Measuring the Demographic Discontinuity

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ABSTRACT ———————————————————————————————————

Nathan Keyfitz emphasized the importance of developing research which is focused on the sudden changes in age-specific growth patterns of population that started around the middle of the 20th century, particularly those in less developed countries. Such sudden changes were identified by Keyfitz as "demographic discontinuity." In reference to this discontinuity he discussed the usefulness of the constancy of intercohort differences to reconstruct the age-time distribution of population and to see what amounts of cohort change are capable of producing the sharp discontinuity. As did Keyfitz in his paper we also used the United Nations world population estimates and projections in an attempt to measure quantitatively the discontinuities or bends appearing in the age distribution by use of first and second differences as well as curvatures and other factors to examine the usefulness of our techniques. The data used are for the world total and five countries in Asia, including China, Indonesia, Japan, the Philippines and Thailand. An earlier version of this paper entitled "A Method of Measuring Demographic Discontinuity" was presented at the IIASA Conference on "Future Changes in Population Age Structures" held in Sopron, Hungary, 18-21 October 1988. The authors have revised it for publishing as a NUPRI Research Series monograph.

I. Introduction

The concept of "demographic discontinuity" and a technique concerning how to measure it were first introduced by Keyfitz (1987, 1989a, 1989b). In the present paper we developed his method for somewhat more detailed measuring of the bend in the curve of age distribution and tested our techniques by applying them to the population data for the world total and several Asian countries. We would like to briefly outline the study of the demographic discontinuity by Keyfitz referring to his paper of 1987. His primary problem in this paper concerned the pattern of expansion of the world population in the present century, which "did not take place as a smooth acceleration, but showed sudden changes in the demographic parameters on at least two occasions" (Keyfitz, 1987:iii). The main change took place in the 1940s in the form of the so-called population explosion. He developed a technique providing new information concerning the onset of that explosion. The material used by Keyfitz (1987) was the age distributions of the world population by 5-year age groups from 1950 to 1980 at five-year intervals estimated by the United Nations. By comparing the age distributions for 1950, 1955, and so on in a graphic manner Keyfitz noticed that the two sharp bends or discontinuities appear corresponding to the persons 5-9 and 30-34 in 1950 which refer to the cohorts of 1940-45 and 1915-20, respectively. Two different ways of cohort arrangement of measuring cohort-to-cohort increases were presented by him to bring out the essential features of the age curves. One is a cohort arrangement of the age-to-age differences and the other a cohort arrangement of the period-to-period increases at given ages. In other words, the former differences are calculated with respect to age and the latter with respect to time. The differences calculated with respect to time are found to produce more consistent results. The below-cited finding based on them by Keyfitz provides an important clue to his main interest in studying demographic discontinuity:

> "When world population growth is seen as a series of period totals it is a relatively smooth curve, until the 1970s rising with more than exponential steepness. When cohort differences are taken the acceleration appears as steps, clearly measurable from age distributions derived from censuses in 1950 and subsequently. There were distinct jumps about 1900, about 1920, and about 1945, each one something like a doubling of the rate of growth from the previous step" (Keyfitz, 1987: 8).

Keyfitz utilized the general tendency of the uniformity of the cohort-to-cohort differences to reconstitute the original age-period data as a test to see "what amounts of cohort change are capable of producing the sharp discontinuity in rates of growth" (Keyfitz, 1987: 13).

We presented in this paper some measures for estimating in somewhat more detail the onset of the "discontinuity" and some other related demographic parameters.

In Section II of this paper, we used the world population estimates and projections as assessed in 1984 by the United Nations (United Nations, 1987) as our base material. Using the age-time distribution of the world total population from 1950 to 1980, we calculated not only the first but also the second differences. Both were calculated with respect to time as well as with respect to age. They are presented in Table 1 for the world total population and in Appendix Tables A and B for Japan and Thailand, respectively. It will be found, for example, that a series of the second differences concerning the cohort born in 1940-45 (persons aged 5-9 in 1950) are distinctly larger than those concerning the neighboring cohorts. This will suggest that the 1940-45 cohort is the cohort just before the onset of the population explosion. This can be similarly recognized by either the differences with respect to time or those with respect to age.

The subject of the present study is on the bend occurring in cohort-to-cohort growth of population. As noted above, we have two ways of observing the bend in cohort-to-cohort growth. One is in the curve of period-to-period growth for different ages and the other in the curve of age-to-age growth, that is, in the curve of age distribution for different points of time. Section II will deal with both ways in discussing the bend. However, in Section III, which deals with how to measure the amount and timing of the bend, we will concentrate only on the bend in the curve of age distribution. Further remarks will be made in Sections II and III below.

The population of an age group is determined by the number of births of that cohort multiplied by the cohort survival probabilities to that age group insofar as we take a closed population assumption. If we calculate the logarithm of the population of an age group, therefore, we can present it as the sum of such fertility and mortality components. For this reason, the first and second differences of the logarithms of the age-time data will be useful in analyzing the fertility and mortality components of the bend. Some equa-

tions in relation to analyzing the bend from this viewpoint are presented, together with several examples of the calculation based on these equations, in the latter part of Section II. The tables concerned are Tables 3 and 5.

In Section III, we tried, after interpolating the 5-year age group data into single years of age, to estimate such points of age as showing sharp bends in the age curve of population. As the measures of the bend, we used the curvature $CV(a,t)$ and the angle $ANG(a,t)$ (where a and t refer to age and year, respectively). Their definitions and ways of calculation were also given. Numerical applications were attempted on the world total and five selected Asian countries, namely China, Indonesia, Japan, the Philippines and Thailand (Tables 6 and 7 and Appendix Table C).

The reason for using the United Nations population estimates and prospects in this paper is simply the convenience of their data arrangements for our techniques inasmuch as they are presented in 5-year age groups at five-year intervals. In many developed countries there are reliable age-time data made available by single years of age on an annual basis. The techniques used in this paper have not been adjusted to such data. It may prove useful if the measures of the bend developed in this paper are also applied to actual census population data of developing countries.

II. First and Second Differences of
Age-Time Population Distribution

1. Cohort-to-cohort growth of population

Let $f(a,t)$ denote, in general, the function of two variables age a and time t. We define the first differences of $f(a,t)$ with respect to a and t, respectively, as follows:

$$\triangle_t f(a,t) = f(a,t) - f(a,t-1) \tag{1}$$

and

$$\triangle_a f(a,t) = f(a,t) - f(a+1,t) \tag{2}$$

We then define the second differences with respect to a and t, respectively, as follows:

$$\triangle_t^2 f(a,t) = f(a,t-1) + f(a,t+1) - 2f(a,t) \tag{3}$$

and

$$\triangle_a^2 f(a,t) = f(a-1,t) + f(a+1,t) - 2f(a,t) \tag{4}$$

The actual functions of a and t used in this paper are $P_{a,t}$, $P'_{a,t}$, B_{t-a}, B'_{t-a}, $L_{a,t}$ and $L'_{a,t}$ as defined and discussed later. The expressions of $f(a,t)$ from (1) to (4) above are all applicable to these six functions.

$P_{a,t}$ denotes the population aged a at last birthday at time t. The present paper uses the U.N. population estimates and projections by age for the period 1950 onward for the world total and several selected countries (United Nations, 1987) as the material to which our techniques are applied. The U.N. age-time data are given by 5-year age groups at five-year intervals. The age-time data dealt with in our analysis range from 0-4 to 75-79 years for age and from 1950 to 2025 for time. We thus take a = 0, 1, 2,... and 15 as corresponding to the age groups 0-4, 5-9,... and 75-79, and t = 0, 1, 2, ... as corresponding to the years 1950, 1955, 1960,.... Age a, therefore, corresponds to the 5-year age group covering ages 5a to 5a + 4 years and time t corresponds to the year 1950 + 5t.

