

# ***Decomposition and Reassembly of the Age-Time Distribution***

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## A B S T R A C T

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To understand how age distribution has changed over time we consider simultaneously the several calendar years for which data are available. Rotating the age-time plane through 45 degrees places cohorts in columns on the tabular presentation and cohorts rather than periods in vertical sections parallel to an axis plane of the three-dimensional chart. When that is done it turns out that the differences between any pair of successive columns are remarkably constant.

The constancy means that the difference in numbers between two successive cohorts comes out nearly the same when those cohorts are independently measured at different times and ages. The average increase can be designated the intercohort difference of the two cohorts in question. Knowing only the profile of intercohort differences plus the numbers of one central cohort, we can to a close approximation reassemble the age-time distribution.

The profile of intercohort differences summarizes the evolution of vital rates through time. Its calculation is shown in detail for Japan, and in summary for 38 other populations. Among other results, the summary brings out the sharp upturn in population increase after the first World War, the baby boom in the developed countries after the second World War, and the sudden onset of the unprecedented population expansion that still continues in the less developed countries.

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## I. Introduction

The essence of the Cartesian method for understanding, no matter what object, is to disassemble the object into its simplest elements, then reconstitute it from those elements. To do that is to create and verify a model that is a simplified representation of the object under investigation, and so lays bare the mechanism by which it operates.

This general statement of the method of science applies to age distribution in particular. A full set of age distributions like that of Table 1 for Japan for the years from 1950 to 1985 with its 120 numbers is difficult to understand; one can speak about it more easily once the age-time pattern is reduced to a small number of parameters. This paper tries to account for age-time sets of data in terms of simpler elements that will lead to understanding of the dynamics that produced the outcome, and so see how changes in those elements would change (or would have changed) the distribution.

The most obvious set of antecedent elements is the series of births, deaths and migration, and one way of proceeding is to build up the age distributions at different dates from these. Only for a relatively few countries, of which Japan is one, is there a complete set of vital statistics, and these countries will test the method whose greatest usefulness will be for less developed countries that have censuses but no vital statistics.

For some 70 percent of the world's population vital statistics of serviceable accuracy are not to be had. Lacking them we must find some way of using the age distributions themselves to derive a set of parameters that will explain the original data. One would like these underlying explanatory parameters to be independently meaningful ("behavioral", as is sometimes said), and seeking such will be the ultimate goal of what follows, but the initial goal is the more modest one of arithmetical decomposition and reconstitution of the age-time table.

## II. The Initial Data

The set of data that is to be explained is typified by the distributions by age for Japan and other countries provided by the UN

Table 1. Original Population Numbers by Age and Time, Japan, 1950-1985  
(Tens of Thousands of Persons)

Age	1950	1955	1960	1965	1970	1975	1980	1985
0	1118	952	801	820	888	994	859	773
5	967	1099	950	800	824	890	999	858
10	880	964	1092	945	795	826	893	997
15	864	870	939	1078	914	801	826	892
20	776	845	836	917	1069	916	791	824
25	619	762	827	842	912	1071	913	789
30	522	611	752	829	841	921	1070	909
35	508	512	603	751	825	842	917	1067
40	449	497	504	594	737	818	834	910
45	403	437	483	492	590	729	806	824
50	339	386	420	468	483	574	713	791
55	276	320	368	400	444	466	557	692
60	230	250	294	336	374	426	446	536
65	178	196	217	256	300	343	394	416
70	129	140	158	175	214	256	300	352

Source: United Nations, 1986

in 1986 in machine-readable form. A specimen is shown in Table 1 above. Like the tables that follow, it is a window on the more complete table that would show five-year age intervals from 0-4 to 80-84, for the years 1950, 1955, ..., and includes projections to 2025.

In Table 1 the window shows the years 1950 through 1985. The full UN table from which it is extracted shows ages to 80+, and the years 1985-2020 as projected. Figure 1 represents a part of Table 1.

### III. A Three-Dimensional Representation

To see in geometrical terms what the arithmetic of this paper achieves consider Figure 2, showing in semi-perspective the Japanese population from the age group 0-4 to 70-74, for calendar years 1950 to 2020, as throughout this paper estimated by the United Nations. It shows a sharp ridge starting at 0,1950 and going to the back of the figure at 70,2020, then a trench ten years later, then another ridge

