
is generally expressed by:

$$\log P = - a \log R + b \qquad (2)$$

where P is the population of a city, R is the rank of the population
P, and a and b are parameters.

The mechanism by which the rank-size rule appears has been stud-
ied by many students. Berry (1961) insisted that when the distribution
of the urban population could be described by a truncated lognormal
distribution, the urban population could be expressed by rank-size
rule (Parr and Suzuki, 1973), and Berry and Garrison (1961) proved
that no great difference exists between Zipf's rank-size rule and
the structure found by Christaller.

According to the studies stated above, many models can be regard-
ed as adequate models for describing the systems of urban population.
Also the rank-size rule can be regarded as an adequate model for
expressing the structure of the population of cities in a country.

In this paper our object is to analyze the regional structure
of urban population in Japan. By applying the rank-size rule, we
will be able to analyze the form quantitatively. However, when we
apply the rank-size rule to the regional structure of population,
we must prove theoretically that the rank-size rule can hold simul-
taneously for the population of the cities in each region in a coun-
try, provided that the rule can hold for the population of cities
in the whole country.

Fortunately, this can be certified as shown in Section IV. There-
fore, in this study the rank-size rule is used for the description
of the urban population in each region in Japan.


II. Methods of Examination


In this study Japan is divided into nine regions. We try to
apply the rank-size rule expressed by the equation (2) to the popula-
tion of cities of each region for 1960, 1965, 1970 and 1975. Also
we examine whether the urban population of each region can be de-
scribed by the rank-size rule.

The values of the parameters of the equation (2) which is applied
to the data are obtained by the least square method.

If we can apply this rule to the population of the cities of

each region, it will be concluded that the regional structure of the urban population in Japan is homogeneous because it can be described by one and the same rule.

## A. Decomposability of Rank-size Rule

When we examine whether the rank-size rule can be applied to the urban population within a region, first of all we must investigate theoretically whether the rule can be applied simultaneously to the population of cities within more than one region.

## B. Theoretical Explanation

Suzuki (1980) discovered that if the urban population in a country observed can be described by the rank-size rule, the population of the cities in a region of the country can be also described by the rule under certain conditions.[1] This hypothesis about the characteristics which are found in the rank-size rule can explained by the following proof.

The basic assumptions used for the proof of the hypothesis are as follows:

1. The population of cities in a country observed P can be described by the rank-size rule expressed by equation (2). The rank of the population in the country is called "the rank in a country" and is denoted by R.

2. The country observed is divided into k regions, the population of a city in the sth region (s = 1, 2, . . . . , k) is denoted by $P_s$ , and the rank of the population of the city in the sth region is denoted by $R_s$, and this rank is called the "rank in the sth region."

Now, if there is a relationship expressed by:

$$R = k R_s - \epsilon \qquad (0 \leq \epsilon < k) \qquad (3)$$

between R and $R_s$, the population of a city in the sth region $P_s$ can be expressed by:

$$\log P_s = - a \log (k R_s - \epsilon) + b \qquad (4)$$

because the population $P_s$ whose rank is $R_s$ is population P whose rank is $(k R_s - \epsilon)$ and when the population P can be described by the rank-size rule, the P whose rank is $(k R_s - \epsilon)$ is obtained by [2]:

$$\log P = - a \log (k \, R_S - \epsilon) + b \tag{5}$$

When the $\epsilon$ is 0, by transformation of equation (3), we can obtain the equation:

$$\log P_S = - a \log R_S + (b - a \log k) \tag{6}$$

This equation can be expressed by:

$$\log P_S = - a' \log R_S + b' \tag{7}$$

where a' and b' are parameters whose values are a and (b - a log k), respectively.

Also when the $\epsilon$ is not 0, equation (5) can be transformed into the following equation:

$$\log P_S = - a \log k \, R_S \left( 1 - \frac{\epsilon}{k \, R_S} \right) + b \tag{8}$$

Therefore, $P_S$ can be expressed by:

$$\log P_S = - a \log R_S + b' - a \log \left( 1 - \frac{\epsilon}{k \, R_S} \right) \tag{9}$$

Here, if the value is so small that we can ignore the value of $\log \left( 1 - \frac{\epsilon}{k \, R_S} \right)$, we can obtain the relationship expressed by:

$$\log P_S \doteqdot - a' \log R_S + b' \tag{10}$$

Equations (7) and (10) are identical with the equation which expresses the rank-size rule because the b' is constant.

For example, if the country observed is divided into three regions and the $\epsilon$ written in equation (3) is constant for each region ($\epsilon$ = 0 for the first region, $\epsilon$ = 1 for the second region, and $\epsilon$ = 2 for the third region), then in the first region, we can find the population of cities whose ranks in a country are 1, 4, 7, 11, . . . . , in the second region the population of cities whose ranks in a country are 2, 5, 8, 12, . . . . , and in the third region the population of cities whose ranks in a country are 3, 6, 9, 13, . . . . . The rank-size rule can be exactly applied to the urban population in the third region. On the other hand, in the first and the second regions the rank-size rule holds approximately.

