Some Simple Method for the Evaluation of Causal Models

Robert W. Hodge
Naohiro Ogawa

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Robert W. Hodge
Professor
Department of Sociology
University of Southern California
Los Angeles, California, U.S.A.

Naohiro Ogawa
Professor
College of Economics
Nihon University
and
Deputy Director
Nihon University
Population Research Institute
Tokyo, Japan

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This paper reviews several simple methods for evaluating causal models. A straightforward method for computing the correlations implied by an estimated casual model is presented. The use of the frequency distribution of model errors and the plot of actual vs. implied correlations to examine the overall performance of a model are discussed. Explicit hypotheses about the sources of model errors are introduced and examined in an illustrative data set. These hypotheses concern the signs and magnitudes of the initial correlations, the variable content of the correlations, and the path distance between *the variables. Small systematic errors are detected in the* illustrative model, despite the fact that its overall fit appears quite good.

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I. Introduction

Causal models are now commonplace in all of the social sciences. Although there are disciplinary distinctions in the history, use, and practice of causal modeling, these differences represent alternative facets of what is basically a common core of ideas and statistical methods. In economics, which has perhaps the longest tradition of causal modeling, the variables are manifest, the data are often time-series, and the relevant causal parameters reflect the scales in which the variables are measured. In psychology, where the tradition is nearly as long, the variables are usually latent and indexed by multiple indicators, the data cross-sectional, and the relevant parameters expressed in a common metric of standardized units. In sociology and political science, the situation is more nearly a blend of what is commonly the case in economics and psychology. The data are typically cross-sectional, the variables a mix of manifest and latent ones, and the causal parameters expressed in standardized units because at least some of the variables have no meaningful unit of measurement such as pounds, bushels, or dollars.

As the development of causal modeling has unfolded, a great deal of attention has been paid to techniques of estimation. Somewhat less attention has been paid to the evaluation of estimated causal models. This is particularly the case in sociology and political science, where causal models or path analyses are often introduced and discussed without any assessment of the consistency between the implications of the model and basic relationships in the underlying data set. Before interpreting causal parameters, it is, however, important to assess the adequacy of the model in which they are embedded. Again, this is especially the case in the nonexperimental sciences, where attributions of causality are made particularly hazardous owing to the inevitable presence of confounding factors.

In sociology, Blalock (1962, 1964) has espoused the method of nonvanishing partial correlations to make causal inferences in nonexperimental situations. Duncan (1966, 1975) introduced the method of path analysis developed by the biologist Sewell Wright (1921, 1934, 1960) to sociology. This method yields direct estimates of causal impacts, rather than qualitative evidence about the presence or absence of causal forces. In this paper, the focus of our concern is on the assessment of models upon which causal inferences rest, rather than either upon the conditions under which such inferences are valid or invalid, or upon techniques of estimation. The paper brings together some considerations which are implicit in much writing about causal models and reviews some procedures which are scattered throughout the published literature. Some of the simple techniques introduced here are, however, not made explicit in major expositions of techniques of causal modeling (see, e.g., Asher, 1976; Duncan, 1975; Heise, 1975); the main contribution of the paper, therefore, is in explicitly laying out strategies of assessment in a single source.

II. Causal Sensibility: The First Test

There is an inherent limitation to all causal modeling which is often ignored in practical applications. That limitation is quite simple: there are <u>no</u> conditions under which one can unambiguously infer causality. This is no less true of experiments than it is of nonexperimental analyses, though the latter circumstance is generally more hazardous. The best one can do is to obtain estimates of causal parameters which are <u>consistent</u> with the presence of a presumed causal force. One can, of course, generate strong evidence that a causal force is inoperative, as happens in the case when the estimate of a causal parameter has the wrong sign or is insignificant statistically, substantively, or both.

Evidently, one would not want to draw evidence in support of causal hypotheses from models of causal effects which were not well grounded in what is known about the temporal arrangement of and theoretical relationships between the variables in a model. We would argue that the first and foremost test of any proposed model is its <u>causal sensibility</u>. Before proceeding with any of the tests proposed herein, one must first be satisfied that the model makes sense--theoretically, temporally, and substantively. When working with nonexperimental data, one cannot estimate a model at all without imposing causal assumptions upon the relationships between the variables. This is perhaps the most important reason why one cannot infer causality from the analysis of such data sets. The best one can do is to demonstrate that the causal parameters are consistent with the causal assumptions upon which they are based. Such a demonstration will, however, itself be meaningless if the causal assumptions--usually the ordering of the variables--are not defensible.

We would argue, therefore, that the first and perhaps the most important step in evaluating a causal model is vindicating the causal assumptions built into its estimation. A causal arrangement of variables and estimates of the causal relationships between them can pass all of the tests proposed herein and *still be intellectually indefensible*. Theory is vital to all causal modeling, and executing statistical investigations of the adequacy of a particular model is no guarantee that it is built on sound foundations. Given that a model is well-grounded, then and only then will the procedures outlined herein provide further information relevant to assessing its causal adequacy.

III. Derivation of Implied Correlations in Recursive Models

Among the class of recursive models, we can distinguish between those which are *fully* or *completely* recursive and those which are *partially* or *incompletely* recursive. In a fully recursive model, all of the measured exogenous variables affect all of the endogenous variables and each of the endogenous variables is affected by all of the endogenous variables which are causally antecedent to it. In a partially recursive model, at least one of the endogenous variables is not affected by all of the exogenous or all of the measured endogenous variables causally prior to it. Stated otherwise, in a partially recursive model, some potential, direct causal impacts are assumed to be equal to zero.

When all of the endogenous and exogenous variables are measured, except for the hypothetical factors introduced to secure determination of the endogenous variables, the distinction between fully and partially recursive models corresponds to the difference between just-identified and overidentified models. We maintain the distinction between fully and partially recursive models, as well as that between just-identified and overidentified models, because when some of the endogenous variables are latent ones, a partially recursive model may be just-identified and a fully recursive one may be underidentified, depending on the outside information available, such as assumptions about the equivalence of certain causal effects.

In any fully recursive model involving only measured endogenous variables and latent exogenous variables to secure their complete determination, all of the information in the covariance matrix of the measured variables is used up in estimating the model. In this situation, the estimated model enables one to retrieve, within rounding error, all of the correlations between the variables. In the case of a partially recursive, overidentified model, some of the covariances between pairs of variables will not be exploited in securing estimates of the postulated causal paths. As many writers have noted, the presence of this "free" information provides a vehicle for making an assessment of the causal adequacy of the estimated model. When the model satisfactorily captures associations which were not utilized in its estimation, the model is an adequate representation of the relationships between the variables; when it fails this test, it is defective in one or another regard. This is the case we consider in the remainder of this paper, illustrating how the "free" information can be used to check the estimated model against the associations observed in the data which underlie its estimation. Evidently, such checks are not possible in the fully recursive, just-identified case. There one can only rely upon the sensibility of the causal arrangement and the plausibility of the estimated causal impacts to evaluate the model.

The first step in evaluating a partially recursive, overidentified model is to retrieve from the estimated causal paths and the known correlations between the exogenous variables, the values of the correlations between all pairs of variables which are implied by the model. This is often done by tracing through all of the legitimate causal linkages between pairs of variables in the path diagram which summarizes the model. Such an operation is tedious, particularly as the number of variables in the model becomes large. It is also error prone, since it is quite easy to miss a causal link when reading indirect causal paths from a path diagram. There is, however, a straightforward manner in which the implied correlations may be computed, without recourse to a path diagram or other visual crutch.

To derive the implied correlations from a recursive model in standardized form, we must first define the structure matrix S. Suppose there are r exogenous variables and t endogenous variables in the model. We begin with the definition of the matrix of coefficients Z which links the exogenous variables to the endogenous variables. This matrix has r rows and t columns, i.e., one row for each exogenous variable and one column for each endogenous variable. The entries of $Z = [z_{ij}]$ are defined as follows:

$z_{ij} = 0$, if the ith exogenous variable does not have a direct causal impact on the jth endogenous variable,

and $z_{ij} = p_{\gamma_j x_i}$, if the ith exogenous variable has a direct causal impact on the jth endogenous variable, with the p's being the estimated values of the structural coefficients (in standardized form).