Panel 1 of Table 1 shows the age-time distribution for the world total $P_{a,t}$ for a = 0 to 15 and t = 0 to 6. By applying (1) to (4) we have the first and second differences of $P_{a,t}$ as shown below.

First differences:

$$\triangle_t P_{a,t} = P_{a,t} - P_{a,t-1} \tag{5}$$

$$\triangle_a P_{a,t} = P_{a,t} - P_{a+1,t} \tag{6}$$

Second differences:

$$\triangle_t^2 P_{a,t} = P_{a,t-1} + P_{a,t+1} - 2P_{a,t} \tag{7}$$

$$\triangle_a^2 P_{a,t} = P_{a-1,t} + P_{a+1,t} - 2P_{a,t} \tag{8}$$

The differences with respect to time $\triangle_t P_{a,t}$ and $\triangle_t^2 P_{a,t}$ and those with respect to age $\triangle_a P_{a,t}$ and $\triangle_a^2 P_{a,t}$ given above are both to measure cohort-to-cohort growth of population.

The number of (five-year) birth cohorts involved in (5) to (8) above are three. The eldest cohort is $P_{a,t-1}$ in (5) and $P_{a+1,t}$ in (6). Both are the same cohort born from t-a-1 to t-a. The middle cohort is $P_{a,t}$ in (5) to (8) born from t-a to t-a+1, and the youngest is $P_{a,t+1}$ in (7) and $P_{a-1,t}$ in (8), both which are the same cohort born from t-a+1 to t-a+2. The length of the period of birth is five years in all of the above three cohorts. A schematic diagram showing the above relationship is given in Figure 1.

Figure 1. Schematic Diagram Showing the Components of Differences $\triangle_t^2 P'_{a,t}$ and $\triangle_a^2 P'_{a,t}$
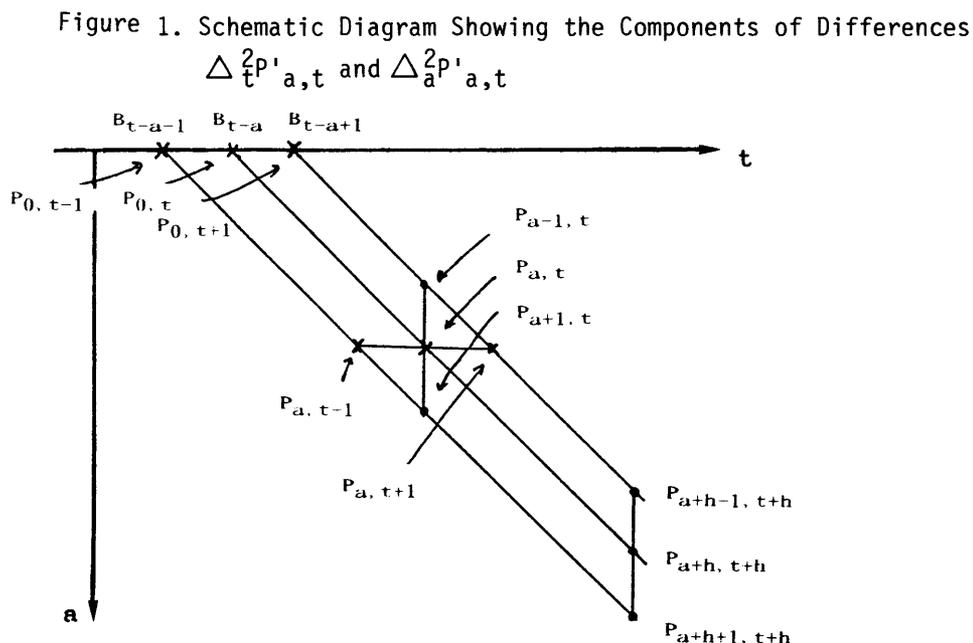
2. Sharp bends in the curve of cohort-to-cohort growth

Panels 2 to 5 of Table 1 present the results of calculation of (5) to (8) for the world total population. [1] Observing the respective rows (age) in Panels 2 and 4 of Table 1, we may notice periods when the values of $\triangle_t P_{a,t}$ and $\triangle_t^2 P_{a,t}$ change suddenly with the progression of time. In other words, the curve of period-to-period growth shows us some sharp bends at such periods. Similarly, when observing the respective columns (times) in Panels 3 and 5, we can notice ages at which the values of $\triangle_a P_{a,t}$ and $\triangle_a^2 P_{a,t}$ show sudden changes with the progression of age. [2] This implies that the curve of age distribution presents some sharp bends at such ages.

Table 1. Population Numbers by Age and Time $P_{a,t}$ and Their First and Second Differences with Respect to Time $\triangle_t P_{a,t}$ and $\triangle_t^2 P_{a,t}$ and with Respect to Age $\triangle_a P_{a,t}$ and $\triangle_a^2 P_{a,t}$: World Total, 1950 to 1980 (in millions)

1. $P_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
0- 4	341	400	429	478	522	544	541
5- 9	270	317	376	407	459	503	526
10-14	258	264	312	370	403	455	499
15-19	239	253	259	307	365	400	451
20-24	220	233	247	254	301	360	394
25-29	195	214	227	242	249	296	354
30-34	165	189	208	222	237	245	292
35-39	159	158	183	203	216	232	241
40-44	145	152	152	177	198	211	228
45-49	126	137	145	145	170	192	204
50-54	106	117	128	137	137	162	184
55-59	89	95	107	119	129	129	153
60-64	73	79	84	95	108	118	118
65-69	55	61	66	71	83	94	103
70-74	38	43	48	53	56	66	77
75-79	22	25	28	32	36	39	48

2. $\triangle_t P_{a,t}$

Age	1955	1960	1965	1970	1975	1980
0- 4	59	30	49	44	22	-3
5- 9	47	59	31	52	45	23
10-14	7	48	58	33	52	44
15-19	13	7	48	58	35	52
20-24	13	14	7	47	58	34
25-29	19	13	15	7	47	58
30-34	24	20	13	15	9	46
35-39	-1	24	20	13	17	8
40-44	7	0	25	21	13	18
45-49	11	8	0	25	22	12
50-54	11	12	9	0	25	22
55-59	6	12	12	10	0	24
60-64	5	5	12	12	10	1
65-69	5	5	5	12	11	9
70-74	5	5	5	3	10	11

3. $\triangle_a P_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
0- 4	71	82	53	71	63	40	14
5- 9	13	53	64	37	56	48	27
10-14	18	12	53	63	38	56	48
15-19	19	19	12	53	64	40	57
20-24	26	19	20	12	53	64	40
25-29	30	25	19	20	12	51	63
30-34	5	30	26	19	21	13	51
35-39	15	6	31	26	18	22	13
40-44	19	15	7	32	28	19	24
45-49	20	20	17	8	33	29	20
50-54	17	22	21	18	9	34	31
55-59	16	17	23	24	21	11	35
60-64	18	18	18	25	25	24	15
65-69	17	18	19	18	27	28	26
70-74	16	18	19	20	20	26	28

4. $\triangle_t^2 P_{a,t}$

Age	1955	1960	1965	1970	1975
0- 4	-29	19	-5	-21	-25
5- 9	12	-28	21	-7	-22
10-14	41	10	-25	19	-8
15-19	-6	41	10	-23	17
20-24	1	-7	40	11	-24
25-29	-6	2	-8	40	11
30-34	-4	-7	2	-6	38
35-39	25	-4	-8	4	-8
40-44	-8	25	-4	-8	5
45-49	-3	-8	25	-3	-9
50-54	1	-3	-8	25	-3
55-59	6	0	-2	-10	24
60-64	0	6	1	-3	-9
65-69	0	-1	7	0	-2
70-74	0	0	-2	7	1