The results expressed by equations (7) and (10) mean that if the population of the cities of a country observed can be described by the rank-size rule, the population of the cities of each region found in the country observed can be also described by the rank-size rule provided that the condition written by equation (3) is satisfied.

This characteristic found in the urban population described by

the rank-size rule was called "decomposability of the rank-size rule" by Suzuki (1980).

Of course, the condition given by equation (3) cannot be necessarily satisfied in the actual population of the cities. However, if such a condition is approximately satisfied, the characteristic called decomposability of the rank-size rule will approximately exist in the actual population of the cities within each region of a country provided that the population of the cities in the country can be described by the rank-size rule.

C.  Examination by Simulation

In this paper, we test the hypothesis mentioned above by the following simulation.  First of all, we suppose that the population of the 644 cities in a country can be exactly described by:

$$\log P = -\ 0.8728 \log R + 6.9325 \tag{11}$$

The total number of the cities (644) and the equation of the rank-size rule written by equation (11) were obtained from the actual population of the cities of Japan in 1975 (Figure 2).

Figure 2.  Application of Rank-size Rule to
Population of Cities of Japan, 1975



$\log P = -\ 0.8728 \log R + 6.9325$

$r = -\ 0.9916$

Number of Cities:  644
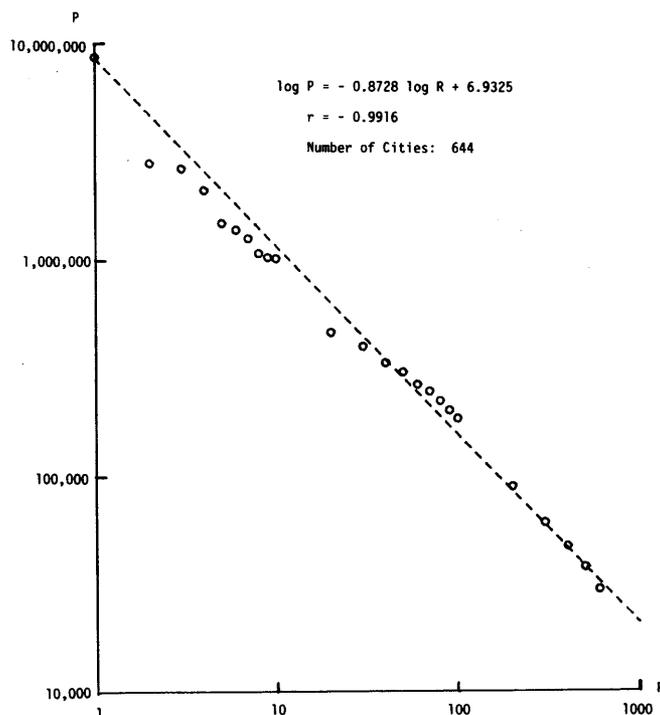
Secondly, we choose at random, by means of the table of random numbers, 32 cities from the 644 cities. The rank of the population of the 32 cities will approximately satisfy the conditions written by the equation (3). Thirdly, we regard these 32 cities as the cities within an imaginary region. Lastly, we try to apply the rank-size rule to the population of the 32 cities.

The results show that the rank-size rule can be applied to the population in the imaginary region as shown in Figure 3.

In the investigation stated above the condition which was written by equation (3) was given. But, if the condition expressed by:
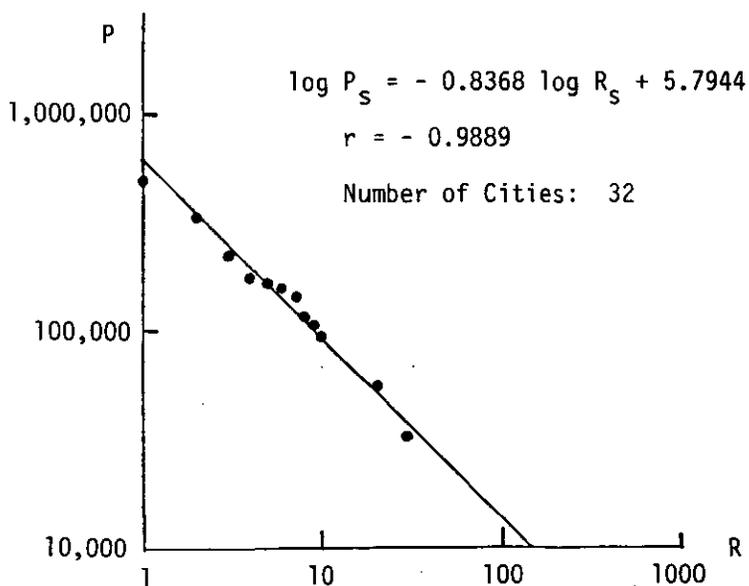
$$R = R_s + \rho \qquad (\rho > 0) \qquad (12)$$

is given, the characteristic given to the rank-size rule stated above, namely decomposability of this rule, disappears where $\rho$ is a constant.