In addition, the columns of Z are arranged so that the first column $Z_{.1}$ refers to the initial endogenous variable, the second column to the second endogenous variable, and so on to the last column of Z, $Z_{.t}$ which refers to the ultimate endogenous variable.

Next, we define the matrix of coefficients C which links the endogenous variables to one another. C is square matrix with dimensions t x t. Its rows and columns are arranged in the same order as the columns of Z. For example, the first row and first column of C, i.e., and $C_{1.}$ and $C_{.1}$ refer to the initial endogenous variable, while the last row and the last column of C, $C_{t.}$ and $C_{.t}$ refer to the ultimate endogenous variable. In the event the model is block recursive, so that some of the endogenous variables do not affect one another and are not causally ordered, then any of the endogenous variables in such a block may be placed in any order when the block is encountered, that is, it does not matter which row and column a variable in a block occupies, so long as the ordering of the block relative to the other endogenous variables is preserved.

The elements of $C = [c_{jk}]$ are defined in following way:

$c_{jk} = 0$, either if $j \geqq k$ or if $j < k$ and the jth endogenous variable does not have a direct causal impact on the

$\underline{k}$th endogenous variable,

and $c_{jk} = p_{Y_k Y_j}$, if the $\underline{j}$th endogenous variable has a direct causal impact on the $\underline{k}$th endogenous variable, where as before the p's are the estimated values of the structural coefficients (in standardized form).

The structure matrix S is formed by stacking Z on top of C, i.e.,

$$S = \begin{bmatrix} Z \\ \hline C \end{bmatrix}.$$

Evidently, S has dimensions ($\underline{r}$ + $\underline{t}$) x $\underline{t}$. The nonzero entries in the $\underline{j}$th column of S are just the coefficients associated with the variables which directly effect the $\underline{j}$th endogenous variable.

To calculate the implied correlations among the first endogenous variable, Y_1, and the exogenous variables, we begin with the matrix of correlations, X, among the exogenous variables. The entries of $X = [x_{ij}] = r_{X_i X_j}$ are just the correlations among the exogenous variables. Evidently, X is symmetrical, i.e., $x_{ij} = x_{ji}$ and $x_{ij} = 1$ if $i = j$, and has dimensions $\underline{r}$ x $\underline{r}$, since there are $\underline{r}$ exogenous variables. Let $S^r_{.1}$ be a column vector which contains the first $\underline{r}$ elements in $S_{.1}$, the first column of the structure matrix. The implied correlations, $R^r_{.1} = [r_{j1}] = [\hat{r}_{Y_1 X_j}]$, between the first endogenous variable and the exogenous variables are given by

$$(X)S^r_{.1} = R^r_{.1} .$$

We proceed to find the implied correlations of the second endogenous variable with the first endogenous variable and all the exogenous variables by augmenting the matrix of correlations among the exogenous variables with an additional row and column. This additional row and column contain the implied correlations just computed, i.e., the elements of $R^r_{.1}$. The augmented matrix of correlations, X^*, has dimensions ($\underline{r}$ + 1) x ($\underline{r}$ + 1) and the following structure:

$$X^* = \begin{bmatrix} X & R^r_{.1} \\ (R^r_{.1})' & 1 \end{bmatrix} ,$$

where $(R^r_{.1})'$ is the transpose of $R^r_{.1}$. Let $S^{r+1}_{.2}$ be a column vector which contains the first $\underline{r} + 1$ elements of $S_{.2}$, the second column of the structure matrix. We simply calculate

$$(X^*)S^{r+1}_{.2} = R^{r+1}_{.2}$$

to find the implied correlations of the second endogenous variable with all the variables causally prior to it.

The process just outlined is repeated until all of the correlations implied by the estimated model have been obtained. For example, in the third step, the new augmented correlation matrix is given by

$$\begin{bmatrix} X^* & R^{r+1}_{.2} \\ (R^{r+1}_{.2})' & 1 \end{bmatrix}.$$

This is just postmultiplied by $S^{r+2}_{.3}$, the column vector containing the first $\underline{r}$ + 2 elements of $S_{.3}$ (the third column of the structure matrix), to find the implied correlations of the third endogenous variable with its causal antecedents. This process may be tiresome if the calculations are done by hand, but it is straightforward. It requires no tracing of causal linkages in a path diagram, since all of these causal linkages are embedded in the implied correlations which are used to augment the matrix of intercorrelations among the exogenous variables. In addition, it can be easily implemented with a computer program and all calculations in this paper were made using a SAS routine.

IV. Illustrative Calculations of Implied Correlations

Although the procedure for obtaining the correlations implied by a causal model is straightforward, it may nonetheless be helpful to have an explicit numerical example. For purposes of illustration, we used estimates of an expanded version of an overidentified model of fertility and reproductive behavior in contemporary Japan which was initially proposed by Ogawa and Hodge (1983). Estimates of the model are derived from the nationwide survey conducted in 1981 with a sample size of 2,648 currently married Japanese women of childbearing age for which observations on all the variables were available. As microlevel sociodemographic models go, this illustrative model is relatively large. It involves four exogenous variables and twelve endogenous variables which are linked by 10 stochastic equations and two accounting identities. The exogenous variables are as follows: a measure of premarital urban exposure of wives (= U), wife's education (= E_W), husband's education (= E_H), and wife's age (= Y). The endogenous variables, arranged in their causal order from that most proximate to that least proximate to the exogenous variables, are as follows: Wife's premarital work experience (= J), a dummy variable for arranged marriages (= M), wife's age at current marriage (= Z), duration of current marriage (=X), patrilocality of residence at marriage (= R), ideal number of children for a Japanese couple (= F), desired number of children (= D), number of pregnancies (= P), number of stillbirths and spontaneous abortions (= S), number of induced abortions (= B), number of children ever born (= C) and a measure of attitudes toward abortion (= A). The matrix of actual correlations between all of the variables is given in Appendix Table A.1; the structure matrix of estimated causal paths is exhibited in Appendix Table A.2.

The upper left-hand corner of Appendix Table A.1 contains the 4 by 4 matrix of intercorrelations among the exogenous variables and the first four entries in the first column of Appendix Table A.2 exhibit estimates of the causal paths which link the exogenous variables to the first endogenous variable, wife's premarital work experience. We obtain the implied correlations of the first endogenous variable with the exogenous variables by multiplying the matrix of intercorrelations among the exogenous variables by the vector of coefficients as follows:

$$\begin{array}{c} \\ U \\ E_W \\ E_H \\ Y \end{array} \begin{array}{c} \begin{array}{cccc} U & E_W & E_H & Y \end{array} \\ \left[\begin{array}{cccc} 1 & .2032 & .2038 & -.1120 \\ .2032 & 1 & .6012 & -.2980 \\ .2038 & .6012 & 1 & -.1843 \\ -.1120 & -.2980 & -.1843 & 1 \end{array}\right] \end{array} \begin{array}{c} J \\ \left[\begin{array}{c} .1589 \\ 0 \\ 0 \\ -.2467 \end{array}\right] \end{array} = \begin{array}{c} \hat{r}_{JX_i} \\ \left[\begin{array}{c} .1865 \\ .1058 \\ .0779 \\ -.2645 \end{array}\right] \end{array} .$$

We now just augment the matrix of correlations between the exogenous variables with the vector of implied correlations. Multiplying the augmented matrix by the vector of coefficients observed in the second column of the structure matrix yields the implied correlations of type of marriage (= M) with its causal antecedents. The calculation is as follows:

$$\begin{bmatrix} 1 & .2032 & .2038 & -.1120 & .1865 \\ .2032 & 1 & .6012 & -.2980 & .1058 \\ .2038 & .6012 & 1 & -.2980 & .0779 \\ -.1120 & -.2980 & -.1834 & 1 & -.2467 \\ .1865 & .1058 & .0779 & -.2645 & 1 \end{bmatrix} \begin{bmatrix} -.1412 \\ 0 \\ 0 \\ .1814 \\ -.1119 \end{bmatrix} = \begin{bmatrix} -.1824 \\ -.0946 \\ -.0709 \\ .2268 \\ -.1862 \end{bmatrix} .$$

The process continues by augmenting the correlation matrix once more with the implied correlations derived in the second stage. The new matrix is postmultiplied by the coefficients from the third column of the structure matrix to obtain the correlations of marital age with its causal antecedents. The process ends when all the implied correlations have been derived.

