New Relational Models of Age-Specific Mortality
—All and Selected Causes of Death—

Zenji Nanjo
Takao Shigematsu

NUPRI Research Paper Series No.63

November 1993

Zenji Nanjo
Professor
Department of Liberal Arts
Tohoku Gakuin University
Sendai City, Japan
and
Affiliated Expert
Population Research Institute
Nihon University
Tokyo, Japan

Takao Shigematsu
Professor
Medical Department
Fukuoka University
Fukuoka City, Japan

CONTENTS

TABLES

I. Introduction

The relational and parameterized models of age-specific mortality are very useful for the forecasting, smoothing and interpolation of mortality for all causes.

The Perks model[7,8] with five parameters, the Heligman-Pollard model[4] with eight parameters and the Rogers-Planck model[10] with nine parameters are well-known among the many parameterized models.

The last two models, which describe mortality probabilities in childhood, middle-life and older ages, are very effective for parameterizing mortality curves for all causes, but not always effective for each cause of death.[11]

First of all, the Brass logit systems of mortality were generalized to obtain effective models for cause-specific mortality.

Applicability of these models to Japanese mortality for all and main causes were tested. Fit by these models with a small number of parameters seems to be much better than that by traditional ones.

Furthermore, our models were also applied to the USSR mortality[12] for all causes of death.

Finally, for the sake of comparison with our results, the Heligman-Pollard model was applied to mortality for all causes of death and the Perks model for all and the three biggest causes of death in Japan.

II. Generalization of the Brass Logit Model (1)

The three-parameter generalized Brass logit models with the standard life table survivorship values $l_{s,x}^{c}$ in place of $l_{s,x}$ were shown, where $l_{s,x}$ stands for the standard life table survivorship value at age x, $l_{s,o}=1$ and c power or exponent.

The logit transformation of a survivorship, $l_x(l_0=1)$, is

$$\text{logit}(l_x) = 1/2 \ln(l_x/(1-l_x)).$$

For convenience of notation, let us define

$$Y_x = \text{logit}(l_x)$$

and

$$Y_{s,c,x} = \text{logit}(l_{s,x}^{c}).$$

Then we want to obtain parameters *a*, *b* and *c* such that minimize

$$\sum_x \{ Y_x - (a+bY_{s,c,x}) \}^2. \qquad (1)$$

For that, the least squares method and simple direct search of optimization method were combinedly used.

In fact, for a fixed *c*, the method of least squares provides values *a* and *b* that minimized (1) and the minimum *s* of (1).

Since *a*, *b* and *s* thus obtained are functions of *c*, let us denote them by $a=f_1(c)$, $b=f_2(c)$, and $s=f_3(c)$, respectively.

If we find c', the value of *c* that minimizes $s=f_3(c)$ for $0.2 \leq c \leq 3.0$, then $a'=f_1(c')$, $b'=f_2(c')$ and c' are the three required parameters.

When we put

$$p_{s,t} = l_{s,t+1} \,/\, l_{s,t} \text{ and } p_{s,t} = 1,$$

then

$$l_{s,t} = \prod_{t=0}^{x-1} p_{s,t} = \prod_{t=0}^{x-1} (1-q_{s,t}).$$

Therefore, we can consider the generalized Brass type logit model

with the standard $l_{s,x}^{(c)} = \prod_{t=0}^{x-1} (1 - q_{s,t}^{c})$ in place of that

with the standard $l_{s,t}^{c} = \prod_{t=0}^{x-1} p_{s,t}^{c}$.

New models with c=1 coincide with the original Brass logit model.

This idea can be used for the four-parameter and the reduced two-parameter models by Ewbank et al. In this case, new models will have five and three parameters, respectively.

Our model can also be used for the survivorship $\overline{l}_x^{(\alpha)}$ ($\overline{l}_0^{(\alpha)} = 1$) for each cause α. Next, we will explain the survivorship $\overline{l}_x^{(\alpha)}$ and mortality $\overline{q}_x^{(\alpha)}$ for cause α at age x to be used in this paper.