With the condition given by equation (12) the population $P_s$ can be expressed as follows:

$$\log P_s = - a \log (R_s + \rho) + b \qquad (13)$$

Therefore, when the condition written by equation (12) is given,

Figure 3. Application of Rank-size Rule
to Population of Cities in
an Imaginary Region



$$\log P_s = - 0.8368 \log R_s + 5.7944$$

$$r = - 0.9889$$

Number of Cities: 32

population $P_s$ can be expressed by the equation:

$$\log P_s = - a \log R_s + b'' \tag{14}$$

$$b'' = b - a \log \left(1 + \frac{\rho}{R_s}\right) \tag{15}$$

In equation (14) the value of $b''$ is not constant, because $\rho$ is not 0, but larger than 0. Consequently, in this case, the population $P_s$ cannot be described by the rank-size rule.

## III. Empirical Results

The results obtained by application of the rank-size rule to the population of the cities within the nine regions of Japan: Hokkaido, Tohoku, Kanto, Chubu, Kinki, Chugoku, Shikoku, Kyushu and Okinawa, for 1960, 1965, 1970 and 1975 are shown in Table 2, where r is the correlation coefficient.

Figure 4 shows the rank-size rule applied to the population of the cities of the nine regions for 1975.

Based on the values of the correlation coefficient r, we can say that the rank-size rule can be successfully applied to the population of the cities within Japan and each region of the country.[3/]

## IV. Discussion of the Results Obtained

By the results obtained which were shown in Section V, we can say that the urban population of Japan and the nine regions in this country can be successfully described by the rank-size rule. The fact that we can arrive at the conclusion stated above means that the structure of the population of the cities within each region in Japan is homogeneous, because the structures of spatial distribution of the population of the cities within the nine regions are described by one and the same rule called the rank-size rule.

Additionally, if we make figures to compare the values of the parameters a or a', the coefficients of the term, log R for Japan and the nine regions, for the four years: 1960, 1965, 1970 and 1975 with each other, the following observations can be made. In the graphs in Figure 5, we can find four straight lines obtained by fitting the rank-size rule to the population of the cities within Japan and the nine regions. The four straight lines found in each graph are those
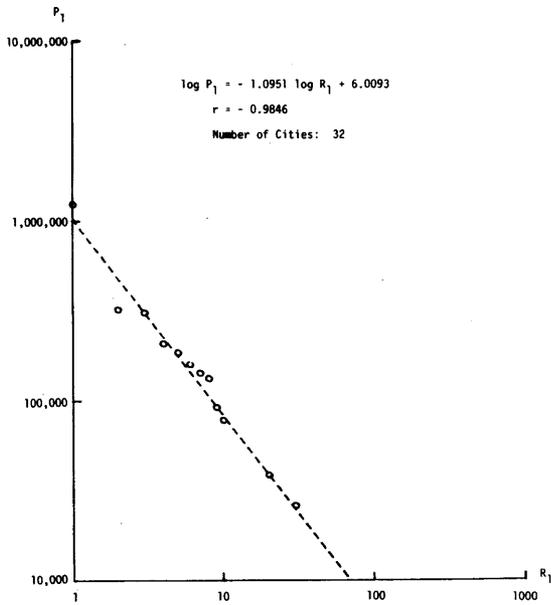
Table 2.  Results from Application of Rank-size Rule
to Urban Population of Regions in Japan