All the correlations implied by the estimates of the illustrative model at hand are reported above the diagonal in Appendix Table A.3. The signed differences between the actual and implied correlations are reported below the diagonal in the same table.

Some of the correlations implied by a model will necessarily be equal (within rounding error) to the observed correlations among the variables. This occurs because of the way the causal paths are estimated. In Appendix Table A.3, the implied correlations which must correspond to the actual correlations have been placed in parentheses. These correlations are obviously not "free" pieces of information and they should be excluded from any subsequent use of the implied correlations to assess the model.

Identifying the implied correlations which must coincide with the actual correlations can be tricky and, indeed, the ones which must exhibit agreement with the actual correlations will depend upon the estimating strategy. There are several feasible ways to estimate overidentified, recursive models. For example, from the derivation of implied correlations given above, it is clear that the implied correlations may be derived at each step in the construction of a causal model. That is, after the equation for the first endogenous variable has been estimated, one can work out its implied correlations with the remaining variables and so on for the second and the remaining endogenous variables. One strategy for estimating causal models is to make estimates of the causal paths consistent with the implied correlations. Thus, when estimating the equation for any particular endogenous variable, one uses the implied rather than the actual correlations among the variables in the equation when making the estimates. Other strategies involve striking averages of the various alternative estimates which are possible whenever a model is overidentified (Boudon, 1965).

None of the above strategies is, however, recommended. The preferred way to estimate an overidentified, recursive model is simply to perform the ordinary least squares regression of all the measured variables on their measured predictors. (Obviously, an alternative strategy must be used when one introduces latent endogenous variables into a model.) In executing these regressions, one uses the actual correlations among the determinants of each endogenous variable, regardless of whether or not they are implied by the estimates of the preceding structural equations. This strategy of estimation makes the estimates of causal paths consistent with the observed data. The method is preferred because it produces estimates of the causal paths which are both unbiased and have the smallest standard errors. Other methods yield unbiased estimates of the causal paths, so the advantage of the recommended strategy is in minimizing the standard errors of the estimates (Goldberger, 1970).

When the preferred method of estimation has been used, it is relatively straightforward to identify the implied correlations which are constrained by the estimating strategy to equal the actual correlations. The estimates of the model necessarily reproduce the correlations between the first endogenous variable and all of the exogenous variables which directly affect it. For subsequent endogenous variables, one examines the variables which enter the equation. If all the implied correlations between those variables have been constrained to equal the actual correlations between them, then of necessity the correlations between the endogenous variable at hand and the variables in the equation for it will also be reproduced. This will not be the case, however, if any one of the implied correlations between the variables in the equation can be at variance with the corresponding actual association.

V. Frequency Distribution of Signed Disparities

Perhaps the simplest way to obtain an overall view of the performance of a model is to inspect the frequency distribution of the signed differences between the actual and implied correlations. For the illustrative model introduced in the previous section, there are exactly 100 actual correlations whose values are not implied by the strategy suggested by Goldberger for estimating such an overidentified, recursive model. The frequency distribution of the signed differences between the actual and implied correlations, reported below the diagonal in Appendix Table A.3, is given in Table 1.

In general, when a model fits the data well, the distribution of signed disparities will exhibit four desirable properties. First, the distribution will be centered close to zero, indicating that the model does not systematically tend to produce positive or negative errors. Second, the distribution will be unimodel, indicating that the errors are piled up around the center of the distribution. Were this not the case, the distribution might have a desirable mean near zero, but an underlying distribution indicative of a good average fit achieved by compensating errors of alternative signs. Third, there should be no evidence of heaping in the tails of the distribution. The presence of heaping may indicate that one or more causal paths have been erroneously excluded from the model. Finally, of course, there should be no extreme observations, which fall well beyond the discrepancies observed for the vast majority of the contrasts between the actual and implied associations. The occurrence of such "outliers" is a certain sign that a significant causal path has been deleted from the estimated model, usually those linking the pairs of variables involved in the generation of the extreme observations.

The frequency distribution from the illustrative set conforms quite well to the above criteria. As can be seen from Table 1, the distribution of "free" differences between the actual and implied

Table 1. Frequency Distributions of Signed Differences between Actual and Implied Correlations in an Illustrative Model of Fertility and Reproductive Behavior.

Interval	Total	Actual Correlation Positive	Actual Correlation Negative
	Raw Frequencies		
Total	100	54	45
≧ .0700	0	0	0
.0500 - .0699	4	4	0
.0400 - .0499	2	1	1
.0300 - .0399	3	1	2
.0200 - .0299	6	4	2
.0100 - .0199	8	4	4
.0050 - .0099	7	5	2
.0000 - .0049	26	17	9
(-.0049) - (.0000)	22	10	12
(-.0099) - (-.0050)	5	2	3
(-.0199) - (-.0100)	6	3	3
(-.0299) - (-.0200)	6	1	5
(-.0399) - (-.0300)	0	0	0
(-.0499) - (-.0400)	2	0	2
(-.0699) - (-.0500)	3	2	1
≦ -.0700	0	0	0

correlations for the illustrative model is unimodal and centered near zero. Nearly half of the disparities--forty-eight out of one hundred --differ from zero by no more than .005 in absolute magnitude and three-fifths are within .01 of zero. There is no evidence of substantial heaping in the tails of the distribution and there are no extreme outliers since the largest disparity is less than .07 in absolute magnitude. This evidence does not mean that the illustrative model "fits" the data, but the simple distribution of signed differences does not, in and of itself, enable one to pinpoint systematic and serious sources of error.

VI. Overstatement and Understatement of Observed Correlations

In the illustrative data set, there is a healthy mix of positive and negative correlations among the observed variables. Of the 100 correlations not reproduced owing to the way the model was estimated, 54 are positive and 46 are negative. This proves to be a source of difficulty when using summary statistics based on the signed differences to assess the model. The reason for this difficulty is quite clear: The meaning of a signed difference depends upon whether or not the actual correlation is itself positive or negative.

When the original correlation is positive, a positive difference between the actual and implied correlations means that the implied association understates the absolute magnitude of the observed association, while a negative difference means the implied association exceeds the actual association in magnitude. Just the reverse is the case when the initial, observed correlation is negative. When the actual correlation is negative, a positive difference occurs when the implied association is greater than the actual association in absolute magnitude and a negative difference is observed when the implied association is less than the actual one in absolute magnitude.

The foregoing point can be illustrated by reference to the disparities reported below the diagonal in Appendix Table A.3. For the 100 instances in which implied correlations are not required by the estimating procedure to equal the actual ones, the mean signed difference is .0024, which is, indeed, quite small. However, the mean of the signed differences can be small because of contrary and counterbalancing signed differences which occur when the initial associations are positive and when they are negative. That is the case in the illustrative model, though not in a substantively very significant way. For the 54 cases in which the original correlation is positive, the mean signed difference is .0063, more than twice as large as the average overall case. For the 46 instances in which the actual correlation is negative, the mean signed difference is -.0021, which is, indeed, contrary to the mean observed for the differences when the initial correlation was positive. Consequently, in this particular model, the implied correlations tend to understate slightly the actual correlations, regardless of whether the initial association was positive or negative.