5. $\triangle_a^2 P_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
5- 9	58	29	-11	35	7	-8	-12
10-14	-6	41	11	-26	17	-8	-21
15-19	-1	-8	40	10	-26	16	-9
20-24	-6	0	-8	40	11	-24	18
25-29	-4	-6	2	-8	41	13	-23
30-34	25	-5	-7	2	-9	37	12
35-39	-9	24	-5	-8	3	-9	38
40-44	-4	-9	24	-5	-10	3	-11
45-49	-1	-5	-10	24	-5	-10	4
50-54	3	-2	-5	-10	24	-4	-12
55-59	1	5	-2	-6	-12	22	-3
60-64	-2	-1	5	-1	-4	-13	20
65-69	1	0	-1	6	-2	-4	-11
70-74	1	0	0	-2	7	2	-2

When we observe the figures of Panel 2 of Table 1 diagonally along the age-time cohort lines, we notice that the differences $\triangle_t P_{a+h,t+h}$ given by (5) are approximately constant for h = 0, 1, 2 ... (Keyfitz, 1987, 1989a, 1989b). Similar properties are also noticed in the differences obtained from (6) to (8) as shown in Panels 3 to 5 of Table 1. If we compare Panels 4 and 5 of Table 1, we find the figures of Panel 5 ($\triangle_a^2 P_{a,t}$) quite close to those of Panel 4 ($\triangle_t^2 P_{a,t}$) for the corresponding a and t except for younger and older age groups where mortality tends to be higher (Table 2). This closeness signifies that in Figure 1 above the shape of the curve of $P_{a,t-1}$ to $P_{a,t}$ to

Table 2.   Comparison of $\triangle_t^2 P_{a,t}$ and $\triangle_a^2 P_{a,t}$
           (in parentheses)

(in millions)

A g e	1955	1960	1965	1970	1975
10-14	41(41)	10(11)	-25(-26)	19(17)	-8(-8)
15-19	-6(-8)	41(40)	10(10)	-23(-26)	17(16)
20-24	1(0)	-7(-8)	40(40)	11(11)	-24(-24)
25-29	-6(-6)	2(2)	-8(-8)	40(41)	11(13)
30-34	-4(-5)	-7(-7)	2(2)	-6(-9)	38(37)
35-39	25(24)	-4(-5)	-8(-8)	4(3)	-8(-9)
40-44	-8(-9)	25(24)	-4(-5)	-8(-10)	5(3)
45-49	-3(-5)	-8(-10)	25(24)	-3(-5)	-9(-10)
50-54	1(-2)	-3(-5)	-8(-10)	25(24)	-3(-4)
55-59	6(5)	0(-2)	-2(-6)	-10(-12)	24(22)

Notes: The figures are from Panels 4 and 5 of
        Table 1. The sum of the absolute values of
        $\triangle_a^2 P_{a,t}$ (=662) is 4.1% larger than that of
        $\triangle_t^2 P_{a,t}$ (=636).

$P_{a,t+1}$ and that of $P_{a-1,t}$ to $P_{a,t}$ to $P_{a+1,t}$ (not given in Figure 1) will be close to each other. In view of the fact that age distribution information available in many developing countries fail to provide sufficient time series data, the above-mentioned differences calculated with respect to age may be regarded as a more convenient measure. The differences calculated with re-spect to time are age-specific. Compared to this, the differences with re-spect to age always give us information on the total pattern of age distribu-

tion of the population concerned.

We will discuss the first and second differences relating to a "sharp" bend by taking two representative cases in Table 1. Firstly, a set of three successive cohorts born in 1935-40, 1940-45 and 1945-50 will be taken. The calculated values of the first difference of the 1940-45 cohort (that is, 1940-45 cohort minus 1935-40 cohort) are fairly constant along the cohort line within a range as small as 7 to 9 as shown in Panel 2 of Table 1 ($\triangle_t P_{a,t}$). In contrast, the first difference of the 1945-50 cohort (that is, 1945-50 cohort minus 1940-45 cohort) shows that the values are also fairly constant along the cohort line, within a range as large as 46 to 48, also as seen in Panel 2. This remarkable contrast implies that the 1940-45 cohort hardly changes in size from the 1935-40 cohort while the 1945-50 cohort shows a sudden increase in size (the beginning of the so-called postwar population explosion). The population size concerned is found in Panel 1 of Table 1. Taking the number of persons aged 10-14 for the respective cohorts, it is 258 million for the 1935-40 cohort (1950), 264 million for the 1940-45 cohort (1955) and 312 million for the 1945-50 cohort (1960). Thus, in order to detect a bend in cohort-to-cohort increase, in general, we need age-time data for a set of at least three successive cohorts, as seen above. Taking the above-mentioned three successive cohorts, 1935-40, 1940-45, and 1945-50, the second differences with respect to time and with respect to age both present a fairly constant series of values as large as 38 to 41 as shown in Panels 4 and 5 of Table 1 (excluding the value which involves the first difference between age groups 0-4 and 5-9 in Panel 5).

Next, we will take another set of three successive cohorts born in 1930-35, 1935-40 and 1940-45. As mentioned above, little change in population is seen between the two cohorts 1935-40 and 1940-45. The first difference of 1935-40 cohort (that is, 1935-40 cohort minus 1930-35 cohort) shows slightly larger values ranging from 13 to 18 along the cohort line as shown in Panel 2 of Table 1. Here, we see an increase from the 1930-35 cohort to the 1935-40 cohort and a stagnation from the 1935-40 cohort to the 1940-45 cohort. The cohort-to-cohort increase is much longer and is a continual one from the 1920-25 cohort to the 1935-40 cohort, as can be seen in Panels 2 and 3 of Table 1.

Other similar but less remarkable bends will be found in relation to the three cohorts born in 1910-15, 1915-20 and 1920-25 and also the three cohorts born in 1905-10, 1910-15 and 1915-20. The First World War is involved in this period.

In summarizing the above, we can say that a sharp bend occurs, for one, in the situation where two or more successive cohorts continue along a more or less stagnant trend from cohort to cohort and then the new cohort comes with a suddenly increased population. This is the upwards concave bend. A bend also occurs in another situation where two or more successive cohorts continue to increase from cohort to cohort and then the new cohort comes with population stagnancy or decrease. This bend is upwardly convex. How to measure the amount and timing of the bend will be dealt with in Section III.

3. Fertility and mortality components of the differences of age-time distribution

B_{t-a} denotes the births that took place at time t-a, and $L_{a,t-a}$ the probability of cohort survival from the time of birth to age a in completed years. Then we have

$$P_{a,t} = B_{t-a} L_{a,t-a} .$$

If we put

$$P'_{a,t} = \log P_{a,t}, \quad B'_{t-a} = \log B_{t-a}, \quad \text{and} \quad L'_{a,t} = \log L_{a,t},$$

then we have

$$P'_{a,t} = B'_{t-a} + L'_{a,t-a} .$$

On the basis of the above and also applying (1) to (4) we have the first and second differences of $P'_{a,t}$ as given below.