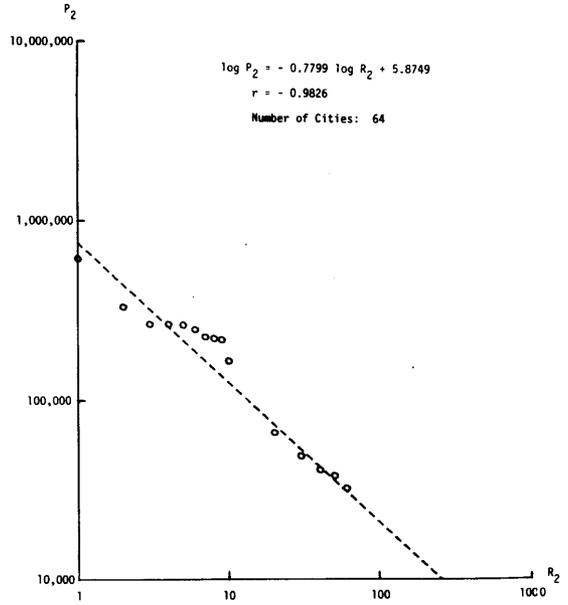| Region | Values | 1960 | 1965 | 1970 | 1975 |
|---|---|---|---|---|---|
| 1.  Hokkaido | a' | 0.85060 | 0.92870 | 0.98881 | 1.09514 |
|  | b' | 5.72957 | 5.82309 | 5.89563 | 6.00933 |
|  | r | -0.9937 | -0.9890 | -0.9883 | -0.9846 |
|  | n | 27 | 28 | 31 | 32 |
| 2.  Tohoku | a' | 0.62106 | 0.67815 | 0.77294 | 0.77993 |
|  | b' | 5.61582 | 5.70247 | 5.84025 | 5.87488 |
|  | r | -0.9896 | -0.9841 | -0.9808 | -0.9826 |
|  | n | 62 | 62 | 59 | 64 |
| 3.  Kanto | a' | 0.86026 | 0.90088 | 0.89203 | 0.89209 |
|  | b' | 6.19438 | 6.34883 | 6.43793 | 6.51552 |
|  | r | -0.9674 | -0.9748 | -0.9810 | -0.9835 |
|  | n | 104 | 109 | 136 | 150 |
| 4.  Chubu | a' | 0.70744 | 0.75158 | 0.80256 | 0.81939 |
|  | b' | 5.93602 | 6.04085 | 6.15778 | 6.21703 |
|  | r | -0.9880 | -0.9898 | -0.9909 | -0.9904 |
|  | n | 119 | 122 | 129 | 132 |
| 5.  Kinki | a' | 0.94814 | 0.98553 | 1.02771 | 1.01372 |
|  | b' | 6.26991 | 6.39981 | 6.52767 | 6.56399 |
|  | r | -0.9911 | -0.9964 | -0.9965 | -0.9948 |
|  | n | 86 | 87 | 93 | 98 |
| 6.  Chugoku | a' | 0.70873 | 0.76356 | 0.87685 | 0.94900 |
|  | b' | 5.70615 | 5.77554 | 5.91628 | 6.03855 |
|  | r | -0.9939 | -0.9939 | -0.9888 | -0.9940 |
|  | n | 49 | 49 | 47 | 48 |
| 7.  Shikoku | a' | 0.75529 | 0.82082 | 0.88402 | 0.91072 |
|  | b' | 5.54548 | 5.60562 | 5.67568 | 5.72653 |
|  | r | -0.9785 | -0.9813 | -0.9807 | -0.9817 |
|  | n | 29 | 29 | 29 | 30 |
| 8.  Kyushu | a' | 0.77007 | 0.87221 | 0.92912 | 0.94381 |
|  | b' | 5.93361 | 6.03998 | 6.10847 | 6.15053 |
|  | r | -0.9914 | -0.9917 | -0.9903 | -0.9885 |
|  | n | 80 | 75 | 76 | 80 |
| 9.  Okinawa | a' | u | u | 0.97490 | 0.99788 |
|  | b' | u | u | 5.25014 | 5.36031 |
|  | r | u | u | -0.9097 | -0.9648 |
|  | n | u | u | 9 | 10 |

Notes:  a' and b' are the parameters a' and b' of equation:  $\log P_s = - a' \log R_s + b'$ which expresses the rank-size rule applied to the sth region (s = 1, 2, . . . , 9).  r is the correlation coefficient between $\log R_s$ and $\log P_s$, and n is the number of cities of the sth region.  u is unavailable.