For the above reason, the frequency distribution of the signed differences is reported separately in Table 1 according to whether the initial correlation was positive or negative. Both of these distributions exhibit properties quite similar to their combined total. They are unimodel and centered close to zero. Neither exhibits a great deal of heaping in the tails, though the distribution observed when the initial correlation was positive shows a bit more variability than does the distribution of signed differences found when the original association was negative.

The complication introduced by the presence of positive and negative associations in the original correlation matrix can be handled in two ways. The most obvious way is simply to take the absolute value of the difference between the actual and implied correlations. This procedure, however, destroys information, since one can no longer tell whether the actual correlation is over- or understated in magnitude by the implied correlation. A preferable method is to change the sign of the differences when (but only when) the initial correlation is negative. This reflects the distribution of differences when the original correlation is negative around the value zero. The resulting distribution of revised differences now has observations whose signs have a common meaning, regardless of the sign of the actual correlation. A positive difference means the original correlations is understated by the implied association, while a negative difference means the original correlation is overstated in magnitude by the implied one. One can, of course, utilize both transformations, as we do in the analyses reported below.

VII. The Shepard Plot

In multidimensional scaling, a triangular matrix of observations which can be interpreted as "distances" among objects is often analyzed. If there are _n_ objects, then technically one may need as many as _n_-1 dimensions to reproduce the actual distances among them. The goal of multidimensional scaling is, of course, to represent the relative locations of the objects by a number of latent dimensions which is substantially less than the number of objects.

The dimensions extracted from a set of distances via multi-dimensional scaling yields, among other parameters of interest, the position of each object on each dimension. These values can be used, along with the standard Pythagorean formula for distance in a multi-dimensional space, to calculate the distance between the objects <u>implied</u> by their location in the reduced space yielded by the multi-dimensional scaling. The plot of the actual distances among the objects against these imputed distances is known as a Shepard diagram.

A visual and often telling overview of the fit of a causal model can be obtained by plotting the actual correlations against the implied ones. Such a plot is analogous to the Shepard diagram which is routinely produced as part of the output of many programs for multi-dimensional scaling. For convenience, and by analogy to multi-dimensional scaling, we may refer to such a scatterdiagram of actual vs. implied correlations as a Shepard plot. For the illustrative model of fertility and reproductive behavior, the Shepard plot is shown in Figure 1.

Evidently, if a model fits the actual data quite well, the points in the Shepard plot will be tightly packed around a line with zero intercept and a slope equal to one. If the intercept is near zero, then there is no particular tendency for the "free" correlations to be over- or understated by those implied by the estimated model. If, in addition, the points fall along a line with slope near unity, then the model tends to over- or understate correlations by roughly the same amount, regardless of their sign and their magnitude.

Visual inspection of Figure 1 reveals that the points are, indeed, tightly packed around the line with zero intercept and a slope of one. Furthermore, what little scatter there is about this line appears to be the same everywhere along the line. Thus, for the illustrative data, the Shepard plot provides no occasion for detecting substantial or systematic irregularities in the fit of the model. The eye, of course, sees what it wants to see and what is a good fit to one observer may be less than satisfactory to another. Consequently, it is advisable to generate some more formal tests based on the Shepard plot.

One simple procedure is to run the regression of the actual correlations on the implied ones. For the illustrative model at hand, this regression is given by

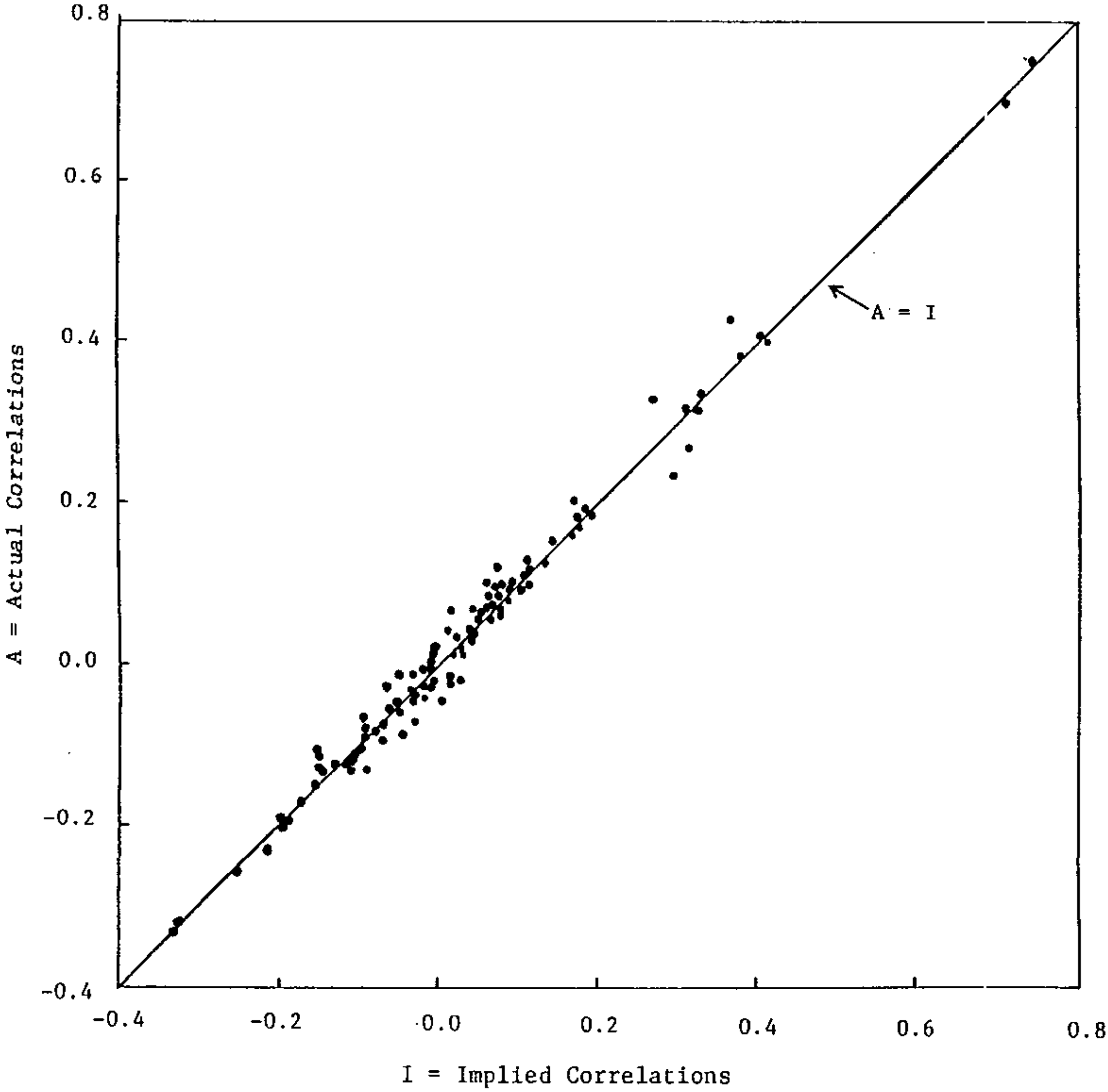

Figure 1. Plot of Actual vs. Implied Correlations for a Model of Reproductive Behavior, Married Japanese Women of Childbearing Age, 1981.

$$\hat{A} = 1.0010(I) + 0.0024,$$
$$(0.0120) \quad (0.0022)$$

where the standard errors of the coefficients are shown in parentheses beneath their estimated values. The intercept of this regression clearly does not differ significantly from zero and, since $[(1.0010) - (1)]/(.0120) < 1$, the hypothesis that the slope is equal to unity can also not be rejected.[1] Thus, the best fitting line does not differ significantly from the line with zero intercept and unit slope which has already been drawn in the Shepard plot. That line appears to fit the data plotted in Figure 1 quite well because for all practical purposes, it is the best fitting line. In addition, the correlation between the actual and implied associations is a healthy .993. Taken together, these are rather impressive pieces of evidence that the illustrative model at hand fits the data upon which it is based quite well.