Figure 1. UN Estimated Age Distributions. 1950-1975

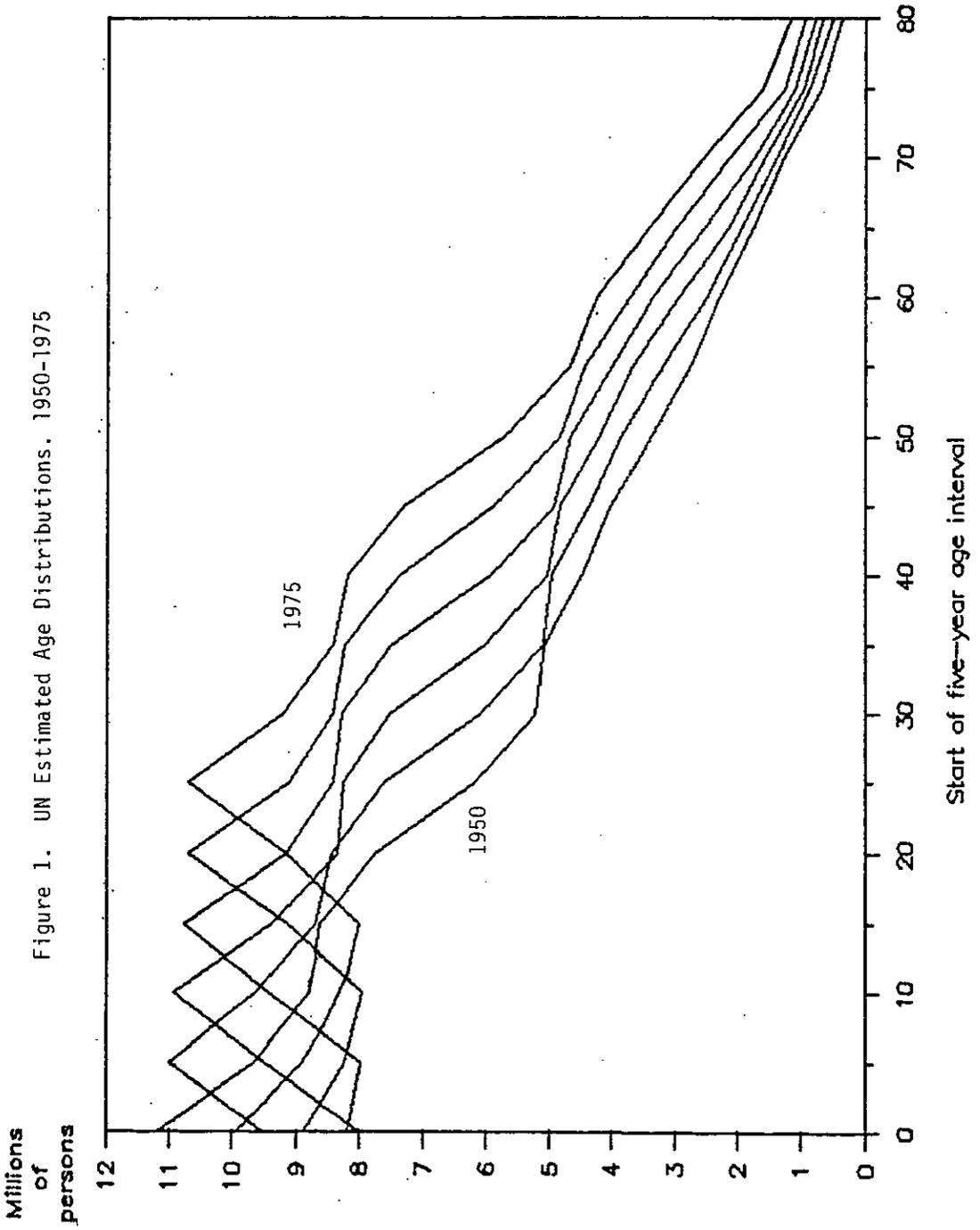


Figure 2. Three Dimensional Representation of the Data of Table 1, Japan, 1950-2020

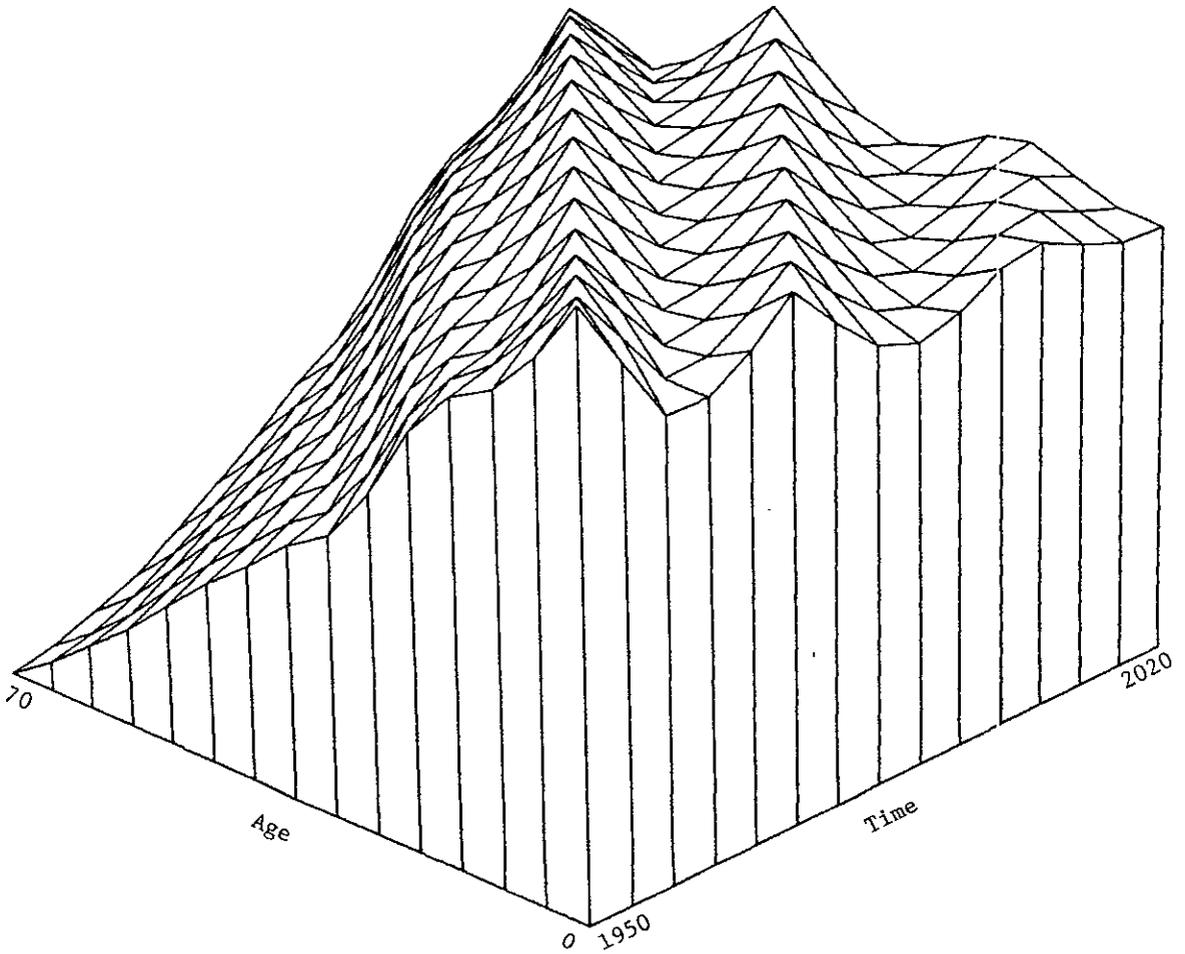


Table 2. Absolute Population Growth by Age and Time, Japan, 1950-1985  
(First Differences of Table 1, Taken Horizontally. Tens of  
Thousands of Persons)