We assume that the diverse causes act independently of each other. When α is the only effective cause of death, if $\overline{l}_x^{(\alpha)}$ is the survivorship for cause α and $\overline{l}_x^{(-\alpha)}$ is that due to all other causes combined, we have

$$l_x = \overline{l}_x^{(\alpha)} \cdot \overline{l}_x^{(-\alpha)}.$$

We put $R = {}_nD_{x,\alpha} / {}_nD_x$, where ${}_nD_{x,\alpha}$ and ${}_nD_x$ stand for the observed number of deaths in the age group $(x, x+n)$ due to cause α and all causes combined, respectively.

Then it follows that

$${}_np_x^{R} = (l_{x+n} / l_x)^R = \overline{l}_{x+n}^{(\alpha)} / \overline{l}_x^{(\alpha)}$$

$$(= {}_n\overline{p}_x^{(\alpha)} = 1 - {}_n\overline{q}_x^{(\alpha)}, \text{ say}).$$

To show the goodness of fit, the averages of the RMSE values (root mean square error) of fit to l_x (or $\overline{l}_x^{(\alpha)}$ for cause of death α) at eight dates, i.e. the years 1950, 1955,..., and 1985, Japan, were calculated.

$$\text{RMSE} = \sqrt{\Sigma\, (\hat{l}_x - l_x)^2 / n}$$

where $\hat{l}_x$ is the estimate of l_x.

The MAE (mean absolute error) values were also calculated when necessary.

$$\text{MAE} = \Sigma \mid \hat{l}_x - l_x \mid / \Sigma l_x$$

Our models with the angular transformation of l_x

$$\text{ang}\ (l_x) = \text{arc}\ \sin\sqrt{l_x},$$

instead of the logit transformation of l_x, are sometimes effective for our purpose.

In comparison with Table 1, Table 2 shows effectiveness of our models. The results obtained by using both the generalized Ewbank et al. model with five parameters and the generalized Brass logit model with the standard $l_{s,x}^{(c)}$ are omitted, because the calculation for their curve-fitting are rather troublesome.

Table 1. The Average of the RMSE(MAE) Values of Fit to l_x for the Years 1950, 1955,..., and 1985, when Using the Original Models (All causes, Japan)

Original Model	RMSE		MAE	
	Female	Male	Female	Male
Brass, two-parameter, logit	0.0115	0.0149	0.0086	0.0126
Ewbank et al. four-parameter	0.0047	0.0051	0.0031	0.0037
Ewbank et al. reduced two-parameter (κ=0.75-0.88β, λ=0.50-0.53β)	0.0041	0.0067	0.0035	0.0057
Brass, two-parameter, angular *1)	0.0288	0.0304	0.0231	0.0285

*1) In the original Brass model, the angular transformation was used in place of the logit transformation. The angular transformation was not always effective in Table 1, but sometimes effective in the other tables, i.e. Tables 6-1, 6-2, 6-3 and 9.

Table 2. The Average of RMSE(MAE) Values of Fit to l_x for the Years 1950, 1955,..., and 1985 when Using Generalized Models (All causes, Japan)

Generalized Model	RMSE		MAE	
	Female	Male	Female	Male
g.*2) Brass, three-parameter logit $(\prod_{0}^{x-1} p_x{}^c)$	0.0063	0.0088	0.0055	0.0082
g. Brass, three-parameter logit $(\prod_{0}^{x-1} q_x{}^c)$	0.0052	0.0067	0.0049	0.0064
g. Ewbank et al. three-parameter (κ=0.75-0.88ß, λ=0.50-0.53ß *3))	0.0020	0.0029	0.0018	0.0024

*2) "g." stands for "generalized."

*3) κ and λ, functions of ß, can be given by the regression method depending on the kind of data for the causes of death in countries in question.

Tables 3-1, 3-2 and 3-3 show the parameters of generalized models for female in Table 2.