First differences:

$$\triangle_t P'_{a,t} = \triangle_t B'_{t-a} + \triangle_t L'_{a,t-a} \tag{9}$$

$$\triangle_a P'_{a,t} = \triangle_a B'_{t-a} + \triangle_a L'_{a,t-a} \tag{10}$$

Second differences:

$$\triangle_t^2 P'_{a,t} = \triangle_t^2 B'_{t-a} + \triangle_t^2 L'_{a,t-a} \qquad (11)$$

$$\triangle_a^2 P'_{a,t} = \triangle_a^2 B'_{t-a} + \triangle_a^2 L'_{a,t-a} \qquad (12)$$

Further we have

$$\triangle_a P'_{a+h,t+h} - \triangle_t P'_{t-a} = \triangle_a L'_{a+h,t+h}$$

$$- \triangle_t L'_{a,t-a} , \qquad (13)$$

and

$$\triangle_a^2 P'_{a+h,t+h} - \triangle_t^2 P'_{t-a} = \triangle_a^2 L'_{a+h,t+h}$$

$$- \triangle_t^2 L'_{a,t-a} . \qquad (14)$$

Panel 1 of Table 3 presents the logarithms of the age-time data given in that of Table 1. Panels 2 to 5 of Table 3 were obtained by (9) to (12), respectively.

The decompositions of the first and second differences of $P'_{a,t}$ given in (9) to (12) tell us that the bend measures can be expressed as a form of the sum of fertility and mortality components.

We will now explain (9) and (12) in more detail. From (9) we have

$$\triangle_t P'_{a,t} = \log \frac{P_{a,t}}{P_{a,t-1}} = \log \frac{B_{t-a}}{B_{t-a-1}} + \log \frac{L_{a,t-a}}{L_{a,t-a-1}} . \qquad (15)$$

If the absolute values of $\log(B_{t-a}/B_{t-a-1})$ are very large compared with those of $\log(L_{a,t-a}/L_{a,t-a-1})$, then we have

$$\triangle_t P'_{a,t} \fallingdotseq \log \frac{B_{t-a}}{B_{t-a-1}} = \triangle_t B'_{t-a} .$$

Table 3. Logarithms of Population Numbers by Age and Time $P'_{a,t}$ and Their
First and Second Differences with Respect to Time $\triangle_t P'_{a,t}$ and
$\triangle_t^2 P'_{a,t}$ and with Respect to Age $\triangle_a P'_{a,t}$ and $\triangle_a^2 P'_{a,t}$: World Total,
1950 to 1980 ($\times 10^{-3}$)

1. $P'_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
0- 4	19647	19806	19877	19985	20072	20114	20108
5- 9	19415	19575	19745	19824	19944	20037	20081
10-14	19368	19393	19559	19730	19815	19936	20029
15-19	19294	19347	19374	19543	19715	19806	19928
20-24	19210	19267	19326	19355	19524	19701	19792
25-29	19087	19181	19241	19304	19332	19506	19686
30-34	18920	19055	19155	19217	19283	19319	19491
35-39	18887	18880	19023	19129	19190	19263	19299
40-44	18790	18840	18838	18990	19103	19165	19245
45-49	18651	18734	18792	18793	18952	19071	19134
50-54	18479	18577	18671	18736	18738	18905	19033
55-59	18306	18370	18489	18595	18673	18673	18846
60-64	18109	18179	18245	18374	18495	18582	18587
65-69	17831	17923	18008	18076	18229	18357	18447
70-74	17462	17574	17677	17776	17834	18002	18153
75-79	16906	17037	17163	17291	17402	17489	17694

2. $\triangle_t P'_{a,t}$

Age	1955	1960	1965	1970	1975	1980
0- 4	158	71	108	87	42	-5
5- 9	160	170	79	120	93	45
10-14	25	166	171	85	122	93
15-19	54	27	169	172	91	122
20-24	57	59	28	169	177	91
25-29	94	60	63	28	174	180
30-34	135	100	62	66	36	173
35-39	-7	143	106	61	74	36
40-44	50	-2	152	113	62	80
45-49	82	59	1	159	120	63
50-54	98	94	65	2	167	128
55-59	65	119	106	77	0	173
60-64	70	66	129	121	87	4
65-69	92	85	69	153	128	90
70-74	111	103	100	58	168	151

3. $\triangle_a P'_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
0- 4	232	231	132	161	129	77	27
5- 9	47	182	186	94	129	100	52
10-14	74	45	185	186	99	130	101
15-19	83	80	48	189	192	105	136
20-24	123	86	85	50	192	195	106
25-29	167	126	86	88	49	187	194
30-34	33	175	132	88	93	55	192
35-39	97	40	185	139	87	98	54
40-44	139	106	46	197	151	94	111
45-49	173	157	122	57	214	166	101
50-54	173	206	182	140	65	232	187
55-59	197	192	244	221	177	91	259
60-64	278	256	238	298	267	226	140
65-69	368	349	331	300	394	354	293
70-74	556	536	514	486	432	514	459

4. $\triangle_t^2 P'_{a,t}$

Age	1955	1960	1965	1970	1975
0- 4	-87	37	-21	-46	-47
5- 9	10	-91	41	-27	-48
10-14	141	5	-86	37	-29
15-19	-27	143	3	-81	31
20-24	3	-31	141	8	-86
25-29	-34	3	-36	146	6
30-34	-35	-39	4	-30	137
35-39	150	-37	-45	13	-38
40-44	-52	154	-39	-51	18
45-49	-24	-58	158	-39	-57
50-54	-4	-29	-63	165	-39
55-59	54	-12	-29	-77	173
60-64	-4	62	-7	-34	-83
65-69	-7	-16	84	-25	-38
70-74	-8	-4	-42	110	-17

5. $\triangle_a^2 P'_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
5- 9	185	48	-54	67	-1	-23	-25
10-14	-26	137	2	-92	30	-30	-49
15-19	-9	-35	137	-3	-93	25	-35
20-24	-40	-6	-38	139	0	-89	30
25-29	-44	-40	-1	-37	143	7	-89
30-34	135	-48	-46	0	-44	132	2
35-39	-64	135	-53	-51	6	-43	138
40-44	-42	-66	139	-58	-65	4	-57
45-49	-34	-51	-76	140	-62	-72	10
50-54	0	-49	-60	-83	148	-65	-86
55-59	-24	15	-62	-81	-112	141	-72
60-64	-81	-64	6	-77	-89	-135	119
65-69	-90	-93	-93	-2	-128	-129	-154
70-74	-188	-187	-183	-186	-37	-159	-166

From (13) we have

$$\Delta_a^2 P'_{a+h,t+h} - \Delta_t^2 P'_{a,t}$$

$$= (\Delta_a^2 B'_{t-a} + \Delta_a^2 L'_{a+h,t-a}) - (\Delta_t^2 B'_{t-a} + \Delta_t^2 L'_{a,t-a})$$

$$= \Delta_a^2 L'_{a+h,t-a} - \Delta_t^2 L'_{a,t-a} \quad (\Delta_a^2 B'_{t-a} = \Delta_t^2 B'_{t-a})$$

$$= \log \frac{L_{a+h-1,t-a+1}}{L_{a,t-a+1}} + \log \frac{L_{a+h+1,t-a-1}}{L_{a,t-a-1}} - 2\log \frac{L_{a+h,t-a}}{L_{a,t-a}} \qquad (16)$$

Two examples of values for $\Delta_a^2 P'_{a+h,t+h} - \Delta_t^2 P'_{a,t}$ of the first expression of (16) are shown below.

(Example 1) If a=1, t=3 and h=2 in (16),

then

$$\Delta_t^2 P'_{1,3} = 41 \text{ (from Panel 4 of Table 3)}$$

and

$$\Delta_a^2 P'_{3,5} = 25 \text{ (from Panel 5 of Table 3)}.$$

Therefore, we have the value of (16) = 25 - 41 = -16.