Age	1950	1955	1960	1965	1970	1975	1980	1985
0	-166	-151	20	68	107	-135	-86	-25
5	132	-149	-150	24	66	108	-141	-86
10	85	128	-148	-149	31	67	104	-141
15	7	69	139	-164	-113	26	66	104
20	69	-9	81	152	-154	-125	34	66
25	144	65	15	70	159	-158	-124	34
30	89	141	77	12	79	150	-161	-123
35	4	91	148	74	17	76	150	-161
40	48	7	90	143	81	16	76	149
45	34	47	9	98	139	77	18	75
50	47	34	47	15	91	139	79	19
55	45	48	32	44	22	91	135	77
60	20	44	41	39	52	20	90	130
65	18	20	39	44	43	51	22	84
70	11	19	16	40	41	45	52	21

Source: Table 1, differenced by rows

25 years after that. A minor bend shows for the 1920s and another for the early 1940s. These correspond to well-known changes in the time series of births, whose effects on age are brought out more clearly in a three- than in any possible two-dimensional picture.

What the arithmetic below will do is analogous to the form of this figure: it will rotate the two axes at the base of Table 1 through 45 degrees. By taking one axis vertically, another parallel, to the paper it simplifies the representation, as will be seen.

#### IV. Absolute Increases over Time

The first step in the arithmetic is to form differences horizontally on Table 1. Table 2, the first differences of Table 1, shows along its main diagonal the five-year increases of the cohort that was 0-4 years of age in 1950, 5-9 in 1955, etc. Those increases (-166, -149, -148, ..., all measured in units of 10,000 persons) are

Table 3. Rearrangement of Table 2 by Cohort, Japan, 1950-1985. First Differences of Table 2, Taken Horizontally, i.e. Cohort Born in Five Years Following Given Date Minus Cohort Born in Five Years Before Given Date (Tens of Thousands of Persons)

	1925	1930	1935	1940	1945	1950	1955	1960	1965	1970	1975
0						-166	-151	20	68	107	-135
5					132	-149	-150	24	66	108	-141
10				85	128	-148	-149	31	67	104	-141
15			7	69	139	-164	-113	26	66	104	-140
20		69	-9	81	152	-154	-125	34	66	104	-140
25	144	65	15	70	159	-158	-124	34	66	103	-140
30	141	77	12	79	150	-161	-123	34	66	103	-139
35	148	74	17	76	150	-161	-123	34	65	103	-139
40	143	81	16	76	149	-159	-121	34	65	102	-138
45	139	77	18	75	148	-157	-120	33	65	102	-136
50	139	79	19	75	146	-154	-118	33	64	100	
55	135	77	19	74	143	-150	-115	32	63		
60	130	75	19	71	138	-144	-110	31			
65	121	71	18	67	130	-135	-103				
70	108	63	17	61	117	-120					

Source: Table 2, with Columns Offset to Place Cohorts in Columns

negative, while the increases (132, 128, 139, ...) of the just preceding cohort are positive. This is an example of the discontinuities that characterize the beginning of the baby boom in the developed countries and the onset of the population explosion in the LDCs.

#### V. Cohort Arrangement

Since the phenomena with which we are concerned show themselves in cohorts the next step is to rearrange the material accordingly. In Table 3, the rows of Table 2 have been offset so that the cohort born in a particular five-year interval appears in a column, i.e. so that the increases for a particular cohort fall under one another.

The decomposition attempted will depend for its success on constancy within the columns of Table 3. Any one column consists of wholly independent estimates of the same entity--the intercohort difference, the absolute amount by which the cohort following the

Table 4. Averages of the Columns of Table 3 (Cohort Born in the Five Years Following the Given Date Minus the Cohort Born Five Years Before the Given Date. Tens of Thousands of Persons)

Year	Average
1900	44
1905	38
1910	49
1915	15
1920	89
1925	135
1930	74
1935	14
1940	74
1945	141
1950	-152
1955	-125
1960	31
1965	65
1970	104
1975	-139
1980	-85

Source: Averages of Columns of Table 3

given year is greater than the cohort preceding, as measured at different ages at the corresponding different times. The small departures from constancy are caused by some combination of three elements:

- 1) Differential errors in the censuses on which successive columns of Table 1 are based, so that what are essentially the same people are counted differently at different dates;
- 2) Net migration, so that some of the people counted at one date are no longer there to be counted at a later date, or else have been added by immigration;
- 3) Changes in the age schedule of mortality.

Table 4 shows averages of the columns of the diamond-shaped table from which Table 3 is an excerpt. The rise at 1945 (i.e. average absolute 5-year increase of individuals born in 1940-45) is conspicuous. Only slightly less conspicuous is the rise at 1925, i.e. persons born 1920-25. Figure 3 plots Table 4, showing in addition the smallest and largest items of each column of Table 3.