Table 3-1. The Parameters *a*, *b* and *c* when Using the Generalized Brass Logit Model with the Standard $l_{s,x}^{c} = \pi\, P_{s,x}^{c}$ (All causes, Japanese female)

Years	*a*	*b*	*c*	RMSE	MAE
1950	0.2231	1.1167	0.6520	0.0035	0.0044
1955	0.6860	1.0829	0.8200	0.0058	0.0057
1960	0.9208	1.1207	0.9160	0.0073	0.0067
1965	0.9160	1.3231	0.7400	0.0084	0.0073
1970	0.8423	1.4461	0.6200	0.0079	0.0063
1975	0.8524	1.4810	0.5500	0.0066	0.0053
1980	0.9126	1.5164	0.5200	0.0061	0.0046
1985	0.5910	1.6375	0.3560	0.0050	0.0038

Table 3-2. The Parameters *a*, *b* and *c* when Using the Generalized Brass Logit Model with the Standard $l_{s,x}^{(c)} = \pi(1-q_{s,x}c)$ (All causes, Japanese female)

Years	*a*	*b*	*c*	RMSE	MAE
1950	1.2209	0.4147	0.6200	0.0039	0.0048
1955	1.5450	0.4087	0.6040	0.0057	0.0062
1960	0.8216	1.1764	1.0600	0.0072	0.0070
1965	2.0576	0.4476	0.5800	0.0074	0.0067
1970	2.2731	0.3705	0.5140	0.0060	0.0050
1975	2.4208	0.3378	0.4900	0.0047	0.0039
1980	2.5753	0.3120	0.4660	0.0043	0.0034
1985	2.7241	0.2466	0.4040	0.0030	0.0024

Table 3-3. The Parameters a, b and c when Using the Generalized Ewbank et al. Model with Three Parameters (All causes, Japanese female)

Years	a	b	c	RMSE	MAE
1950	0.1811	1.1371	0.6200	0.0037	0.0044
1955	0.7740	1.2233	0.9000	0.0025	0.0025
1960	0.9949	1.2787	0.9880	0.0016	0.0014
1965	1.2902	1.4322	1.0980	0.0019	0.0016
1970	1.4491	1.5121	1.1420	0.0014	0.0011
1975	1.6629	1.4732	1.2564	0.0014	0.0012
1980	1.7895	1.5225	1.2520	0.0013	0.0011
1985	0.4659	1.6737	0.3200	0.0045	0.0035

III. Generalization of the Brass Logit Model (2)

The generalized Brass logit model with two-standard survivorship values

$l_{s1,x}$ and $l_{s2,x}$ at two points in time (years) was shown:

$$\text{logit}(l_x) = a + b\,\text{logit}(l_{s1,x}) + c\,\text{logit}(l_{s2,x})$$

The method of least squares gives parameters a, b and c that minimize

$$\sum_x [Yx - \{a + b\,\text{logit}(l_{s1,x}) + c\,\text{logit}(l_{s2,x})\}]^2 .$$

This model for the l_x values for all causes of death is originally by R. Kaneko.[5)]

This model with the angular transformation as well as that with the logit transformation is very useful for our purpose, especially for forecasting mortality for all causes and each cause of death. If the standard $l_{s1,x}$ and $l_{s2,x}$ are suitably chosen, this model will be more effective.

Table 4. The Average of the Six RMSE(MAE) Values of Fit to l_x for the Years 1950, 1955, 1965, 1970, 1975 and 1980, when Using the Generalized Brass Model with Two Standards $l_{s1,x}$ for 1960 and $l_{s2,x}$ for 1985 (All causes, Japan)

Generalized Model	RMSE		MAE	
	Female	Male	Female	Male
g. Brass, two-standard, logit	0.0044	0.0038	0.0048	0.0041
g. Brass, two-standard, angular	0.0061	0.0054	0.0067	0.0063

Tables 5-1 and 5-2 are the parameters of generalized models for female in Table 4.