(Example 2) If $2 \leq a \leq 7$ and h=0, the absolute values of (16) are very small. Therefore, we have

$$\Delta_t^2 P'_{a,t} = \Delta_a^2 P'_{a,t} \text{ (cf. Panels 4 and 5 of Tables 3)}.$$

As a special case of (16), we have

$$\Delta_a^2 P'_{a,t-a} - \Delta_t^2 P'_{0,t} = \Delta_a^2 L'_{a,t} - \Delta_t^2 L'_{0,t} \qquad (17)$$

From Panels 4 and 5 of Table 3, we have the value of (17), which are denoted by α. Using (17) and

$$\Delta_t^2 P'_{0,t} = \Delta_t^2 B'_t + \Delta_t^2 L'_{0,t} , \qquad (18)$$

we have

$$\Delta_a^2 P'_{a,t+a} = \Delta_t^2 P'_{0,t+a}$$

$$= \Delta_t^2 B'_t + \Delta_t^2 L'_{0,t+a}. \tag{19}$$

If the absolute values of $\Delta_t^2 B'_t$ are very large compared with those of $\Delta_t^2 L'_{0,t+a}$, then we have

$$\Delta_a^2 P'_{a,t+a} \fallingdotseq \Delta_t^2 B'_t .$$

The following example (Example 3) is an application of (18) and (19).

(Example 3) Using (18) we can get the values of $\Delta_t^2 L'_{0,t}$ (Table 4) by subtracting the values of $\Delta_t^2 B'_t$ (calculated by using U.N. estimated births (1986)) from the values of $\Delta_t^2 P'_{0,t}$ shown in Panel 4 of Table 3. From (19), with a=2 and t=4, we have the values of $\Delta_a^2 P'_{2,6}$ with their fertility and mortality components (Table 5).

Table 4. Examples of the Calculation of $\Delta_t^2 L'_{0,t}$

$(\times 10^{-3})$

Country	$\Delta_t^2 L'_{0,t}$	=	$\Delta_t^2 P'_{0,t}$	−	$\Delta_t^2 B'_t$
(1) t=4: 1965-1970 Cohorts					
World total	−17	=	−46	−	(−29)
Indonesia	−3	=	−20	−	(−17)
Japan	−22	=	34	−	56
(2) t=5: 1970-1975 Cohorts					
World total	−1	=	−47	−	(−46)
Indonesia	−1	=	−104	−	(−103)
Japan	44	=	−256	−	(−303)

Table 5. Examples of the Calculation of Formula (15) for
Cohorts Born in the Five-Year Period 1965 to 1970

$(\times \ 10^{-3})$

Country	$\triangle_a^2 P'_{2,6}$	=	$\triangle_t^2 B'_4$	+	(	$\triangle_t^2 L'_{0,4}$	$+\alpha$)
World Total	-49	=	-29	+	(	-17	-3)
Indonesia	-23	=	-17	+	(	-3	-3)
Japan	33	=	56	+	(	-22	-1)

III. Quantitative Measuring of the Bend by Curvature and Angle

In our analysis we need population data by single years of age at five-year intervals. Greville's interpolation or cubic spline interpolation formulas were used to obtain that data. Figure 2 shows age distributions calculated for the world total. Hereafter, the unit for age is a single year and that for time is a one-year interval.

Figure 2. Changing Age Distributions by Single Years of Age from
1950 to 2025: World Total
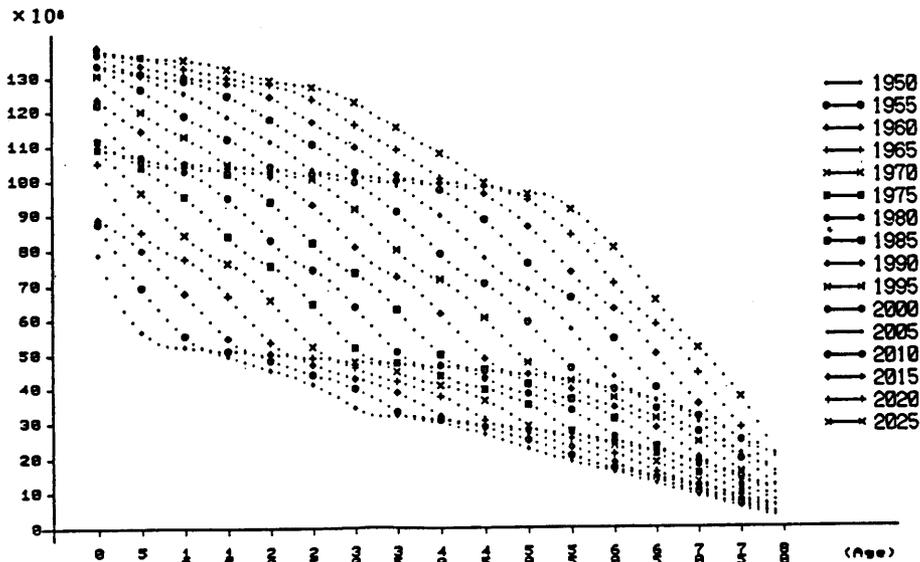
1. The bend in the curve of age distribution

Figure 2 is a schematic diagram for explaining several bends in an age distribution of population expressed by a function p=f(a) at a fixed point of time t. This function reaches a local minimum value at point A and a local maximum value at point B. The function decreases monotonically at the right part of point B, but its slope shows a series of sudden changes at points C, D, E and F. In terms of curvature, the curvature is locally maximum at the three points A, C and E and locally minimum at the other three points, B, D and F.

Figure 3. Bends in the Curve of Age Distribution
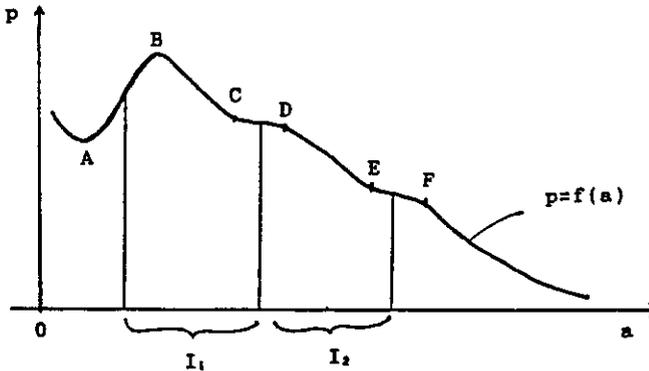
Age intervals where the curve of age distribution changes smoothly are in general outside the object of this paper. We are interested in age intervals where sudden changes of the age curve are found. In order to see when and with what extent sudden changes in the demographic parameters occurred, we need to examine the amount of changes in sharp bends in cohort age-time process and the ages at which such bends occur.

Of the aforementioned points in the curve of age distribution, points A, C and E correspond to the points of time when a sudden change (for example, increase of births) starts. The other points, B, D and F, correspond to the points of time when the change terminates. The amount of the bend roughly indicates the extent of the change. In interval I_1 in Figure 3, the change starts at the point of time corresponding to point C and terminates at point

B. In interval I_2 it starts and terminates at points E an D, respectively. Using our method, we can find such points as A, B, C, D, E and F in the age distribution and examine the amounts and changes of curvature, etc., at these points along the age-time cohort line.

2. Formulas for calculation

In the age distribution at time t by single years of age, the size of population in age a years and its proportion to the population in all ages (in percentage) are denoted here by $P_{a,t}$, and $Q_{a,t}$, respectively. Then we have

$$Q_{a,t} = (P_{a,t} / \sum_{a=0}^{\omega} P_{a,t}) \times 100.$$

Using $Q_{a,t}$ we have formulas for measuring curvature CV(a,t) and angle ANG(a,t).