Figure 3. Intercohort 5-year Increase, 1890-1970

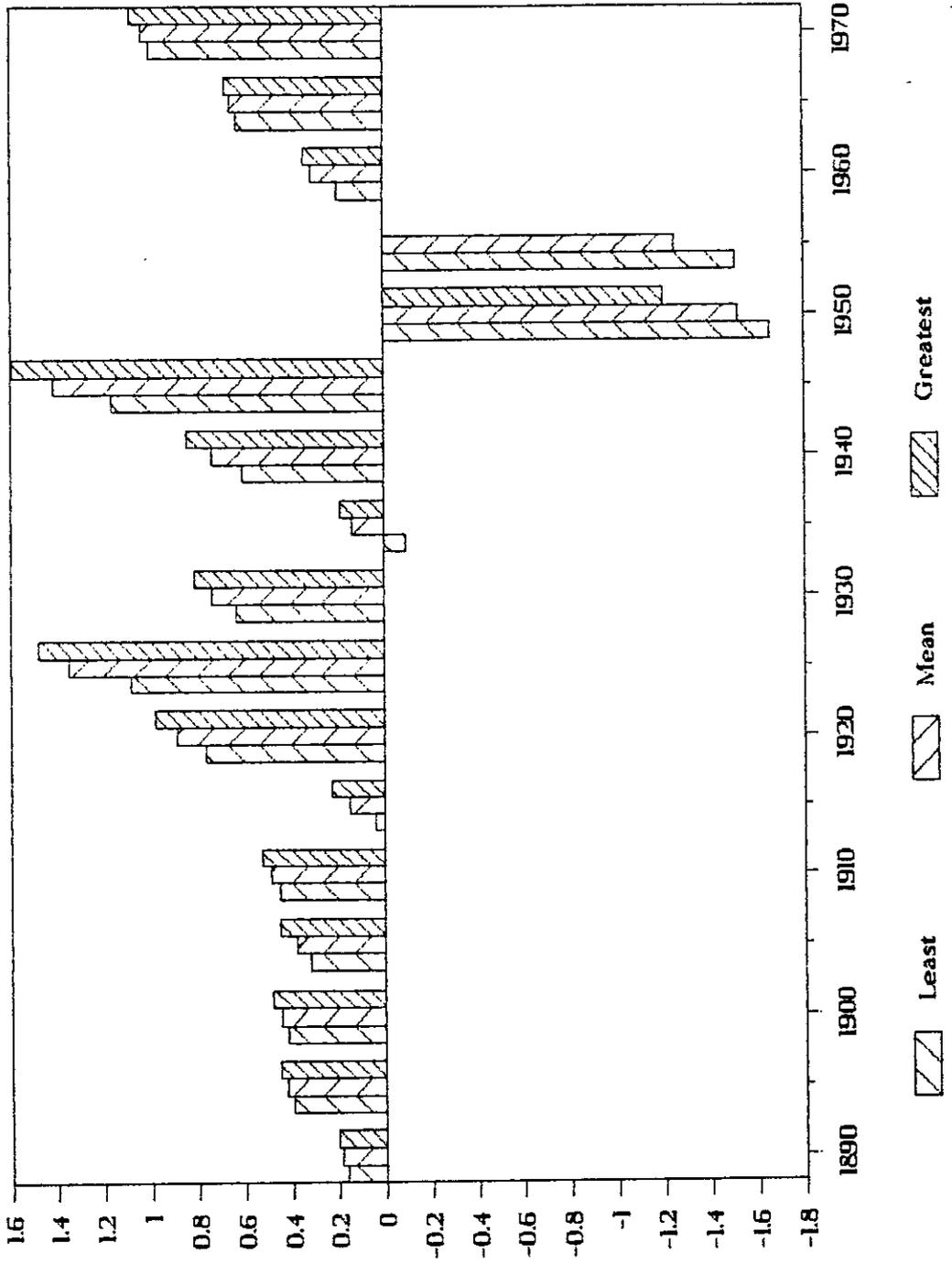


Table 5. Middle Cohort (Born in 1945-50)  
to Serve as Base for Reconstruction  
(Tens of Thousands of Persons)

Age	Number
0	1118
5	1099
10	1092
15	1078
20	1069
25	1071
30	1070
35	1067
40	1059
45	1048
50	1031
55	1004
60	968
65	908
70	814

Source: First Diagonal of Table 1

We are working with the concept of cohort increase, which is different from the increase over time of a particular age or all ages together. If the life table is constant and there is no migration, then the cohort increase depends only on the increase of births. A similar statement can be made for a calculation confined to adult ages if mortality at those ages is constant, with only births and child mortality changing over time. But if mortality is changing in general fashion, then the cohort increase is awkward to define, since it is different when measured at different ages. One might argue that the first age group is privileged, and so its increase ought to be taken as the measure of increase for the cohort. However, what has been done above--measuring the increase of the cohort on the averages of its several ages and times--seems more broadly representative, and it avoids eccentricities due to peculiarities of enumeration in the 0-4 age group. That still leaves errors of course, but they are measurable by virtue of the independence of items in each column.

Table 6. Reconstruction: Cohort Arrangement, Japan, 1915-1980  
(Tens of Thousands of Persons)

	1915	1920	1925	1930	1935	1940	1945	1950	1955	1960	1965	1970	1975	1980
0								1118	966	841	871	937	1040	902
5							958	1099	947	823	853	919	1022	883
10						877	951	1092	940	815	846	911	1015	876
15					849	863	936	1078	926	801	832	897	1001	862
20				767	840	854	928	1069	917	793	823	889	992	853
25			634	769	842	856	930	1071	919	795	825	891	994	855
30		544	633	768	841	855	929	1070	918	793	824	890	993	854
35	526	541	630	764	838	852	925	1067	915	790	821	886	990	851
40	518	533	622	756	830	844	917	1059	907	782	813	878	982	843
45	507	522	611	745	819	833	906	1048	896	771	802	867	971	
50	490	505	594	728	802	816	889	1031	879	754	785	850		
55	463	478	567	702	775	789	863	1004	852	728	758			
60	427	442	531	666	739	753	827	968	816	691				
65	367	382	471	605	679	693	766	908	756					
70	273	288	377	512	585	599	673	814						

Source: Table 5, Adding and Subtracting Elements of Table 4

#### VI. The Reconstitution of Table 1

Using only the base cohort (that born 1945-50), recopied as Table 5, and the average increases of Table 4, we can reconstitute the entire age-time distribution. Working forward from the base gives the cohorts born 1950-55 to 2020; working backwards gives the cohorts born 1880-85 to 1940-45. Nothing more than addition and subtraction is required. Thus from each item of the 1118, 1099, etc. of Table 5 we add -152 as shown in Table 4 opposite 1950, etc. By adding these constant numbers forward, and subtracting backwards, the entire cohort table is obtained. Table 6 is the result.