Table 5-1. The Parameters *a*, *b* and *c* when Using the Generalized Brass Model(Logit) with the Two Standards $l_{s1,x}$ for 1960 and $l_{s2,x}$ for 1985 (All causes, Japanese female)

Years	*a*	*b*	*c*	RMSE	MAE
1950	-0.0782	1.3817	-0.4332	0.0172	0.0211
1955	-0.0203	1.0470	-0.1045	0.0036	0.0037
1960	-0.0000	1.0000	0.0000	0.0000	0.0000
1965	-0.0039	0.8632	0.2053	0.0015	0.0012
1970	-0.0109	0.7177	0.3678	0.0024	0.0016
1975	0.0026	0.4430	0.6006	0.0006	0.0004
1980	0.0117	0.2295	0.7961	0.0011	0.0008
1985	0.0000	0.0000	1.0000	0.0000	0.0000

Table 5-2. The Parameters *a*, *b* and *c* when Using the Generalized Brass Model(Angular) with the Two Standards $l_{s1,x}$ for 1960 and $l_{s2,x}$ for 1985 (All causes, Japanese female)

Years	*a*	*b*	*c*	RMSE	MAE
1950	0.0049	1.1334	-0.4067	0.0210	0.0264
1955	0.0016	1.0403	-0.0703	0.0061	0.0064
1960	0.0000	1.0000	0.0000	0.0000	0.0000
1965	-0.0008	0.8912	0.1356	0.0024	0.0021
1970	-0.0007	0.7491	0.2819	0.0022	0.0020
1975	-0.0006	0.4460	0.5727	0.0025	0.0019
1980	-0.0006	0.2126	0.7981	0.0023	0.0017
1985	0.0000	0.0000	1.0000	0.0000	0.0000

IV. Generalized Models and Cause-Specific Mortality

Tables 6-1, 6-2 and 6-3 are the examples of application of our models to mortality for the three biggest causes of death in Japan. These show rather close fit to survivorship values.

Table 6-1. The Average of the Six RMSE(MAE) Values of Fit to l_x for the Years 1950, 1955, 1965, 1970, 1975, and 1980, when Using the Generalized Brass Model with Two Standards $l_{s1,x}$ for 1960 and $l_{s2,x}$ for 1985 (Malignant neoplasms, Japan)

Model	RMSE		MAE	
	Female	Male	Female	Male
g. Ewbank et al. three-parameter	0.0089	0.0079	0.0055	0.0046
g. Brass, two-standard, three-parameter, logit	0.0016	0.0032	0.0008	0.0016
g. Brass, two-standard, three-parameter, angular	0.0014	0.0020	0.0007	0.0011

Table 6-2. The Average of the Six RMSE(MAE) Values of Fit to l_x for the Years 1950, 1955, 1965, 1970, 1975, and 1980, when Using the Generalized Brass Model with Two Standards $l_{s1,x}$ for 1960 and $l_{s2,x}$ for 1985 (Heart disease, Japan)

Model	RMSE		MAE	
	Female	Male	Female	Male
g. Ewbank et al. three-parameter	0.0030	0.0039	0.0019	0.0035
g. Brass, two-standard, three-parameter, logit	0.0016	0.0051	0.0008	0.0023
g. Brass, two-standard, three-parameter, angular	0.0022	0.0019	0.0010	0.0010

Table 6-3. The Average of the Six RMSE(MAE) Values of Fit to l_x for the Years 1950, 1955, 1965, 1970, 1975, and 1980, when Using the Generalized Brass Model with Two Standards $l_{s1,x}$ for 1960 and $l_{s2,x}$ for 1985 (Cerebrovascular disease, Japan)

Model	RMSE		MAE	
	Female	Male	Female	Male
g. Ewbank et al. three-parameter	0.0045	0.0048	0.0023	0.0044
g. Brass, two-standard, three-parameter, logit	0.0083	0.0079	0.0034	0.0033
g. Brass, two-standard, three-parameter, angular	0.0038	0.0038	0.0018	0.0019

V. Substitution of Parameterized Models of Mortality

It is sometimes difficult to estimate the parameters of parameterized models, such as the Heligman-Pollard model and others. Some substitution of these models may be effective for practical use. This is also one of the problems concerning data reduction (for example, determining knots of variation diminishing spline for electrocardiogram data). Two examples are given that estimate all the l_x values from the limited number of l_x values by the combined use of three interpolatory functions. The procedure of fitting this model to l_x is very easy and the RMSE values are very small, as Table 7 shows. And the yearly changes of the l_x values are rather smooth in a certain period.