Formula 1: CV(a,t) based on curvature

Let t be fixed. Putting $S(a) = Q_{a,t}$, we define CV(a,t) to be a curvature of the curve p=S(a) at age a. Then we have

$$CV(a,t) = \frac{S''(a)}{\left[1 + \{S'(a)\}^2\right]^{3/2}}.$$

When applying fourth degree polynomial to p=S(a), we have the following formulas for first and second derivatives, respectively:

$$S'(a) = \{S(a-2) - 8S(a-1) + 8S(a+1) - S(a+2)\}/12,$$

and

$$S''(a) = -S(a-2) + 16S(a-1) - 30S(a) + 16S(a+1) - S(a+2).$$

Formula 2: ANG(a,t) based on angle

We take three points on the curve p=S(a) at the same interval h, A(a-h,S(a-h)), B(a,S(a)) and C(a+h, S(a+h)) where h is taken here as 3 years of age.[3] Using angle θ_2 between the positive a-axis and line AB, and the angle θ_1 between the positive a-axis and line BC, the angle formed by line AB

and BC which is denoted by ANG(a,t) is calculated by the following formula:

$$\text{ANG}(a,t) = \tan(\theta_1 - \theta_2) = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}.$$

3. Use of the formulas

How to measure sharp bends in reference to points of time t by use of Formulas 1 and 2 will be discussed below. Years 1950, 1955, 1960, ... are denoted by t=0, 5, 10.... Within the age range which we deal with we take small age intervals (s+t,s+t+d), where s+t $\geqq$ 0, d $>$ 0, and where both s and d are given in advance.

We calculated local maximum and minimum values as well as local maximum and minimum points (ages) of both CV(a,t) and ANG(a,t) which are functions of a in the small age interval for fixed t. Some examples are given in Table 6 for the world total. In the middle panel of Table 6, we use S=0 and d=19 when t=0.[4] In this case, therefore, local maximum and minimum values and points of the functions are calculated in the respective small age intervals (0,19), (5,24), (10,29),..., according to t=0, 5, 10,.... In the lowest panel of Table 5 where t=30, s=-30 and d=19, such small age intervals as (0,19), (5,24),..., corresponding, respectively, to t=30, 35,... are taken.

In estimating years at which sharp bends occur, it is desirable to use the results obtained by Formulas 1 and 2 in a combined manner as far as possible.[5] Table 6 shows years of birth of cohorts at the onset of upward concave bends estimated by observing the regular changes in a series of values obtained in Table 6. The figures in Table 7 are presented by single years, but inasmuch as our original age-time data are given in terms of 5-year age groups at five-year intervals, it will be appropriate to return our data in Table 6 to the respective five-year intervals, that is, for example, 1918 to 1915-20, 1943 to 1940-45 and so on irrespective of whether the data are for developing or developed countries.

Depending on the kinds of interpolation formulas used, some limitations may need to be considered in interpreting the results.[6]

Table 6. Local Maximum and Minimum Values and Points (Ages) of
Curvature CV(a,t) and Angle ANG(a,t) for Each Fixed t:
World Total

Year	Max.CV	(Age)	Min.CV	(Age)	Max.ANG	(Age)	Min.ANG	(Age)
(1) From earlier birth cohorts
1950	22.10	(32)	-11.83	(37)	48.76	(32)	-25.57	(37)
1955	19.67	(37)	-10.45	(42)	43.53	(37)	-22.55	(42)
1960	18.10	(42)	-9.90	(47)	39.95	(42)	-21.41	(47)
1965	16.18	(47)	-8.66	(52)	35.62	(47)	-18.98	(52)
1970	15.13	(52)	-9.33	(57)	33.29	(52)	-20.49	(57)
1975	13.02	(57)	-8.51	(62)	28.38	(57)	-18.51	(62)
1980	10.28	(62)	-6.88	(67)	22.47	(62)	-15.30	(67)
(2) From birth cohorts around the mid-century
1950	28.95	(7)	-17.44	(12)	79.00	(6)	-32.58	(12)
1955	24.43	(12)	-13.92	(17)	59.17	(12)	-28.01	(17)
1960	25.29	(17)	-13.12	(22)	57.46	(17)	-26.82	(22)
1965	23.06	(22)	-11.61	(27)	51.85	(22)	-23.92	(27)
1970	20.95	(27)	-11.51	(32)	47.28	(27)	-23.70	(32)
1975	17.09	(32)	-9.60	(37)	38.65	(32)	-19.86	(37)
1980	16.64	(37)	-10.32	(42)	37.47	(37)	-21.59	(42)
(3) From later birth cohorts
1980	0.00	(4)	-6.56	(12)	0.00	(4)	-15.81	(12)
1985	12.90	(6)	-12.02	(12)	34.64	(6)	-25.47	(12)
1990	13.75	(12)	-10.60	(17)	31.18	(12)	-23.71	(17)
1995	12.95	(17)	-9.50	(22)	28.55	(17)	-21.32	(22)
2000	12.21	(22)	-8.65	(27)	26.76	(22)	-19.55	(27)

Table 7. Examples of the Estimation of Birth Years of
Cohorts at the Onset of Sharp Bends

Country	Birth	Years	of	Cohorts
World Total	1918,	1943,	1978	
China	1948			
Indonesia	1923,	1948		
Japan	1918,	1943		
Philippines	1918,	1943		
Thailand	1918,	1938		

IV. Final Comments

A few remarks will be made by comparing the age-time distributions of the world total in Table 1 and Japan in Appendix Table A. The world total population continued period-to-period growth from 1950 to 1980 in all age groups. The single minor exception was the decrease seen in age group 35-39 from 159 to 158 million between 1950 and 1955. Without exception, the younger age groups always had larger populations. As the first differences of the age-time data indicate, the 1945-50 and all the later cohorts have much larger population in comparison with the 1940-45 and earlier cohorts. This means that the upper-right part of the age-time population table formulates a distinct highland as a whole, when we take the 1940-45 cohort as the boundary. In contrast, in Japan, the 1945-50 cohort is the only cohort jutting out like a mountain range along the cohort line from the upper-left corner down to the lower-right. The upper-right part of the age-time distribution does not formulate a continuing highland area. The 1945-50 cohort was the only baby-boom cohort (actually 1947-49) until the 1970-75 second baby boom. Demographic implications of the bend in the curve of age distribution are found not only in the occurring individual bends but also in the overall pattern of post-bend demographic dynamics.

1. In Panels 2 and 4 of Table 1, the columns are headed by calendar years (1950, 1955,...) as in the other panels, but what they actually signify are the first and second differences of the years concerned. In Panels 3 and 5 of Table 1, similarly, the rows are headed by 5-year age groups as in the remaining panels, but what they actually signify are the first and second differences of the age groups concerned.

2. When "a" denotes an age showing a bend for a fixed t, we find the following relationship between the first and second differences: if $\triangle_t P_{a,t}$ and $\triangle_a P_{a,t}$ as the functions of "a" reach a local minimum at a_0, then $\triangle_t^2 P_{a,t}$ and $\triangle_a^2 P_{a,t}$ reach a local maximum at a_0.

3. Parameter h may take any one of the four values 2, 3, 4 and 5.

4. In many cases d=19 is used, but we also use d=9 or 14 when necessary.

5. When using formulas 1 and 2, we can obtain cohort information, even if curves of age distribution are available only at a few points of time.

6. To get age distributions by single years of age, Greville's interpolation formula is used instead of cubic spline interpolation. This is because more desirable results can be obtained when we use the formulas shown in III.2.