VIII. Some Hypotheses about the Structure of Errors

Even when there is evidence that the general fit of a model to the underlying data is satisfactory, small systematic errors can still be present. Thus, the analysis of the fit of a model should not end with the examination of the Shepard plot and its characteristics. Instead, a more detailed search should be made for particular sources of error. In this section, we examine four specific hypotheses about the error structure of the data.

Positive vs. Negative Initial Correlations. The conformity of an estimated model with the underlying associations among the variables entering the model should not be affected by the signs of the actual correlations. One check on a model, therefore, is to see if the error structure is conditional upon the sign of the underlying correlations among the variables. Any model which exhibits an error structure which is related to the signs of the underlying covariances is a candidate for revision.

The data required to test relationship of the pattern of errors to the signs of the initial associations in the illustrative model is already contained in Table 1. Before using these data, the frequency distribution of the differences between the actual and the implied correlations must be reflected around zero if the initial association is negative. As noted above, this reflection is necessary if the signed differences between the actual and implied correlations are to have the same meaning without regard to the sign of the actual correlations. After reflecting the frequency distribution of differences observed for initially negative correlations and collapsing categories, one obtains the following reduced table:

Direction and Magnitude of Error	Initial Correlation	
	Positive	Negative
	Frequencies	
Actual > Implied		
≧ .0100	14	11
.0000 - .0099	22	15
Actual < Implied		
(-.0099) - (.0000)	12	11
≦ (-.0100)	6	9

The value of χ^2 for the above table is just 1.687, so there is no evidence that the illustrative model is any better or worse at reproducing "free" positive than "free" negative associations.

Variable Content of Correlations. In any causal model, there are three kinds of correlations: (i) those among the exogenous variables, (ii) those among the endogenous variables, and (iii) those linking the exogenous to the endogenous variables. All of the correlations among the exogenous variables are taken as given, so they cannot be a source of model error. However, in overidentified recursive models, there are usually some "free" associations among the endogenous variables as well as among the correlations relating the exogenous variables to the endogenous ones. As a recursive model unfolds, each successive endogenous variable has one additional, intervening endogenous variable standing between it and the exogenous factors. As the endogenous variables become increasingly further removed from the exogenous launching pad of the model, it is increasingly difficult for the exogenous variables to exhibit direct impacts on the endogenous variables because the number of intervening variables which must be taken into account becomes progressively larger. Such small impacts as the exogenous variables may have on causally removed exogenous variables are apt to be small, statistically insignificant, and, hence, deleted from the estimated model. A reasonable hypothesis about the error structure of the model is, therefore, that the model will do somewhat better at reproducing the associations between the endogenous variables than it does at reproducing the associations linking the exogenous variables to the endogenous variables.

The tabulation for the illustrative model of the reflected differences between the actual and implied correlations by the variable content of the correlations is exhibited in Table 2. Inspection of the frequency count reveals that the reflected disparities which involve an exogenous and an endogenous variable are somewhat more likely to be relatively larger than the reflected disparities between actual and implied correlations which pertain to a pair of endogenous variables. This is precisely what was expected, but the value of X^2 in Table 2 is just 3.425. With three degrees of freedom, the probability of observing a chi-square this large or larger is in excess of .3. Thus, while the association is in the expected direction, there is no convincing evidence that the illustrative model does appreciably worse at capturing "free" associations between exogenous and endogenous variables than it does between pairs of endogenous ones.

Magnitude of Actual Correlations. Another factor which can be implicated in the error structure of an estimated model is the absolute size of the initial associations. When a "free" observed correlation is small to begin with, there is not much chance of making a substantial error so long as the implied correlation achieves the proper sign. When the initial associations are large, however, there is more leeway for error. Consequently, it is reasonable to postulate that "free" large associations are more likely to be understated by wider margins by the implied correlations than are "free" small correlations.

Obviously, it is highly undesirable to have an estimated model

Table 2. Tabulation of Reflected Differences between Actual and Implied Correlations by Type of Initial Correlation, for an Illustrative Model of Fertility and Reproductive Behavior.

Direction and Magnitude of Error	Total	Type of Correlation: Between Endogenous Variables	Type of Correlation: Between Exogenous and Endogenous Variables
	Frequencies		
Total	100	64	36
Actual > Implied			
≧ .0100	25	15	10
.0000 - .0099	37	27	10
Actual < Implied			
(-.0099) - (.0000)	23	15	8
≦(-.0100)	15	7	8

which does a good job at retrieving small associations, but misses the main associations by a wide margin. One of the purposes of causal modeling, especially with nonexperimental data, is to display the main effects which are presumed to operate in a particular, substantive domain. A model which fails to capture those primary impacts is one which has failed in a very fundamental way.

To this juncture, the model we have been examining for illustrative purposes has performed quite well. However, as can be seen from the Shepard plot in Figure 1 or by inspection of Appendix Table A.1, the model is based on a data set in which most of the associations are quite weak. If the model is performing well because it captures all of these small correlations, while missing the few large associations in the data set, then its performance is not nearly so salutary as the results to this point indicate.

For the illustrative model, the tabulation of the reflected differences between the actual and implied correlations by the absolute magnitude of the initial correlations is given in Table 3. As was the case in the previous analysis, the frequency counts are consistent with the hypothesis at hand. When the initial correlations are relatively large, the reflected differences between the actual and implied correlations are themselves somewhat more likely to be relatively large. However, the association is quite modest. The value of X^2 in Table 3 is 1.335, which is not even close to significance at any plausible level. Thus, the illustrative model at hand does about as well at retrieving "free" large associations as it does at retrieving "free" small ones.

Path Distance. The path distance between a pair of variables in a causal model is the minimum number of causal links through which one has to pass in moving from one variable to the other. Stated otherwise, the path distance between a pair of variables is the least complicated way in which they are related. Evidently, the path distance is one if a pair of variables are joined by a direct causal path from one to the other. This is the only case, excluding the trivial instance of pairs of exogenous variables whose correlations are assumed, in which the path distance is one. Variables which are related to one another indirectly can have path distance between them of varying length, depending upon the shortest legitimate causal connection between them.

Errors have the nasty habit of accumulating as one moves through a causal model. Small errors between causally proximate variables tend to be expanded as variables become causally more remote from one another. An important source of error in causal models can, therefore, be the path distance between variables. In general, one expects the errors to be relatively larger when the path distance between a pair of variables is large. Also, one generally expects models in which the average path distance is small to perform better overall than models in which the average path distance is large.

For the illustrative model, the tabulation of the reflected disparities between the actual and implied correlations by the path distance between variables is shown in Table 4. It is readily apparent from Table 4 that path distance is a significant factor in the

Table 3. Tabulation of Reflected Differences between Actual and Implied Correlations by Absolute Magnitude of Initial Correlation, for an Illustrative Model of Fertility and Reproductive Behavior.

Direction and Magnitude of Error	Total	Absolute Magnitude of Initial Correlation	
		> .1000	< .1000
	Frequencies		
Total	100	57	43
Actual > Implied			
≧ .0100	25	16	9
.0000 - .0099	37	21	16
Actual < Implied			
(-.0099) - (.0000)	23	11	12
≦ (-.0100)	15	9	6

error structure of the illustrative model. Virtually all of the relatively large disparities occur between pairs of variables which are causally linked by indirect mechanisms. The value of χ^2 for Table 4 is 33.87, which is significant by any reasonable criterion.