Example 1. Given the six values of l_1, l_{10}, l_{15}, l_{40}, l_{65} and l_{90}, we can get all the l_x by using the Weibull function for (l_0), l_1 and l_{10}, the Gompertz-Makeham function for l_{15}, l_{40}, l_{65} and l_{90}, and the spline interpolatory function for l_8, l_9, l_{10}, l_{16}, and l_{17} obtained above.

Example 2. Given the nine values of l_1, l_{10}, l_{15}, l_{20}, l_{25}, l_{30}, l_{50}, l_{70} and l_{90}, we can get all the l_x, using the Weibull function for (l_0), l_1 and l_{10}, the Gompertz-Makeham function for l_{30}, l_{50}, l_{70} and l_{90}, and the spline function for l_8, l_9, l_{10}, l_{30}, l_{31} and l_{32} plus l_{15}, l_{20} and l_{25}. In this example, the accident hump of the l_x-curve in ages 20-30 is shown.

Table 7. The Average of the Eight RMSE Values of Fit to l_x by the Combined Use of Interpolatory Functions, for the Years 1950, 1955,..., and 1985, Japan

	Values of l_x	The Number of Values of l_x	Female	Male
Ex.1	(l_0), l_1, l_{10}, l_{15} l_{40}, l_{65}, l_{90}	6	0.0017	0.0022
Ex.2	(l_0), l_1, l_{10}, l_{15}, l_{20} l_{25}, l_{30} l_{50}, l_{70}, l_{90}	9	0.0030	0.0037

The use of this method for projecting or forecasting mortality schedule may come into question. But this idea will be useful for interpolating mortality schedule as a time series and for estimating all the values of life table function (for example, l_x, e°_x etc.) using the minimum number of values of life table function (for example, l_x, e°_x etc.).

VI. Application of Generalized Models to the USSR Data

The USSR life tables recently received by us are for the years 1896-97, 1927-29, 1938-39, 1958-59, 1968-71, and 1984-85. Some of our models were applied to these six life tables.

In Tables 8 and 9, the USSR life tables by sex at three dates subsequent to the year 1950 were used.

Table 8. The Average of the Three RMSE Values of Fit to l_x for the Years 1958-59, 1968-71 and 1984-85, USSR

	Model	Female	Male
Original models	Brass, two-parameter, logit	0.0082	0.0251
	Ewbank et al., four-parameter	0.0047	0.0056
	Ewbank et al., reduced two-parameter (κ=0.75-0.88β, λ=0.50-0.53β)	0.0040	0.0150
Generalized models	g. Brass, three-parameter, logit(π P_{sx}^c)	0.0073	0.0115
	g. Ewbank et al., three-parameter (κ=0.75-0.88β, λ=0.50-0.53β)	0.0024	0.0106

Table 9. The RMSE Values of Fit to l_x for the Years 1968-71 when Using the Three-Parameter Brass Model with the Two Standards $l_{s1,x}$ for the Years 1958-59 and $l_{s2,x}$ for the Year 1984-85

Generalized Model	Female	Male
g. Brass, two-standard, logit	0.0020	0.0058
g. Brass, two-standard, angular	0.0022	0.0038

From Tables 8 and 9, we can see that the generalized Ewbank et al. model and the generalized Brass models with two standards are very effective for the USSR female life tables.

Table 10 is the parameters of the generalized Ewbank et al. model (three-parameter) in the last row of Table 8.

Table 10. The Parameters *a*, *b* and *c*, and the RMSE and MAE Values of Fit to l_x when Using the Generalized Ewbank et al. Model with Three Parameters (Female, USSR)

Years	Parameter			RMSE	MAE
	a	*b*	*c*		
1896-97	-0.2529	0.7112	1.01	0.0154	0.0254
1927-29	0.0573	0.5993	0.91	0.0081	0.0105
1938-39	0.1513	0.5291	0.94	0.0106	0.0120
1958-59	0.7597	1.1203	0.74	0.0016	0.0015
1968-71	1.0560	1.3307	0.90	0.0029	0.0025
1984-85	1.0399	1.3677	0.91	0.0027	0.0027

Table 10 also shows that the values of RMSE and MAE are very small in the period after the year 1950.