# Figure 4. Application of Rank-size Rule to Population of Cities within Nine Regions in Japan, 1975
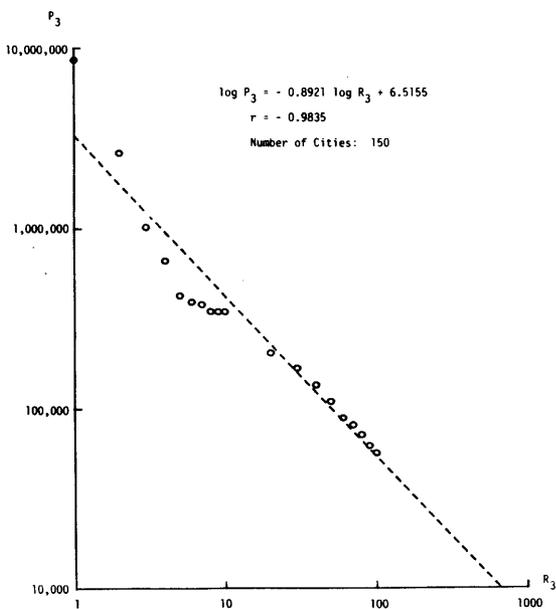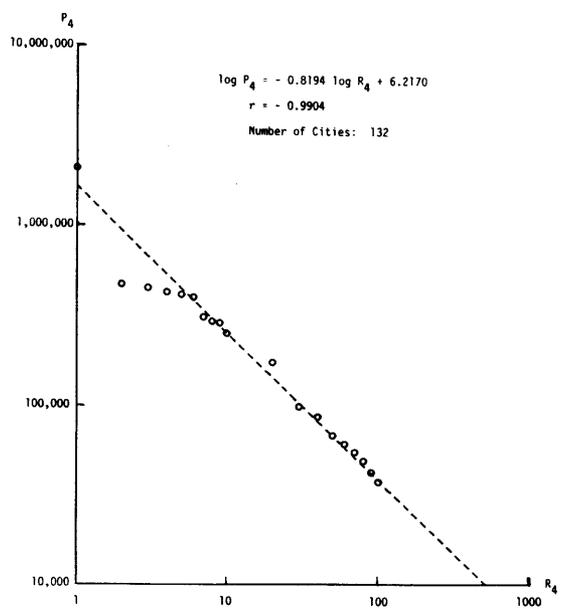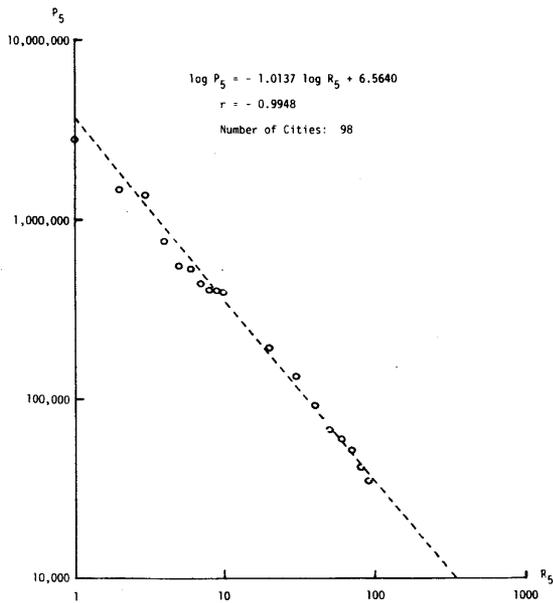
## 1. Hokkaido

$$\log P_1 = -1.0951 \log R_1 + 6.0093$$
$$r = -0.9846$$

Number of Cities: 32

## 2. Tohoku

$$\log P_2 = -0.7799 \log R_2 + 5.8749$$
$$r = -0.9826$$

Number of Cities: 64

## 3. Kanto

$$\log P_3 = -0.8921 \log R_3 + 6.5155$$
$$r = -0.9835$$

Number of Cities: 150

## 4. Chubu

$$\log P_4 = -0.8194 \log R_4 + 6.2170$$
$$r = -0.9904$$

Number of Cities: 132

## 5. Kinki

P$_5$

10,000,000

$\log P_5 = -1.0137 \log R_5 + 6.5640$

r = - 0.9948

Number of Cities: 98

1,000,000

100,000

10,000

1          10          100          1000

R$_5$

## 6. Chugoku

P$_6$

10,000,000

$\log P_6 = -0.9490 \log R_6 + 6.0385$

r = - 0.9940

Number of Cities: 48

1,000,000

100,000

10,000

1          10          100          1000

R$_6$

## 7. Shikoku

P$_7$

10,000,000

$\log P_7 = -0.9107 \log R_7 + 5.7265$

r = - 0.9817

Number of Cities: 30

1,000,000

100,000

10,000

1          10          100          1000

R$_7$

## 8. Kyushu

P$_8$

10,000,000

$\log P_8 = -0.9438 \log R_8 + 6.1505$

r = - 0.9885

Number of Cities: 80

1,000,000

100,000

10,000

1          10          100          1000

R$_8$

- 11 -

## 9. Okinawa

$P_9$

log $P_9$ = - 0.9979 log $R_9$ + 5.3603

r = - 0.9648

Number of Cities: 10

10,000,000

1,000,000

100,000

10,000
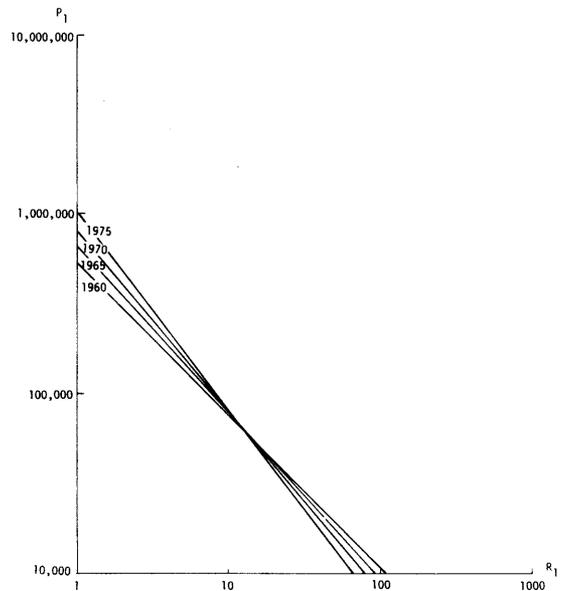
1     10     100     1000    $R_9$