The particular way in which we have been tabulating the reflected discrepancies between the actual and implied associations can be reinterpreted as embodying three separable hypotheses, rather than a single hypothesis about the position of the reflected discrepancies on the line of real numbers and the variable of interest. There are three degrees of freedom in the tables we have been examining; each of these degrees of freedom can be associated with a different hypothesis. The tabulated distribution of the reflected frequencies keeps track of their sign, i.e., whether or not the implied correlation

Table 4. Tabulation of Reflected Differences between Actual and Implied Correlations by Path Distance, for an Illustrative Model of Fertility and Reproductive Behavior.

Direction and Magnitude of Error	Total	Path Distance	
		One	Two or Three
	Frequencies		
Total	100	35	65
Actual > Implied			
≧ .0100	25	1	24
.0000 - .0099	37	24	13
Actual < Implied			
(-.0099) - (.0000)	23	10	13
≦ (-.0100)	15	0	15

under- or overstates the observed association. The first hypothesis embedded in the tables concerns, therefore, the direction of the disparities. In addition, for disparities of alternative directions, the tabulated distribution keeps track of the magnitude of the disparity. The second hypothesis involves, therefore, the magnitude of the reflected discrepancies, without regard to sign. The third hypothesis is rooted in the interaction between the two previous hypotheses. This interaction can be stated in several ways, two common ways being that (i) the impact of the variable of interest (path distance in Table 4) on the magnitude of the reflected disparities depends upon the sign of the disparity or (ii) vice versa.

In the previous tables, there was no point in examining these separable hypotheses because there was no statistical evidence of association. That is not the case, however, with path distance. We can examine the relationship of path distance to the magnitude of the reflected discrepancies in the illustrative model by folding the distribution of disparities about the value zero. This amounts to adding together the bottom and top rows in the body of Table 4 and adding together the two middle rows. The resulting 2 by 2 table is as follows:

Absolute Magnitude of Error	Path Distance	
	One	2 or 3
≧ .0100	1	39
.0000 - .0099	34	26

The value of X^2 in this reduced table is 30.95, which leaves no doubt that path distance has a potent impact upon the magnitude of the errors in the illustrative model. The difference between the chi-square in the larger table and that in this reduced table is given by 33.87 - 30.95 = 2.92, which is itself distributed as chi-square with two degrees of freedom. This value of chi-square is not significant at the .05 level, even with one degree of freedom. Consequently, we may conclude (i) that path distance does not affect the direction of the model errors and (ii) there is no interaction of the relationship between path distance and the magnitude of the error with the sign of the disparity, i.e., with whether or not the model error involves an over- or understatement of an observed correlation.

Even though the illustrative model at hand performs quite well in general, we have been able to demonstrate that the model errors are related to the path distances in the model. The concept of path distance also illuminates why the model has performed well in every other regard examined to this point. If one examines the structure matrix for the model reported in Appendix Table A.2, it is readily apparent that, despite the relatively large number of endogenous variables, one or more of the exogenous variables tends to exhibit a direct causal link with most of the endogenous variables. The persistence of the causal linkages between the exogenous and endogenous variables effectively interlaces all of the variables in the model.

Indeed, the largest path distance in the model is only three and there are only four pairs of variables which are separated by a path distance that large. In these four cases, the actual correlation is always understated in magnitude by the implied correlation and the average reflected disparity is--as this model goes--a fairly healthy .0278. This illustrative model does relatively well overall in large measure because the path distances in it are quite small; the average path distance is appreciably less than two.

IX. Multivariate Analysis

As with the analysis of any set of nonexperimental data, the analysis of the error structure of a causal model is beset with the problem of multicollineation between the various sources of error. Consequently, it is advisable to examine the error structure in a multivariate context, since it is entirely possible that the gross impacts of the various sources of error misrepresent their net contributions. Here, we use regression methods to analyze, for the illustrative model, both the signed, reflected differences and their absolute values.

In studying the indicators of model error, we utilize four variables. These are: a dummy variable which takes on the value 1 if an actual correlation is between a pair of endogenous variables and the value 0, otherwise; the measure of path distance; the signed value of the original correlations and the absolute value of the initial correlations. For both the signed reflected differences and their absolute values, two regressions were calculated. These differ by whether or not the value of the actual correlations, as well as their absolute value, is entered into the regression equation.

The results of the regression analyses are summarized in Table 5, where it can be seen that only the measure of path distance is consistently significant. It is the only variable which bears any relationship to the signedreflected differences and it is not a very powerful predictor of them. The variables at hand do a somewhat better job of capturing the absolute magnitude of the errors in the illustrative model and, in the final equation, all of the variables are at least marginally significant.

The second equation for the absolute magnitudes of the reflected differences which includes both the actual correlations and their absolute magnitude as predictors, indicates that the errors tend, as expected, to be large when the path distance between a pair of variables is likewise great. The equation also reveals, again as expected, that the model is a little better at reproducing the magnitudes of correlations involving pairs of endogenous variables than it is at retrieving correlations involving both an exogenous and an endogenous variable. Understanding the coefficients associated with the signed value of the actual correlations (= A) and their absolute values (= $|A|$) is aided by a little algebra.

We may begin by introducing a dummy variable P which takes on the value 1 if the sign of an actual correlation is positive and the value 0, otherwise. We then define N = 1 - P, so that N + P = 1. With these dummy variables, we may express the actual correlations and their absolute values in the following ways:

$$A = P(|A|) - N(|A|)$$

and
$$|A| = P(|A|) + N(|A|).$$

If b and c are the regression coefficients of $|A|$ and A, respectively, we can rearrange the regression equation as follows:

$$\begin{aligned} b|A| + cA &= b[P(|A|) + N(|A|)] + c[P(|A|) - N(|A|)] \\ &= bP(|A|) + bN(|A|) + cP(|A|) - cN(|A|) \\ &= (b + c) P(|A|) + (b - c) N(|A|). \end{aligned}$$

Thus, b, the coefficient of $|A|$, is the common impact of the magnitude of the initial correlations--regardless of their sign--on the magnitude of the errors. The coefficient of A (= c) simply allows the impact of the magnitude of the initial correlations on the magnitude of the errors to vary according to the sign of the initial correlations. The results for the final regression equation indicate that the magnitude of the errors increases with the magnitude of the initial correlations when the initial correlation is positive. This impact is, however, inverted when the initial association is negative, i.e., the model reproduces large negative correlations more accurately than small negative ones.

Table 5. Summary of Regression Analyses of Reflected Differences and the Absolute Value of Reflected Differences Between Actual and Implied Correlations in an Illustrative Model of Fertility and Reproductive Behavior, for Currently Married Japanese Women of Childbearing Age, 1981.

Independent Variables or Parameter	Dependent Variables			
	Signed Reflected Differences		Absolute Value of Reflected Differences	
	Model I	Model II	Model I	Model II
	Regression Coefficients in Raw Score Form			
Intercept	-0.0110	-0.0104	-0.0096*	-0.0082*
Type of Correlation (Endogenous Only =1)	0.0034	0.0026	-0.0026	-0.0047*
Path Distance	0.0074**	0.0077**	0.0145**	0.0155**
Actual Correlation (Signed)	. . .	0.0113	. . .	0.0285**
Absolute Value of Actual Correlation	0.0049	-0.0041	0.0009	-0.0217*
	Standard Errors of Coefficients			
Intercept	0.0084	0.0084	0.0059	0.0057
Type of Correlation (Endogenous Only =1)	0.0044	0.0046	0.0031	0.0031
Path Distance	0.0040	0.0040	0.0028	0.0027
Actual Correlation (Signed)	. . .	0.0154	. . .	0.0105
Absolute Value of Actual Correlation	0.0159	0.0201	0.0112	0.0136
	Coefficients of Determination			
Adjusted R^2	.0075	.0027	.2178**	.2670**

* Significant at .10 level with a one-tail test.
** Significant at .05 level with a one-tail test.