VII. Application of the Heligman-Pollard Model and the Perks Model to Japanese Mortality

To compare the above results with those by parameterized models, the Heligman-Pollard model was applied to mortality for all causes, and the Perks model for selected causes of death.

Heligman-Pollard model:

$$qx = A^{(X+B)^C} + D \cdot exp\,[\,-E(\ln x - \ln F)^2] + GH^X / (1-GH^X)$$

Perks model:

$$\overline{q}x^{(\alpha)} = \frac{A + BC^X}{KC^{-X} + 1 + DC^X}$$

Nonlinear curve-fitting techniques by the program UNABR (Mortpak, U.N.) and the program NLIN(SAS) were used for our computation. Table 11 and 12 show the averages of MAE and RMSE values describing goodness of fit. These values in Tables 11 and 12 are only little smaller than those by new relational models.

This means that new relational models with the small number of parameters are very effective. Tables A and B in Appendix show the parameters and goodness of fit of parameterized models. We could not get the close fit to l_x for all causes combined after the year 1975. This is because the age pattern of Japanese mortality has slightly changed recently. Therefore, we may have to modify slightly the parameterized models for these data.[12)]

Table 11. The Average of the Eight MAE(RMSE) Values of Fit to q_x and l_x for the Years 1950, 1955,..., and 1985 when Using the Heligman -Pollard Model (All causes, Japan)

Sex	q_x	l_x	
	MAE	MAE	RMSE
Female	0.0566	0.0023	0.0036
Male	0.0394	0.0025	0.0036

Table 12. The Average of the Eight MAE(RMSE) Values of Fit to $\bar{q}_x^{(\alpha)}$ and $\bar{l}_x^{(\alpha)}$ for the Years 1950, 1955,..., and 1985 when Using the Perks Models (Malignant neoplasms, heart disease and cerebrovascular disease, Japan)

Cause of death	Sex	$\bar{q}_x^{(\alpha)}$	$\bar{l}_x^{(\alpha)}$	
		MAE	MAE	RMSE
Malignant neoplasms	F	0.0821	0.0010	0.0015
	M	0.0784	0.0016	0.0032
Heart disease	F	0.1500	0.0018	0.0042
	M	0.0818	0.0015	0.0031
Cerebrovascular disease	F	0.0544	0.0014	0.0027
	M	0.0521	0.0016	0.0026

VIII. Conclusion

The use of parameterized models, such as the Heligman-Pollard model, is desirable for projecting or forecasting mortality schedules, but it is sometimes difficult to find the set of suitable parameters for mortality for all causes and much more for mortality for each cause of death. This paper, therefore, has attempted to obtain relational models that provide a closer fit to l_x. Of these generalized models, the useful ones are: the generalized three-parameter Brass model, the generalized three-parameter Ewbank et al. model and the generalized three-parameter model with two standards using the logit (or angular) transformation. If we use suitable standards $l_{s,x}^{(\alpha)}$ for each cause of death, our new models may be more effective for our purpose. The work reported in this paper is only the first step in a sequence that should make operational the use of relational models in modeling cause-specific mortality.

Finally, we need to study what kinds of causes of death should be taken, in appling our new models to cause-specific mortality in the future study of mortality.

Appendix Table A. Parameters of the Heligman-Pollard Mortality Schedule (All causes, Japan, 1950-1985)