Figure 5.   Comparison between Regression Equations expressing Rank-size Rule for Population of All Cities within Japan and Nine Regions Obtained at Four Points of Time by means of Regression Line
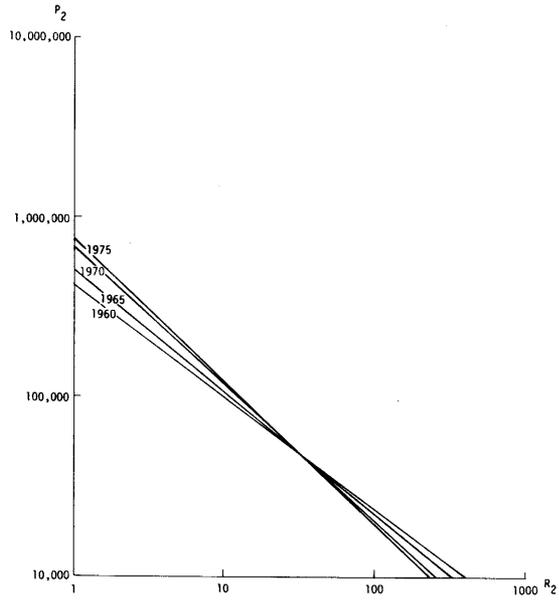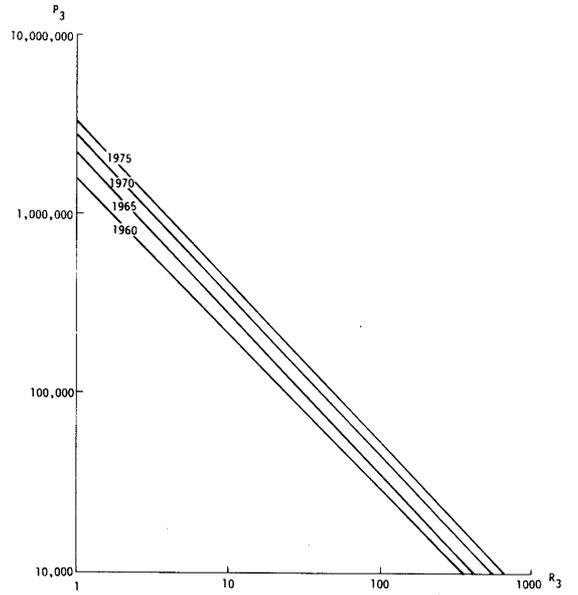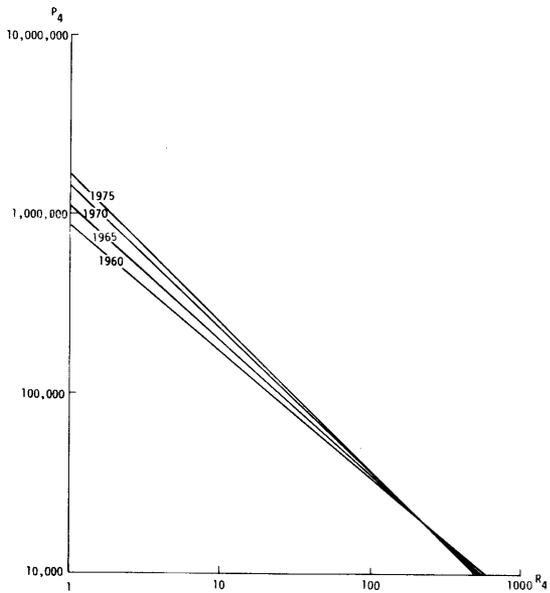
## 1. All Japan

P

10,000,000

1970   1975

1965

1960

1,000,000

100,000

10,000

1     10     100     1000    R

## 2. Hokkaido

$P_1$

10,000,000

1,000,000

1975

1970

1965

1960

100,000

10,000

1     10     100     1000    $R_1$
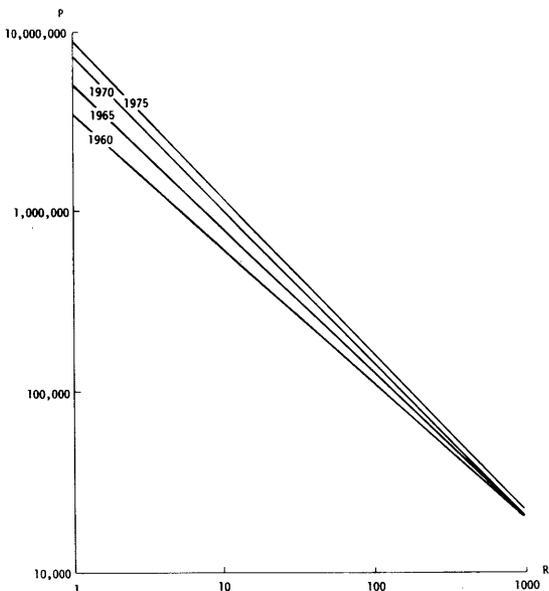
## 3. Tohoku