In summary, then, the multivariate analysis reveals that there is little structure to the signed model errors. The variables at hand largely fail to pinpoint the areas where the estimated model understates and overstates the "free" observed associations. The sources of error studied here do, however, enable us to identify places where the magnitudes of the model error are relatively small and relatively large. The estimated, illustrative model fits the data relatively poorly when the initial correlations are large and positive or small and negative, when they involve both an exogenous and an endogenous variable, and especially when the path distance between the variables implicated in the correlation is great. These are important and general sources of error which are likely to be operative across a wide range of circumstances.

X. Concluding Discussion

In this paper, we have set forth a simple procedure for computing the correlations implied by an estimated causal model and illustrated how frequency distributions of the errors and the Shepard plot of actual vs. implied correlations can be utilized to obtain an impression of the overall fit of a causal model to the underlying data upon which it is based. We have also introduced explicit hypotheses about the structure of errors in causal models and shown, for an illustrative case, the presence of systematic errors in a model which exhibits a generally good fit to the data.

Our concerns in this paper have focused on techniques for evaluating causal models, rather than upon strategies for coping with systematic errors once they have been revealed. The latter topic is no less important than the focus of this paper. However, in general we do not believe there is any fixed strategy for handling revealed errors. Instead, we espouse the view that strategies for revising causal models in the light of errors should themselves be dependent upon a variety of factors, including the substantive domain, the results of prior research, the nature of the data--particularly, whether it is macro- or micro-level, and the status of the theory embedded in the causal model. Two features which will often govern the choice of alternative strategies for dealing with error are parsimony and the magnitude of the errors themselves. One may well decide to maintain a parsimonious model in the face of errors, especially in circumstances where one is less interested in fitting the data than in seeing how well it can be explained by the main features of a theoretically driven and parsimonious model. Obviously, one would ordinarily be more tolerant of small than of large errors. We have demonstrated for the illustrative model studied in this paper that systematic model errors are present. However, all of these errors are small and the overall fit is reasonable. Thus, on the one hand, little is lost by ignoring the errors, having acknowledged their presence. On the other hand, the model is already complicated by the presence of numerous small causal effects; adding a few more to improve the fit would not destroy a parsimonious treatment of the main effects present in the illustrative data set. What one does in a situation like this is not dictated by any scientific principle known to the present authors. Instead, there is room for the operation of research taste. We would not quarrel with a decision to leave the model as it stands, despite detectable and systematic model errors, on the grounds that the model is already so complicated that the addition of further causal paths runs a high risk of modeling sample errors rather than theoretically based, causal relations.

We may close this paper by noting that our exposition has been based on the analysis of correlations rather than regressions. That will be a comfortable strategy for some social scientists, but others may--as, indeed, we do--prefer the analysis of regressions. There is nothing inherent about the methods used and hypotheses studied in this paper which demands the examination of correlations. The entire exposition could have focused upon zero order regressions. All one needs to do, once the matrix of implied correlations has been derived, is to multiply the actual and implied correlations by the ratio of the standard deviation of the causally dependent variable to that of the predictor or causally prior variable in each correlation. The analysis can then be based on the resulting actual and implied zero order regressions.

NOTE

Statistical tests should be used with caution when analyzing data of the present kind. First, the data in some sense exhaust the relevant universe, so treating the observations as a sample is contrived in the first place. Second, and perhaps more important, there is good reason to believe the observations are not independent of one another. For actual correlations, there are mathematical constraints which allow one to infer the sign of the correlation between A and C if the correlations between A and B and between B and C are sufficiently large. Here, the correlations are too small for these mathematical constraints to come into play. However, with a causal model in hand, the sign of the correlation between pairs of variables which are causally related only indirectly can be inferred when all of the indirect causal linkages between them imply the same sign. See Duncan (1963) and Costner and Leik (1964). The main consequence of the potential dependence among the observations is to reduce the effective degrees of freedom. One can guard against premature rejection of the null hypotheses in these circumstances by picking a small value for alpha. These comments also apply to the analysis of disparities between actual and implied correlations, though in a less obvious way.

Appendix Table A.1. Product Moment Correlations Between Selected Social and Demographic Variables, for Married Women of Childbearing Age, Japan, 1981.

Variable Description and Symbol	Variable Symbol															
	U	E_W	E_H	Y	J	M	Z	X	R	F	D	P	S	B	C	A
Urban Experience (= U)	1.0000	0.2032	0.2038	-0.1120	0.1865	-0.1824	0.1128	-0.1560	-0.1850	-0.1090	-0.0239	-0.0716	0.0425	-0.0047	-0.1331	0.0704
Wife's Education (= E_W)	0.2032	1.0000	0.6012	-0.2980	0.0973	-0.0757	0.1134	-0.3366	-0.1291	-0.1029	-0.0121	-0.1883	0.0271	-0.1291	-0.1904	0.0747
Husband's Education (= E_H)	0.2038	0.6012	1.0000	-0.1843	0.0817	-0.0748	0.1248	-0.2312	-0.0902	-0.1020	0.0021	-0.1463	0.0586	-0.1174	-0.1505	0.0623
Age of Women (= Y)	-0.1120	-0.2980	-0.1843	1.0000	-0.2645	0.2268	0.1438	0.9092	0.1351	0.0790	0.0371	0.3368	-0.0342	0.2354	0.3287	-0.0490
Work Experience (= J)	0.1865	0.0973	0.0817	-0.2645	1.0000	-0.1863	0.1514	-0.3202	-0.0994	-0.0016	0.0170	-0.1249	-0.0050	-0.0465	-0.1534	0.0729
Type of Marriage (= M)	-0.1824	-0.0757	-0.0748	0.2268	-0.1863	1.0000	0.0807	0.1860	0.1665	0.0095	-0.0154	0.0595	-0.0058	-0.0241	0.1233	-0.0169
Marital Age (= Z)	0.1128	0.1134	0.1248	0.1438	0.1514	0.0807	1.0000	-0.2813	-0.0972	-0.0172	-0.1128	-0.1911	0.0429	-0.0809	-0.2509	-0.0256
Marital Duration (= X)	-0.1560	-0.3366	-0.2312	0.9092	-0.3202	0.1860	-0.2813	1.0000	0.1719	0.0838	0.0835	0.4070	-0.0512	0.2623	0.4243	-0.0367
Patrilocal Residence (= R)	-0.1850	-0.1291	-0.0902	0.1351	-0.0994	0.1665	-0.0972	0.1719	1.0000	0.0752	0.0905	0.0997	0.0039	0.0227	0.1367	-0.0408
Ideal Family Size (= F)	-0.1090	-0.1029	-0.1020	0.0790	-0.0016	0.0095	-0.0172	0.0838	0.0752	1.0000	0.3812	0.1534	0.0157	0.0363	0.2039	-0.0767
Desired Children (= D)	-0.0239	-0.0121	0.0021	0.0371	0.0170	-0.0154	-0.1128	0.0835	0.0905	0.3812	1.0000	0.3125	0.0689	0.0632	0.4071	-0.0479
Number of Pregnancies (= P)	-0.0716	-0.1883	-0.1463	0.3368	-0.1249	0.0595	-0.1911	0.4070	0.0997	0.1534	0.3125	1.0000	0.3221	0.7468	0.7092	0.1205
Spontaneous Abortions (= S)	0.0425	0.0271	0.0586	-0.0342	-0.0050	-0.0058	0.0429	-0.0512	0.0039	0.0157	0.0689	0.3221	1.0000	0.0659	-0.0622	-0.0223
Number of Abortions (= B)	-0.0047	-0.1291	-0.1174	0.2354	-0.0465	-0.0241	-0.0809	0.2623	0.0227	0.0363	0.0632	0.7468	0.0659	1.0000	0.1807	0.1881
Children Ever Born (= C)	-0.1331	-0.1904	-0.1505	0.3287	-0.1534	0.1233	-0.2509	0.4243	0.1367	0.2039	0.4071	0.7092	-0.0622	0.1807	1.0000	0.0196
Abortion Attitude (= A)	0.0704	0.0747	0.0623	-0.0490	0.0729	-0.0169	-0.0256	-0.0367	-0.0408	-0.0767	-0.0479	0.1205	-0.0223	0.1881	0.0196	1.0000

Appendix Table A.2. Structure Matrix for an Illustrative Model of Fertility and Reproductive Behavior, for Japanese Women of Childbearing Age, 1981.