The discrepancies from Table 1 are small, and they exist at all only insofar as the columns of Table 3 are not quite constant. If we had used the full information of Table 3, we would reproduce in Table 6 the original material of Table 1; there would be no decrease in the number of parameters and no gain in the calculation. We could do something intermediate between retaining only the average intercohort difference as Table 6 does, and using the entire detail of Table 3. Such an intermediate number of parameters might be obtained by

Table 7. Reconstruction of Table 1, Japan, 1950-1985

Age	1950	1955	1960	1965	1970	1975	1980	1985
0	1118	966	841	871	937	1040	902	817
5	958	1099	947	823	853	919	1022	883
10	877	951	1092	940	815	846	911	1015
15	849	863	936	1078	926	801	832	897
20	767	840	854	928	1069	917	793	823
25	634	769	842	856	930	1071	919	795
30	544	633	768	841	855	929	1070	918
35	526	541	630	764	838	852	925	1067
40	469	518	533	622	756	830	844	917
45	420	458	507	522	611	745	819	833
50	359	403	441	490	505	594	728	802
55	291	333	377	414	463	478	567	702
60	236	254	296	341	378	427	442	531
65	157	175	194	236	280	318	367	382
70	53	63	82	101	143	187	224	273

Source: Period Arrangement of Table 6

using a linear regression of the several columns of Table 3 on the base cohort born 1945-50. This would provide an alternative Table 6 that would incorporate two constants rather than one of Table 4 for each column. On a later occasion that will be done.

All that now remains is to offset the rows of the reconstituted Table 6 so as to convert back to the usual period form of Table 1. The offset version is shown as Table 7, and its discrepancies from Table 1 as Table 8. Table 9 gives the same discrepancies as percentages of the original Table 1.

#### VII. Accuracy Attained in the Reconstitution

Note that the errors shown in Table 9 for many ages are only about 1 or 2 percent. (The zero along the main diagonal does not count, since that diagonal of the reconstruction was taken from the original input for the cohort of 1945-50, and necessarily agrees.) It is where mortality variation is important and the denominator populations diminish, as beyond age 50, that we find gross percentage

Table 8.. Discrepancy of Reconstruction: Period Arrangement,  
Japan, 1950-1985

Age	1950	1955	1960	1965	1970	1975	1980	1985
0	0	14	40	51	49	46	42	43
5	-9	0	-3	23	29	29	24	26
10	-3	-14	0	-4	20	20	18	18
15	-15	-8	-3	0	12	0	5	5
20	-9	-5	18	11	0	2	2	-1
25	16	7	15	14	18	0	6	6
30	23	22	16	12	14	8	0	9
35	18	29	27	14	13	10	8	0
40	20	21	29	27	19	12	10	8
45	17	21	23	30	20	16	13	8
50	20	17	21	22	22	20	16	11
55	15	12	9	15	19	12	10	10
60	5	5	2	5	4	1	-4	-5
65	-21	-21	-23	-20	-19	-25	-27	-35
70	-76	-76	-77	-74	-72	-69	-76	-79

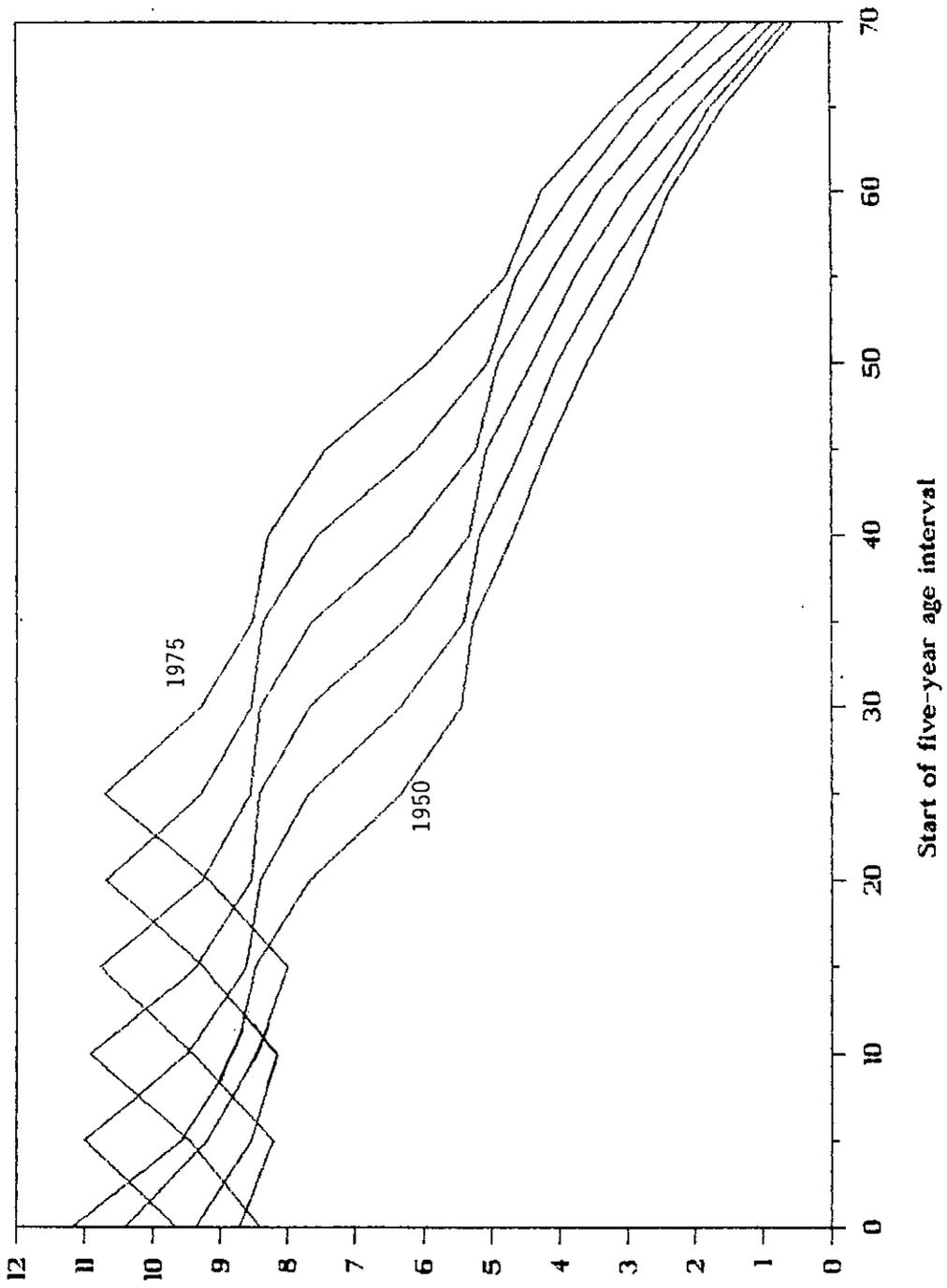
Source: Excess of Table 7 over Table 1.