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Table A. Population Numbers by Age and Time $P_{a,t}$ and Their First and Second Differences with Respect to Time $\triangle_t P_{a,t}$ and $\triangle_t^2 P_{a,t}$ and with Respect to Age $\triangle_a P_{a,t}$ and $\triangle_a^2 P_{a,t}$: Japan, 1950 to 1980 (in millions)

1. $P_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
0- 4	112	95	80	82	89	99	86
5- 9	97	110	95	80	82	89	100
10-14	88	96	109	94	80	83	89
15-19	86	87	94	108	91	80	83
20-24	78	85	84	92	107	92	79
25-29	62	76	83	84	91	107	91
30-34	52	61	75	83	84	92	107
35-39	51	51	60	75	82	84	92
40-44	45	50	50	59	74	82	83
45-49	40	44	48	49	59	73	81
50-54	34	39	42	47	48	57	71
55-59	28	32	37	40	44	47	56
60-64	23	25	29	34	37	43	45
65-69	18	20	22	26	30	34	39
70-74	13	14	16	17	21	26	30
75-79	7	9	10	11	13	16	20

2. $\triangle_t P_{a,t}$

Age	1955	1960	1965	1970	1975	1980
0- 4	-17	-15	2	7	11	-14
5- 9	13	-15	-15	2	7	11
10-14	8	13	-15	-15	3	7
15-19	1	7	14	-16	-11	3
20-24	7	-1	8	15	-15	-12
25-29	14	7	1	7	16	-16
30-34	9	14	8	1	8	15
35-39	0	9	15	7	2	8
40-44	5	1	9	14	8	2
45-49	3	5	1	10	14	8
50-54	5	3	5	2	9	14
55-59	4	5	3	4	2	9
60-64	2	4	4	4	5	2
65-69	2	2	4	4	4	5
70-74	1	2	2	4	4	4

3. $\triangle_a P_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
0- 4	15	-15	-15	2	6	10	-14
5- 9	9	14	-14	-14	3	6	11
10-14	2	9	15	-13	-12	3	7
15-19	9	3	10	16	-16	-11	4
20-24	16	8	1	7	16	-16	-12
25-29	10	15	8	1	7	15	-16
30-34	1	10	15	8	2	8	15
35-39	6	1	10	16	9	2	8
40-44	5	6	2	10	15	9	3
45-49	6	5	6	2	11	16	9
50-54	6	7	5	7	4	11	16
55-59	5	7	7	6	7	4	11
60-64	5	5	8	8	7	8	5
65-69	5	6	6	8	9	9	9
70-74	6	5	6	6	9	9	10

4. $\triangle_t^2 P_{a,t}$

Age	1955	1960	1965	1970	1975
0- 4	1	17	5	4	-24
5- 9	-28	0	17	4	4
10-14	4	-28	0	18	4
15-19	6	7	-30	5	14
20-24	-8	9	7	-31	3
25-29	-8	-5	6	9	-32
30-34	5	-6	-7	7	7
35-39	9	6	-7	-6	6
40-44	-4	8	5	-6	-7
45-49	1	-4	9	4	-6
50-54	-1	1	-3	8	5
55-59	0	-2	1	-2	7
60-64	2	0	0	1	-3
65-69	0	2	0	0	1
70-74	1	0	2	0	0

5. $\triangle_a^2 P_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
5- 9	6	-28	-1	17	4	4	-24
10-14	7	4	-30	-1	15	4	4
15-19	-7	7	5	-29	4	14	3
20-24	-7	-6	9	9	-31	4	16
25-29	6	-7	-7	6	9	-31	3
30-34	8	5	-7	-7	5	7	-31
35-39	-5	8	5	-8	-7	6	7
40-44	1	-5	8	5	-6	-7	6
45-49	-2	1	-4	8	4	-7	-7
50-54	0	-2	1	-4	7	5	-6
55-59	2	0	-2	0	-3	7	5
60-64	-1	2	0	-2	0	-4	6
65-69	0	0	2	0	-1	0	-4
70-74	-1	1	0	2	0	-1	0

Table B. Population Numbers by Age and Time $P_{a,t}$ and Their First and Second Differences with Respect to Time $\triangle_t P_{a,t}$ and $\triangle_t^2 P_{a,t}$ and with Respect to Age $\triangle_a P_{a,t}$ and $\triangle_a^2 P_{a,t}$: Thailand, 1950 to 1980 (in tens of thousands)

1. $P_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
0- 4	372	424	478	558	636	626	605
5- 9	273	350	404	460	541	621	633
10-14	219	268	344	399	455	536	611
15-19	205	215	263	339	394	451	529
20-24	179	199	210	258	333	388	457
25-29	155	173	193	205	253	328	375
30-34	123	149	167	188	200	248	288
35-39	107	117	144	162	183	195	236
40-44	88	101	112	138	157	178	218
45-49	73	83	96	107	133	151	183
50-54	60	68	77	90	101	126	150
55-59	52	54	61	71	83	94	113
60-64	44	45	47	54	63	75	88
65-69	34	36	37	39	46	54	68
70-74	25	25	27	28	30	36	50
75-79	15	16	16	18	19	21	29

2. $\triangle_t P_{a,t}$

Age	1955	1960	1965	1970	1975	1980
0- 4	52	54	80	78	-9	-21
5- 9	76	54	56	81	80	12
10-14	49	76	55	57	81	75
15-19	10	49	76	55	57	78
20-24	20	11	48	75	55	69
25-29	18	20	12	48	75	48
30-34	27	18	20	12	48	41
35-39	11	26	18	21	13	41
40-44	13	11	26	19	21	40
45-49	10	13	11	26	19	31
50-54	8	10	13	11	25	24
55-59	2	8	9	12	11	19
60-64	1	2	8	9	12	13
65-69	2	1	2	7	8	14
70-74	0	2	1	2	6	14

3. $\triangle_a P_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
0- 4	99	75	74	98	95	6	-27
5- 9	54	82	60	62	86	84	21
10-14	15	53	81	60	61	85	82
15-19	25	16	54	81	61	62	71
20-24	24	26	17	53	81	61	82
25-29	32	24	26	17	53	80	87
30-34	16	32	24	26	17	53	52
35-39	18	16	31	24	26	18	18
40-44	15	18	17	31	24	27	36
45-49	14	15	19	17	31	25	32
50-54	8	14	16	19	18	32	37
55-59	8	9	15	17	20	19	25
60-64	10	9	10	15	17	21	20
65-69	9	11	10	11	15	18	18
70-74	10	10	11	10	11	15	21

4. $\triangle_t^2 P_{a,t}$

Age	1955	1960	1965	1970	1975
0- 4	2	26	-2	-87	-12
5- 9	-22	2	25	-1	-67
10-14	28	-21	2	25	-6
15-19	38	27	-21	2	21
20-24	-9	38	27	-20	14
25-29	2	-8	37	27	-27
30-34	-8	2	-8	36	-7
35-39	16	-8	2	-8	29
40-44	-2	15	-7	2	20
45-49	3	-2	14	-7	13
50-54	2	3	-1	14	-1
55-59	6	2	3	-1	8
60-64	2	5	1	3	2
65-69	0	1	4	2	6
70-74	1	0	1	4	9