Variable Symbol	Endogenous						Variables					
	J	M	Z	X	R	F	D	P	S	B	C	A
U	.1589	-.1412	.0876	0	-.1381	-.0838	0	0	0	.0404	0	.0493
E_W	0	0	.1519	0	0	-.0364	0	-.0408	0	0	0	.0733
E_H	0	0	0	0	0	-.0523	0	-.0356	.0708	0	0	0
Y	-.2467	.1814	.2303	.9697	.1066	.0535	0	0	-.1862	0	0	-.0787
J	0	-.1119	.1985	0	0	.0407	0	0	0	0	0	0
M	0	0	.0929	0	.1257	0	0	0	0	-.0811	0	0
Z	0	0	0	-.4208	-.1071	0	-.1012	-.0531	.1391	.0802	0	0
X	0	0	0	0	0	0	0	.3469	0	0	0	0
R	0	0	0	0	0	.0471	.0534	0	0	0	0	0
F	0	0	0	0	0	0	.3755	0	0	0	0	0
D	0	0	0	0	0	0	0	.2772	-.0449	-.1945	0	-.0696
P	0	0	0	0	0	0	0	0	.4358	.9003	1.6177	.0480
S	0	0	0	0	0	0	0	0	0	-.2164	-.5179	0
B	0	0	0	0	0	0	0	0	0	0	-.9933	.1848
C	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	0	0	0	0	0	0

N.B. See text or stub of Appendix Table A.1. for variable definitions.

Appendix Table A.3. Implied Correlations (Above Diagonal) and Differences Between Actual and Implied Correlations (Below Diagonal) in a Causal Model of Selected Social and Demographic Variables, for Married Women of Childbearing Age, Japan, 1981.

Variable Description and Symbol	Variable Symbol															
	U	E_W	E_H	Y	J	M	Z	X	R	F	D	P	S	B	C	A
Urban Experience (= U)		(0.2032)	(0.2038)	(-0.1120)	(0.1865)	(-0.1824)	(0.1128)	(-0.1561)	(-0.1851)	(-0.1090)	-0.0620	-0.0929	0.0133	-0.0102	-0.1470	0.0710
Wife's Education (= EW)	(0.0000)		(0.6012)	(-0.2980)	0.1058	-0.0946	0.1133	-0.3366	-0.0839	-0.1005	-0.0536	-0.1999	0.0291	-0.1508	-0.1886	0.0730
Husband's Education (= EH)	(0.0000)	(0.0000)		(-0.1843)	0.0779	-0.0709	0.0756	-0.2105	-0.0648	-0.1010	-0.0490*	-0.1508	0.0521	-0.1174	-0.1542	0.0431
Age of Women (= Y)	(0.0000)	(0.0000)	(0.0000)		(-0.2645)	(0.2268)	(0.1438)	(0.9092)	(0.1352)	(0.0790)	0.0222	0.3326	-0.0353	0.2914	0.2669	-0.0378
Work Experience (= J)	(-0.0000)	-0.0085	0.0038	(-0.0000)		(-0.1862)	0.1527	-0.3207	-0.0937	-0.0014	-0.0209*	-0.1323	-0.0193*	-0.0843	-0.1402	0.0173
Type of Marriage (= M)	(-0.0000)	0.0189	-0.0039	(-0.0000)	(-0.0001)		0.0778	0.1872	0.1668	0.0348	0.0140*	0.0711	-0.0061	-0.0196	0.1376	-0.0350
Marital Age (= Z)	(0.0000)	0.0001	0.0492	(0.0000)	-0.0013	0.0029		(-0.2814)	-0.0976	-0.0082	-0.1094	-0.1883	0.0405	-0.0784	-0.2478	-0.0134
Marital Duration (= X)	(0.0001)	0.0000	-0.0207	(0.0000)	0.0005	-0.0012	(0.0001)		0.1722	0.0800	0.0675	0.4018	-0.0513	0.3156	0.3631	-0.0310
Patrilocal Residence (= R)	(0.0001)	-0.0452	-0.0254	(-0.0001)	-0.0057	-0.0003	0.0004	-0.0003		0.0725	0.0895	0.0954	-0.0058*	0.0409	0.1167	-0.0200
Ideal Family Size (= F)	(-0.0000)	-0.0024	-0.0010	(0.0000)	-0.0002	-0.0253	-0.0090	0.0038	0.0027		0.3801	0.1413	0.0215	0.0407	0.1770	-0.0311
Desired Children (= D)	0.0381	0.0415	0.0511	0.0149	0.0379	-0.0294	-0.0034	0.0160	0.0010	0.0011		0.3104	0.0675	0.0579	0.4096	-0.0527
Number of Pregnancies (= P)	0.0213	0.0116	0.0045	0.0042	0.0074	-0.0116	-0.0028	0.0052	0.0043	0.0121	0.0021		0.3231	0.7454	0.7100	0.1187
Spontaneous Abortions (= S)	0.0292	-0.0020	0.0065	0.0011	-0.0243	0.0003	0.0024	0.0001	0.0097	-0.0058	0.0014	-0.0010		0.0656	-0.0605	-0.0285*
Number of Abortions (= B)	0.0055	0.0217	0.0000	-0.0560	0.0378	-0.0045	-0.0025	-0.0533	-0.0182	-0.0044	0.0053	0.0014	0.0003		0.1785	0.1821
Children Ever Born (= C)	0.0139	-0.0018	0.0037	0.0618	-0.0132	-0.0143	-0.0031	0.0612	0.0200	0.0269	-0.0025	-0.0008	-0.0017	0.0022		-0.0035*
Abortion Attitude (= A)	-0.0006	0.0017	0.0192	-0.0112	0.0556	0.0181	-0.0122	-0.0057	-0.0208	-0.0456	0.0048	0.0018	-0.0508	0.0060	0.0231	

* Implied correlation had wrong sign.

References

Asher, Herbert B. 1976. Causal Modeling. Beverly Hills: Sage Publications.

Blalock, Hubert M., Jr. 1962. "Four Variable Causal Models and Partial Correlations," American Journal of Sociology, 68: 182-194.

________. 1964. Causal Inferences in Nonexperimental Research. Chapel Hill: University of North Carolina Press.

Boudon, Raymond. 1965. "A Method of Linear Causal Analysis: Dependence Analysis," American Sociological Review, 30: 365-374.

Costner, Herbert L., and Robert K. Leik. 1964. "Deductions from 'Axiomatic Theory,'" American Sociological Review, 29: 819-834

Duncan, Otis Dudley. 1963. "Axioms or Correlations," American Sociological Review, 28: 452.

________. 1966. "Path Analysis: Sociological Examples," American Journal of Sociology, 72: 1-16.

________. 1975. Introduction to Structural Equation Models. New York: Academic Press.

Goldberger, Arthur S. 1970. "On Boudon's Method of Linear Causal Analysis," American Sociological Review, 35: 97-101.

Heise, David R. 1975. Causal Analysis. New York: John Wiley and Sons.

Ogawa, Naohiro, and Robert W. Hodge. 1983. "Towards a Causal Model of Childbearing and Abortion Attitudes in Contemporary Japan," Population Research Leads, Series No. 15. Bangkok: ESCAP, Population Division, 32 pp.

Wright, Sewell. 1921. "Correlation and Causation," Journal of Agricultural Research, 20: 557-585.

________. 1934 "The Method of Path Coefficients," Annals of Mathematical Statistics, 5: 161-215.

________. 1960 "Path Coefficients and Path Regressions: Alternative or Complementary Concepts?" Biometrics, 16: 189-202.