## 4. Kanto



## 5. Chubu



## 6. Kinki

## 7. Chugoku

$P_6$

10,000,000

1,000,000
- 1975
- 1965
- 1960 1970

100,000

10,000

1          10          100          1000  $R_6$

## 8. Shikoku

$P_7$

10,000,000

1,000,000

- 1975
- 1970
- 1965
- 1960

100,000

10,000

1          10          100          1000  $R_7$

## 9. Kyushu

$P_8$

10,000,000

1,000,000
- 1975
- 1970
- 1965
- 1960

100,000

10,000

1          10          100          1000  $R_8$

## 10. Okinawa

$P_9$

10,000,000

1,000,000

- 1975
- 1970

100,000

10,000

1          10          100          1000  $R_9$
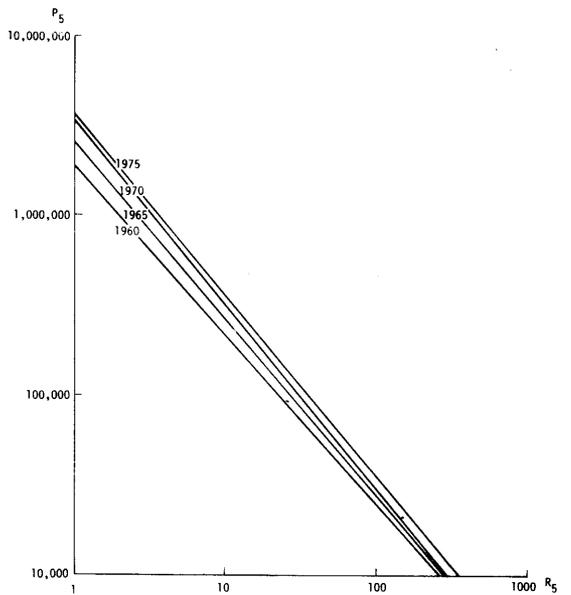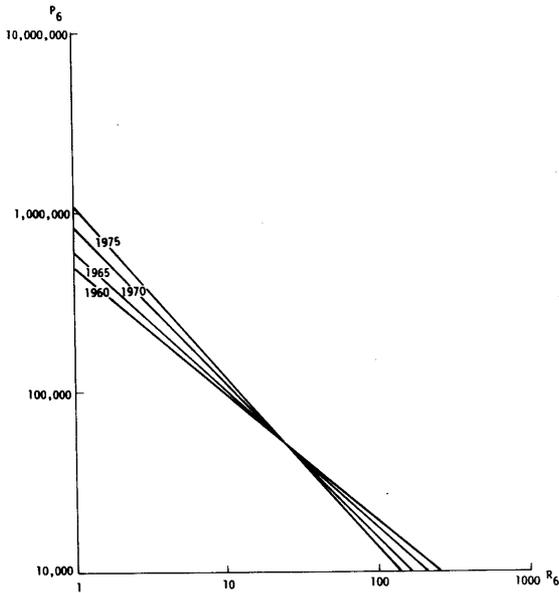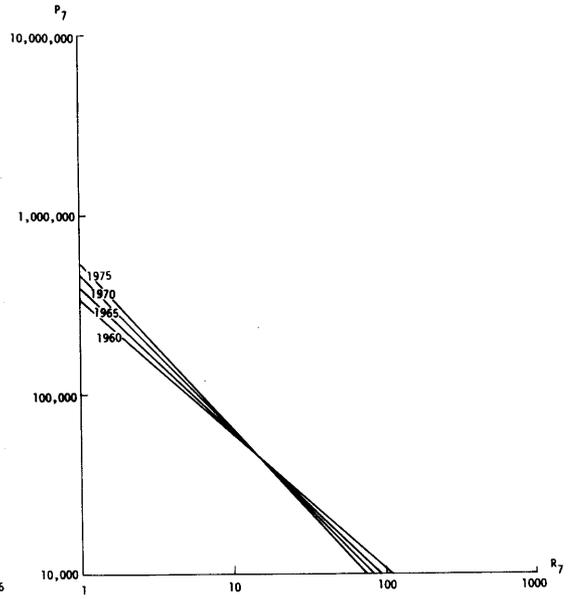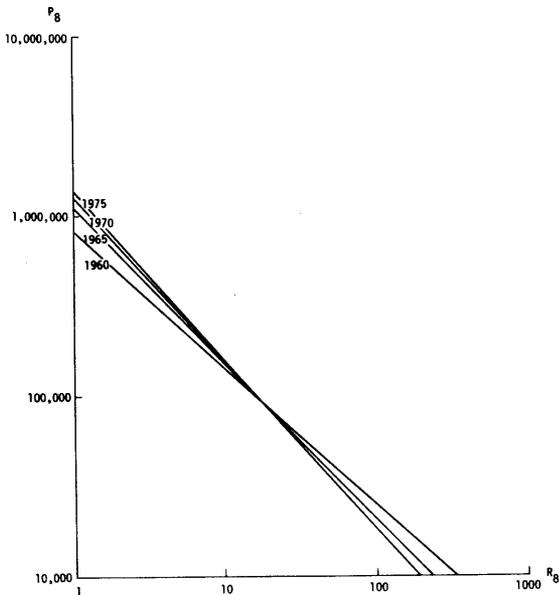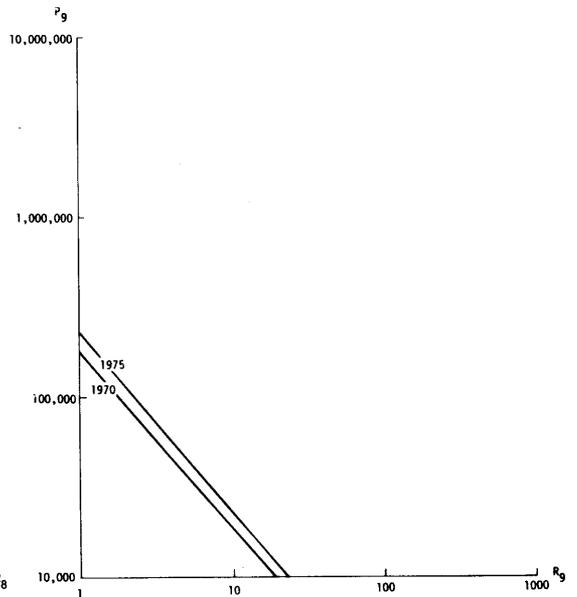
for four years: 1960, 1965, 1970 and 1975. The inclination of the straight lines found for the urban population of Japan which shows that the value of the parameter a becomes steeper as the time goes by. Generally, the same tendency can be found for the inclination of the lines in the graphs for the nine regions which show the value of the parameter a'.