5. $\triangle_a^2 P_{a,t}$

Age	1950	1955	1960	1965	1970	1975	1980
5- 9	45	-7	14	36	9	-79	-49
10-14	39	29	-21	2	25	-1	-61
15-19	-11	37	27	-22	1	23	11
20-24	2	-10	37	28	-20	2	-11
25-29	-8	2	-9	37	28	-19	-4
30-34	16	-8	2	-9	36	27	35
35-39	-2	16	-8	2	-9	35	34
40-44	3	-2	15	-7	2	-9	-17
45-49	2	3	-2	14	-7	1	3
50-54	6	1	3	-2	13	-7	-4
55-59	0	5	1	3	-2	13	12
60-64	-2	0	5	1	3	-1	6
65-69	1	-2	0	4	2	3	1
70-74	0	1	0	1	4	3	-3

Table C. Local Maximum and Minimum Values and Points (Ages) of
Curvature CV(a,t) and Angle ANG(a,t) for Each Fixed t:
China, Indonesia, Japan, Philippines and Thailand

Year	Max.CV	(Age)	Min.CV	(Age)	Max.ANG	(Age)	Min.ANG	(Age)
(1) China
1950	29.55	(5)	-37.15	(17)	88.87	(5)	-79.13	(17)
1955	23.86	(5)	-29.09	(22)	71.54	(5)	-62.51	(22)
1960	43.83	(12)	-23.00	(27)	90.44	(12)	-49.58	(27)
1965	39.07	(17)	-18.08	(32)	88.23	(17)	-38.58	(32)
1970	33.87	(22)	-13.65	(37)	75.76	(22)	-28.97	(37)
1975	29.31	(27)	-10.19	(42)	65.75	(27)	-21.93	(42)
1980	26.37	(32)	-7.96	(47)	58.96	(32)	-16.87	(47)
(2) Indonesia: from earlier birth cohorts
1950	12.15	(28)	-10.79	(37)	26.40	(27)	-18.86	(37)
1955	11.36	(33)	-7.41	(43)	22.34	(32)	-15.02	(42)
1960	9.88	(37)	-9.01	(47)	20.10	(37)	-14.21	(47)
1965	8.02	(43)	-5.84	(52)	17.75	(42)	-11.52	(52)
1970	6.17	(48)	-5.33	(57)	14.13	(47)	-9.98	(57)
1975	7.49	(53)	-4.79	(62)	12.04	(52)	-7.86	(62)
1980	4.86	(57)	-3.64	(68)	10.16	(57)	-5.74	(67)
(3) Indonesia: from later birth cohorts
1950	18.34	(6)	-17.92	(12)	42.75	(7)	-40.65	(12)
1955	36.11	(6)	-17.14	(13)	101.67	(6)	-29.65	(17)
1960	32.05	(12)	-12.82	(21)	74.85	(12)	-24.26	(22)
1965	30.98	(17)	-11.99	(27)	67.26	(17)	-21.18	(27)
1970	26.47	(22)	-8.66	(32)	59.30	(22)	-18.01	(32)
1975	23.59	(27)	-7.68	(37)	51.60	(27)	-15.48	(37)
1980	20.54	(32)	-6.96	(42)	44.82	(32)	-13.25	(42)
(4) Japan: from earlier birth cohorts
1950	17.94	(31)	-19.03	(37)	41.45	(32)	-35.87	(37)
1955	18.00	(36)	-16.14	(42)	39.34	(37)	-33.03	(42)
1960	15.23	(42)	-13.02	(47)	34.72	(42)	-29.40	(47)
1965	15.25	(46)	-11.80	(53)	33.03	(47)	-27.30	(52)
1970	12.38	(51)	-10.54	(58)	27.48	(52)	-19.17	(57)
1975	10.98	(57)	-10.69	(63)	25.41	(57)	-22.42	(62)
1980	9.70	(62)	-10.77	(67)	21.12	(62)	-21.40	(67)
(5) Japan: from later birth cohorts
1950	16.04	(11)	-17.44	(17)	40.26	(12)	-36.67	(17)
1955	24.68	(12)	-41.29	(7)	41.94	(12)	-106.14	(6)
1960	22.05	(17)	-64.74	(12)	45.34	(17)	-143.12	(12)
1965	30.51	(22)	-60.00	(17)	62.70	(22)	-136.18	(17)
1970	29.63	(27)	-62.78	(22)	63.58	(27)	-142.50	(22)
1975	26.23	(32)	-59.03	(27)	52.90	(32)	-130.02	(27)
1980	23.19	(38)	-55.22	(32)	49.65	(37)	-124.99	(32)

Table C. (Continued)

Year	Max.CV	(Age)	Min.CV	(Age)	Max.ANG	(Age)	Min.ANG	(Age)

(6) Philippines: from earlier birth cohorts

Year	Max.CV	(Age)	Min.CV	(Age)	Max.ANG	(Age)	Min.ANG	(Age)
1950	16.62	(33)	-20.68	(27)	24.32	(32)	-42.16	(27)
1955	13.97	(37)	-18.51	(33)	22.36	(37)	-33.54	(32)
1960	8.36	(40)	-24.49	(37)	17.92	(42)	-32.25	(37)
1965	10.77	(48)	-21.55	(42)	14.36	(47)	-25.65	(42)
1970	9.10	(51)	-12.87	(45)	12.43	(52)	-18.64	(47)
1975	6.26	(68)	-7.64	(53)	9.40	(57)	-16.45	(52)
1980	7.24	(74)	-6.90	(68)	12.42	(72)	-10.35	(67)

(7) Philippines: from later birth cohorts

Year	Max.CV	(Age)	Min.CV	(Age)	Max.ANG	(Age)	Min.ANG	(Age)
1950	30.40	(5)	-31.62	(18)	72.97	(5)	-63.24	(17)
1955	17.82	(6)	-26.20	(22)	44.71	(5)	-50.30	(22)
1960	15.53	(12)	-23.00	(27)	31.06	(17)	-40.62	(27)
1965	16.93	(17)	-16.93	(31)	28.73	(22)	-35.91	(32)
1970	17.53	(27)	-17.09	(37)	25.74	(27)	-31.07	(37)
1975	12.33	(27)	-12.33	(42)	24.28	(27)	-25.84	(42)
1980	11.38	(33)	-8.97	(42)	20.69	(32)	-16.56	(42)

(8) Thailand: from earlier birth cohorts

Year	Max.CV	(Age)	Min.CV	(Age)	Max.ANG	(Age)	Min.ANG	(Age)
1950	20.50	(32)	-16.40	(26)	42.64	(32)	-27.88	(27)
1955	18.23	(37)	-13.94	(32)	35.75	(37)	-25.74	(32)
1960	16.44	(43)	-13.34	(47)	27.29	(42)	-18.61	(37)
1965	13.87	(47)	-7.47	(40)	23.47	(47)	-13.87	(42)
1970	11.91	(52)	-9.17	(47)	19.25	(52)	-12.83	(47)
1975	10.87	(57)	-11.48	(51)	16.11	(57)	-10.47	(52)
1980	6.99	(57)	-5.37	(74)	11.46	(57)	-2.87	(74)

(9) Thailand: from later birth cohorts

Year	Max.CV	(Age)	Min.CV	(Age)	Max.ANG	(Age)	Min.ANG	(Age)
1950	37.31	(11)	-27.06	(18)	72.16	(12)	-42.64	(17)
1955	32.89	(18)	-23.24	(22)	62.92	(17)	-37.18	(22)
1960	23.88	(21)	-20.78	(27)	53.35	(22)	-48.39	(12)
1965	19.74	(27)	-24.27	(17)	43.74	(27)	-39.47	(17)
1970	17.64	(33)	-15.12	(21)	37.58	(32)	-30.25	(22)
1975	18.12	(37)	-15.70	(27)	32.21	(37)	-27.38	(27)
1980	13.79	(32)	-13.79	(42)	27.23	(37)	-25.08	(42)