Table 9. Percent Discrepancy of Reconstruction, Japan, 1950-1985

Age	1950	1955	1960	1965	1970	1975	1980	1985
0	0	1	5	6	6	5	5	6
5	-1	0	0	3	4	3	2	3
10	0	-1	0	0	3	2	2	2
15	-2	-1	0	0	1	0	1	1
20	-1	-1	2	1	0	0	0	0
25	3	1	2	2	2	0	1	1
30	4	4	2	1	2	1	0	1
35	3	6	4	2	2	1	1	0
40	4	4	6	5	3	1	1	1
45	4	5	5	6	3	2	2	1
50	6	4	5	5	5	4	2	1
55	5	4	2	4	4	3	2	1
60	2	2	1	2	1	0	-1	-1
65	-12	-11	-10	-8	-6	-7	-7	-8
70	-59	-55	-48	-42	-33	-27	-25	-22

Millions  
of  
persons

Figure 4. Reconstructed Age Distributions, Japan, 1950-75



differences. Between ages 5 and 50 the discrepancies are less than the enumeration errors to be found even in good censuses.

Figure 4 shows the reconstruction whose numbers appear in Table 7. The resemblance to Figure 1 is striking. If one cannot see the difference between original and reconstructed curves in the charts, and given that appearance on the age chart is the primary indication that discontinuities exist in the intercohort differences, then those discontinuities have been tracked back to the variables that underlie the reconstruction. The original and the reconstructed charts are in fact close enough that for the most part discrepancies become visible only when one is laid on the other.

In calculations not shown here an estimate was made of the average percent departure of the reconstituted age distributions from the original. For each of the 39 countries and regions of Table 10, numbers corresponding to Table 9 were calculated, and the percentages averaged as a crude measure of fit. The fit for Japan, at an average of 1.8 percent error for all ages, turned out better than the mean of the 39 countries and regions, but not by much. That Japan, Europe, and the United States are the closest suggests that the quality of census taking (or at least its consistency from period to period) is a main factor. That both Germanies and Canada show relatively poor fits suggests that migration is important. For reasons not easily stated Thailand is one of the worst fits, while India is good. To save space the table containing these numbers has been omitted.

### VIII. The Algebraic Interpretation

In another place the arithmetic described above is traced out in algebraic form, and this part of the argument will not be repeated here. It will suffice to show one formula:

$$D_{t-a-1} = \Delta_t B_{t-a} \left( \frac{\sum_{u=\alpha}^{\beta-1} L_{u,t-a}}{\beta - \alpha} \right) + B_{t-a+1} \left( \frac{\sum_{u=\alpha}^{\beta-1} \Delta_t L_{u,t-a}}{\beta - \alpha} \right)$$

where  $D(t-a-1)$  is the intercohort difference between the cohort just before time  $t-a$  and that just after;  $B(t-a)$  is the births that took place at time  $t-a$ ;  $L(u,t-a)$  is the probability of survival from age 0

to age  $u$  according to the cohort life table starting  $t-a$  years back. The unit for both age and time is five years.

What the formula says is that the intercohort difference is equal to the difference of the relevant births times the probability of surviving averaged over ages, plus the absolute births times the average difference of the two successive life tables. Where births are increasing and mortality falling both terms of the formula will be positive.

The regularity of the departures, whether expressed in absolute or in percentage form, suggests that we can do better if we are willing to allow for smooth mortality changes. Moreover, to decompose the intercohort differences into a part due to mortality and a part due to fertility changes would provide valuable historical information when applied in countries where direct birth and death data are lacking. To accomplish the decomposition we would go back to Table 2, inflate the cohorts by suitable life tables, carry through the reconstruction as done here, then deflate the reconstruction by applying the life tables in reverse. Unfortunately this requires either life table data, which are lacking for most countries, or else assumptions on mortality and the use of model tables. Much as one would like to have a decomposition of the intercohort differences into mortality and fertility elements, where life tables are lacking one hesitates to introduce the arbitrariness of assumed levels of mortality. .

#### IX. Disaggregation of World Intercohort Differences by Continents and Countries

Table 10 summarizes intercohort differences for the world as a whole and 38 countries or regions, showing absolute values for the periods around the 12 points of time from 1910 to 1965. The first number in the row for Japan is 487,000 for 1910, i.e., for the difference between the 1905-10 cohort and the 1910-15 cohort. That is the same number as is given in Table 4 as 490,000. (Table 4 was rounded to 10,000s and Table 10 to 1000s for the sake of the smaller countries.)

Thus Table 10 shows how the discontinuity is distributed among continents and among countries. These are absolute numbers, and they

are necessarily additive because the calculation of the intercohort differences is linear. We started with United Nations populations for the world and its subdivisions, that are certainly additive, and then calculated a profile of increases corresponding to Table 4 using nothing but addition and subtraction, so it too must be additive; and hence also increases of the increases are additive.