On the other hand, when we compare the value of the parameter a in equation (2) with the value of the parameter a' in equations (7) and (10) which is obtained by the theoretical examination of the character of the rank-size rule, we find that the values of the parameters a and a' are equal to each other. From this fact we can say that if the condition expressed by equation (3) is satisfied for the structure of the urban population in a country, the rank-size rule can be found for the city population of each region in the country by the characteristic of the rank-size rule called decomposability of the rule, and for the values of the parameters a and a' found in the regression equations expressing the rank-size rule which is applied to the populations of the cities in the country and in each region. Consequently, if we can find the decomposability of the rank-size rule in the system of the urban population in a country, the value of the parameter a' found in the regression line expressing the rank-size rule applied for each region in the country will become larger when the value of the parameter a found in the regression equation expressing the rank-size rule applied to the country increases.

Therefore, it can be said that the reason why we can find the facts on the inclination of the regression lines expressing the rank-size rule applied to the population of Japan and the nine regions of this country which were stated above will be that in the population of the cities of each region in Japan, the condition expressed by equation (3) which is needed for the appearance of the characteristic called decomposability of the rank-size rule is approximately satisfied.

According to the examination of the work by Auerbach, Gibrat, Christaller, Zipf, Berry and Garrison, we find that the rank-size rule is an adequate rule to describe the distribution of the urban population in a country. By the characteristic of the rule called "decomposability of the rank-size rule" found by Suzuki, we can also apply this rule to the regional distribution of the urban population

in a country.

From a theoretical examination of the characteristics of the rank-size rule, we can obtain the following hypothesis:

> "When we find that the rank of the urban population in each region in a country satisfies the condition written by equation (3), we can say that the structure of the population of the cities in all the regions in a country is homogeneous because the urban population of each region in a country can be described by one and the same rule called the rank-size rule by the characteristic called decomposability of the rank-size rule."

If we utilize this hypothesis, we can investigate the regional structure of the urban population in Japan by applying the rank-size rule to the distribution of the population of the cities within the regions in this country.

As a result of the application of the rank-size rule to the distribution of the urban population within the nine regions in Japan, the rule fits very well. We also found that the population of the cities in each region of Japan could be described by "one and the same" rule called the rank-size rule. Therefore, we can conclude that in Japan we can find a homogeneous structure in the regional structure of the population of cities.

## V. Acknowledgements

## Notes

1/ This condition is expressed by equation (3).

2/ The P in equation (5) is regarded as $P_S$. Therefore, if we substitute $P_S$ for P in equation (5), we can obtain equation (4).

3/ Otomo (1979) applied the rank-size rule to the population of urban areas within the three major metropolitan areas of Japan: Keihin, Keihanshin and Chukyo metropolitan areas and found that the rule could be successfully applied to those groups.

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