Sex	Years	Parameters								q_x	l_x	
		A	B	C	D	E	F	G	H	MAE	MAE	RMSE
	1950	0.024052	0.348615	0.271943	0.004328	6.17394	25.80774	0.000142	1.092191	0.01156	0.00110	0.00110
	1955	0.008933	0.129279	0.192979	0.001990	8.23111	24.37535	0.000090	1.097154	0.01543	0.00109	0.00113
	1960	0.005100	0.064459	0.159959	0.001555	7.86940	23.80105	0.000064	1.102944	0.01176	0.00070	0.00093
M	1965	0.002656	0.033820	0.125310	0.000955	5.99626	24.67790	0.000048	1.106583	0.01559	0.00127	0.00148
	1970	0.002017	0.040827	0.120855	0.000843	6.92934	22.51011	0.000051	1.103596	0.03164	0.00267	0.00331
	1975	0.001716	0.070896	0.130659	0.000713	13.45540	20.66833	0.000051	1.100364	0.07315	0.00471	0.00715
	1980	0.001404	0.095673	0.133417	0.000606	14.64712	20.48907	0.000042	1.101557	0.07481	0.00440	0.00691
	1985	0.001059	0.110432	0.129880	0.000645	16.09204	20.11137	0.000039	1.100827	0.08106	0.00387	0.00678
	1950	0.026120	0.453257	0.282254	0.004226	3.66608	27.31114	0.000071	1.097855	0.02625	0.00181	0.00193
	1955	0.008782	0.165736	0.200368	0.001694	3.84285	28.84212	0.000044	1.102285	0.02443	0.00155	0.00194
	1960	0.004519	0.085049	0.164469	0.001104	3.98817	28.23432	0.000030	1.108221	0.04336	0.00277	0.00381
F	1965	0.002208	0.050349	0.132424	0.000792	2.69647	33.80133	0.000015	1.117626	0.04610	0.00242	0.00386
	1970	0.001638	0.053679	0.123641	0.000582	2.45462	32.36588	0.000014	1.116936	0.04427	0.00231	0.00383
	1975	0.001365	0.074575	0.126952	0.000550	1.62365	38.85328	0.000008	1.122231	0.05836	0.00244	0.00462
	1980	0.000960	0.079821	0.127807	0.000397	1.42104	40.00000	0.000007	1.120644	0.09112	0.00249	0.00402
	1985	0.000862	0.137368	0.147415	0.000304	0.93781	40.00000	0.000007	1.115927	0.11896	0.00256	0.00475

Note: Mean absolute error: $$MAE = \frac{\Sigma \mid \hat{f}_x - f_x \mid}{\Sigma f_x},$$

Root mean square error: $$RMSE = \sqrt{\frac{\Sigma(\hat{f}_x - f_x)^2}{n}},$$

where $\hat{f}_x$ is the fitted value and f_x is the actual value, both at age x.

Appendix Table B. Parameters of the Perks Mortality Schedule for the Three Biggest Causes of Death in Japan, 1950-1985