Consider the first three lines of Table 10, in which the LDCs and the MDCs add to the total. The cohort increase for the world around 1940 (i.e. the five years after minus the five years after 1940) is 7770 (the unit is thousands), and this is the net of an increase of 12606 in the LDCs and a decrease of 4836 in the MDCs. The net for the world rises to 44684 around 1945, for LDCs, 30837. Taking second differences, we find an increase of the increase from 1945 to 1950 of 36914, of which LDCs account for 18231 and MDCs for 18683. Thus the discontinuity around the year 1945 is equally distributed between the LDCs and the MDCs--what was called the population explosion in the former and the baby boom in the latter. Five years later the LDCs showed a further spurt, while the MDCs sank back as the baby boom died away.

As among continents, the 1945 discontinuity occurred primarily in Asia, with Europe next. The USSR made a contribution to it as it recovered from wartime hardships. Africa, with the poorest data, shows little sign of such a discontinuity as it continues steady and very rapid population increase. Latin America shows the discontinuity only hesitatingly. Within Asia the dominating element is China, showing 13019 of the 24906 first differences around 1945. So much for the allocation of the discontinuity among countries, expressed in additive form in Table 10.

While Table 10 shows how the world total for any particular cohort increase is broken down among countries, we cannot expect it to show the relative sharpness of a bend from one country to another, since those with more population will necessarily show bigger differences. Thus it is one thing to see the contribution of the several countries to the discontinuity for the world as a whole; it is quite another to compare the sharpness of the bend among countries. For example Table 10 will not show whether intercohort changes for Japan are more or less meaningful than those for other countries.

Table 10. First Differences of Birth Cohorts, i.e. Cohort Born in Five Years Following Given Date Minus Cohort Born in Five Years Before Given Date (Thousands of Persons)

	1910	1915	1920	1925	1930	1935	1940	1945	1950	1955	1960	1965
World	8610	307	23346	20315	12242	16082	7770	44684	55318	33028	51405	43984
LDCs	6911	7284	11337	13299	14735	10437	12606	30837	48841	28299	50535	50512
MDCs	1698	-6977	12009	7016	-2493	5644	-4836	13847	6478	4729	870	-6529
Africa	905	1087	1336	1753	1894	2136	2379	3614	4622	5725	6333	8074
Ethiopia	70	81	105	124	151	181	216	256	303	361	426	501
Nigeria	158	195	239	288	349	419	482	492	639	1031	1238	1760
Zaire	53	63	74	94	109	132	157	187	222	261	306	413
South Africa	55	74	90	104	123	146	160	200	369	347	409	479
Asia	5433	5612	9404	11454	12285	6720	8892	24906	38071	16695	39075	39977
Bangladesh	-92	-93	173	299	286	301	377	443	703	1804	1113	1546
Burma	69	127	73	68	2	100	299	425	250	457	325	475
China	1880	2542	3370	3488	5097	842	2290	13019	22100	-3484	20747	20690
India	1828	2014	2517	3082	3244	2566	2744	4687	7513	8736	7859	8126
Indonesia	297	136	508	930	818	765	308	336	1926	2386	1360	2379
Iran	48	85	101	116	147	164	284	519	613	475	607	643
Japan	487	150	889	1348	735	138	736	1414	-1520	-1247	307	655
Korea	126	112	232	188	396	384	107	306	767	1476	462	15
Malaysia	4	42	20	62	45	127	30	196	209	219	204	104
Pakistan	153	134	207	398	173	286	284	595	1712	1028	1290	1436
Philippines	144	132	171	150	262	246	290	529	763	637	551	603
Thailand	117	104	208	209	215	204	283	538	648	673	691	771
Turkey	38	-156	301	472	239	-15	71	497	619	603	681	497
Vietnam	163	-10	127	147	106	61	97	120	817	1342	1250	498
Latin America	1051	717	1468	1418	1277	1695	2054	3681	4595	4574	5375	3036
Argentina	91	70	134	110	20	26	115	206	189	87	60	115
Brazil	485	266	572	559	385	698	632	1426	1682	1564	1778	206
Colombia	83	79	81	80	93	122	184	288	395	355	439	171
Mexico	141	55	322	154	389	316	399	659	1007	1022	1147	1335
Europe	-551	-5370	7273	531	-69	736	-1091	4604	1309	1046	1158	-439
France	-209	-751	1221	8	-19	-231	-33	1203	3	0.	84	-103
East Germany	-223	-410	247	37	53	228	-178	-260	360	8	102	-197
West Germany	-244	-1126	1111	62	-7	968	-546	-346	338	381	579	-306
Italy	155	-767	1042	43	-39	152	-321	517	-236	197	441	-10
Poland	-70	-341	610	223	7	-111	-263	847	543	-22	-580	-248
Spain	53	4	295	147	107	-191	-65	278	90	280	260	111
UK	-36	-352	321	-323	-162	64	208	635	-382	262	451	-151
North America	852	650	934	-319	-786	575	2580	3864	2277	2175	-238	-2937
Canada	137	82	143	45	-4	58	262	400	205	223	22	-366
USA	714	568	791	-365	-782	516	2316	3462	2070	1950	-261	-2570
USSR	818	-2432	2803	5428	-2328	4108	-7236	3695	4318	2613	-452	-3853

## X. International Comparison of Intercohort Differences

We still want to know how Japan's profile of intercohort differences shown as Table 4 above compares with other countries in relative amount. For comparison we need not the absolute numbers of Table 10, but ratios to population, and these are shown for the same countries and regions in Table 11.

Deciding what population to divide by is not easy when we deal with cohorts, since a cohort does not have an obvious total in the sense of a population counted at one period. But since the object was merely to make the several countries and regions comparable it seemed sufficient to divide the numbers of Table 10 by the 1950 population under 5 years of age for each country, then multiply by 1000. The results given in Table 11 are in this degree roughly comparable from country to country.

The way to think about Table 11 is as crude estimates of growth or decline per thousand population over five-year intervals. For instance the 121 for 1925 for Japan means that between 1920-25 and 1925-30 the cohort increased by 12.1 percent, i.e., about 2.5 percent per year. It bears repeating that what is referred to by cohort is not the number born, but the number surviving to some average middle age. The denominator in Table 11 is the 1950 population aged 0-4, we repeat, and so the ratios are hardly precise.

Evidently the discontinuities in the LDCs and the MDCs are of roughly equal magnitudes, but they do not always occur at the same time. Thus there was a very large increase for the MDCs about 1920, and again about 1945, while the LDCs showed nothing much in the former period, and reached their largest post World War II differences somewhat later than the MDCs, about 1950, 1960 and 1965.

Yet there are large variations within each of these two categories, with each country having its own story to tell. The largest positive Japanese intercohort differences of 127, occurring about the year 1945, compare with the largest difference of 211 for the United States and 244 for Canada, both also about 1945. Certain LDCs showed much larger peaks than this--China is conspicuous with 291 for 1950, Vietnam with 351 about 1955, and East Germany with 333 about 1950.

Further study will be carried out on a more precise model, but

Table 11. First Differences of Birth Cohorts, i.e., Cohort Born in Five Years Following Given Date Minus Cohort Born in Five Years before Given Date, per Thousand Persons 0-5 in 1950

	1910	1915	1920	1925	1930	1935	1940	1945	1950	1955	1960	1965
World	25	1	68	60	36	47	23	131	162	97	151	129
LDCs	27	28	44	52	58	41	49	121	191	111	197	197
MDCs	20	-82	141	82	-29	66	-57	163	76	56	10	-77
Africa	23	28	34	45	49	55	61	93	119	147	163	207
Ethiopia	19	22	29	34	42	50	60	71	84	100	118	139
Nigeria	26	32	39	47	57	68	78	80	104	167	201	286
Zaire	24	28	33	42	49	59	70	84	99	117	137	185
South Africa	21	28	34	40	47	56	61	77	141	133	157	184
Asia	27	28	47	57	61	33	44	124	189	83	194	199
Bangladesh	-15	-15	28	49	47	49	62	72	115	295	182	253
Burma	24	45	26	24	1	35	105	149	88	160	114	167
China	25	33	44	46	67	11	30	171	291	-46	273	273
India	33	37	46	56	59	47	50	85	137	159	143	148
Indonesia	26	12	45	82	72	67	27	30	169	209	119	209
Iran	18	31	37	42	53	60	103	189	224	173	221	234
Japan	44	13	80	121	66	12	66	127	-136	-112	27	59
Korea	27	24	49	40	84	81	23	65	162	312	98	3
Malaysia	4	42	20	61	44	124	30	193	205	215	200	102
Pakistan	26	22	35	67	29	48	48	100	288	173	217	242
Philippines	39	36	46	41	71	67	79	144	207	173	150	164
Thailand	31	28	56	56	58	55	76	145	174	181	186	207
Turkey	12	-51	98	153	78	-5	23	162	201	196	221	162
Vietnam	43	-3	33	38	28	16	25	32	214	351	327	130
Latin America	39	27	55	53	48	63	77	137	172	171	201	113
Argentina	47	36	69	57	10	14	59	106	97	45	31	59
Brazil	54	29	63	62	43	77	70	157	186	173	196	23
Colombia	40	38	40	39	45	59	90	140	192	172	213	83
Mexico	29	11	67	32	81	66	83	138	210	213	239	279
Europe	-15	-149	202	15	-2	20	-30	128	36	29	32	-12
France	-53	-190	309	2	-5	-59	-8	304	1	0.	21	-26
East Germany	-207	-380	229	34	49	211	-165	-241	333	8	94	-183
West Germany	-71	-326	322	18	-2	281	-158	-100	98	111	168	-89
Italy	36	-179	243	10	-9	35	-75	120	-55	46	103	-2
Poland	-24	-116	207	76	2	-38	-89	287	184	-7	-197	-84
Spain	20	1	108	54	39	-70	-24	102	33	103	95	41
UK	-8	-81	74	-74	-37	15	48	146	-88	60	104	-35
North America	47	36	52	-18	-44	32	143	214	126	120	-13	-163
Canada	84	50	87	28	-2	36	160	244	126	136	14	-224
USA	44	35	48	-22	-48	31	141	211	126	119	-16	-157
USSR	43	-129	149	288	-124	218	-384	196	229	139	-24	-205

irrespective of what may turn up in the future, what has now been found suffices to show that the current world population expansion goes by cohorts rather than by periods, and it was initiated with the cohorts of the 1940s. That it started suddenly (not in time but in cohort) is an aspect measured by the numbers of Tables 10 and 11. Where the data are conspicuously bad, no discontinuity appears; tropical African countries, for instance, show a smooth acceleration of their population growth. With presently available material there is no way of checking, but one suspects that the discontinuity exists in Africa as well as elsewhere, and would be revealed if only one could find an accurate enough record. What is now needed even with less than ideal data is study of the suddenness of the expansion in those populations where it does appear, and especially its effect on employment and other aspects of development.

## XI. Conclusion

The appearance of any set of age distributions given at a number of consecutive points of time in three dimensions plainly suggests the possibility of condensation. If we turn the base of the diagram through 45 degrees we have on one dimension almost constant levels, at least through the considerable intervals of age in which mortality is relatively low. On the other dimension we have ups and downs for those countries in which fertility has been changing that represent the changing cohort sizes.

The constancy in one direction suggests that a considerable redundancy exists in any such table--i.e., it ought to be representable with far fewer numbers than are commonly shown. In the rotated version it would need only one element for each row, and one element for each column. If the original version has  $n$  ages shown for each of  $n$  time periods, then the  $n$ -squared numbers can be reconstructed from knowledge of just  $2 \times n$  parameters. Any study of what is happening to ages over time will be much simplified by such a reconstruction, and that has been the subject of the present paper.

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