Causes of Death	Sex	Years	Parameters A	B	C	D	K	$\bar{q}_x^{(\alpha)}$ MAE	$\bar{l}_x^{(\alpha)}$ MAE	RMSE
Malignant neoplasms	M	1950	0.000015	0.000000	1.171834	0.000128	-0.705822	0.15854	0.00363	0.00740
		1955	0.000028	0.000001	1.165448	0.000112	-0.509735	0.11129	0.00217	0.00443
		1960	0.000028	0.000001	1.153096	0.000131	-0.605532	0.08458	0.00200	0.00411
		1965	0.000038	0.000002	1.145911	0.000137	-0.333929	0.06134	0.00128	0.00263
		1970	0.000033	0.000002	1.139942	0.000146	-0.596636	0.04937	0.00100	0.00208
		1975	0.000033	0.000002	1.138764	0.000123	-0.502134	0.03421	0.00053	0.00105
		1980	0.000039	0.000001	1.145722	0.000080	0.042179	0.04292	0.00091	0.00142
		1985	0.000037	0.000001	1.151073	0.000057	-0.057490	0.08463	0.00137	0.00249
	F	1950	-0.000001	0.000003	1.140838	0.000594	-0.949602	0.11194	0.00166	0.00244
		1955	0.000015	0.000001	1.159583	0.000308	-0.733719	0.10419	0.00120	0.00181
		1960	0.000014	0.000003	1.135361	0.000539	-0.794188	0.08652	0.00099	0.00150
		1965	0.000015	0.000004	1.127751	0.000579	-0.726146	0.07868	0.00091	0.00140
		1970	0.000013	0.000005	1.121869	0.000561	-0.750262	0.07060	0.00078	0.00123
		1975	0.000011	0.000005	1.119463	0.000516	-0.733610	0.06881	0.00085	0.00128
		1980	-0.000007	0.000009	1.099724	0.000539	-0.951436	0.03321	0.00060	0.00083
		1985	0.000011	0.000003	1.119141	0.000326	-0.693082	0.10273	0.00102	0.00165
Heart disease	M	1950	0.000128	0.000003	1.117277	0.000244	2.350824	0.04925	0.00074	0.00140
		1955	0.000006	0.000009	1.093368	0.000007	-0.698362	0.12598	0.00219	0.00463
		1960	-0.000059	0.000010	1.090976	-0.000352	-5.781881	0.16567	0.00364	0.00783
		1965	-0.000005	0.000007	1.096935	-0.000221	-0.176221	0.12020	0.00246	0.00532
		1970	-0.000004	0.000004	1.106857	-0.000057	-0.916820	0.05114	0.00091	0.00164
		1975	-0.000004	0.000004	1.103619	-0.000103	-0.997483	0.04427	0.00064	0.00115
		1980	-0.000003	0.000005	1.103965	-0.000084	-0.988248	0.04732	0.00072	0.00149
		1985	-0.000003	0.000004	1.103248	-0.000115	-0.990845	0.05039	0.00058	0.00109
	F	1950	-0.000059	0.000047	1.062364	-0.001681	-1.243179	0.12553	0.00178	0.00343
		1955	-0.000013	0.000030	1.066155	-0.002528	-0.187953	0.15034	0.00189	0.00394
		1960	-0.000022	0.000016	1.076300	-0.001412	-1.329907	0.17433	0.00210	0.00467
		1965	-0.000007	0.000006	1.090494	-0.000461	-0.928839	0.15160	0.00210	0.00488
		1970	-0.000003	0.000004	1.095648	-0.000302	-0.107165	0.12701	0.00158	0.00374
		1975	-0.000002	0.000002	1.099992	-0.000251	-0.995579	0.18029	0.00191	0.00503
		1980	0.000002	0.000001	1.112557	-0.000073	-0.968158	0.11423	0.00126	0.00322
		1985	0.000001	0.000001	1.107141	-0.000135	-0.979626	0.17656	0.00174	0.00469
Cerebro-vascular disease	M	1950	-0.000004	0.000001	1.160542	0.000019	-0.669370	0.02349	0.000711	0.00111
		1955	-0.000001	0.000000	1.162429	0.000018	-1.494993	0.04055	0.00218	0.00353
		1960	-0.000000	0.000000	1.164941	0.000012	-1.066831	0.03764	0.00374	0.00569
		1965	-0.000002	0.000001	1.157207	0.000014	-1.704481	0.01610	0.00077	0.00130
		1970	-0.000001	0.000001	1.152284	0.000012	-1.284152	0.02865	0.00095	0.00166
		1975	-0.000001	0.000001	1.151130	0.000010	-1.310797	0.07055	0.00159	0.00247
		1980	-0.000001	0.000001	1.145052	0.000007	-1.299901	0.06165	0.00139	0.00218
		1985	0.000000	0.000000	1.146270	0.000008	-0.922559	0.13822	0.00142	0.00274
	F	1950	-0.000000	0.000001	1.144909	0.000003	-1.096907	0.03346	0.00224	0.00343
		1955	-0.000000	0.000000	1.163925	0.000019	-0.961367	0.05006	0.00103	0.00163
		1960	-0.000000	0.000000	1.161215	0.000012	-1.086613	0.02630	0.00072	0.00124
		1965	-0.000001	0.000001	1.155042	0.000008	-1.315841	0.05434	0.00254	0.00500
		1970	-0.000001	0.000001	1.154283	0.000004	-1.230642	0.06489	0.00200	0.00468
		1975	-0.000000	0.000001	1.143731	0.000003	-0.992463	0.08008	0.00126	0.00326
		1980	-0.000001	0.000000	1.141115	-0.000002	-1.282975	0.05360	0.00062	0.00125
		1985	-0.000001	0.000001	1.128386	-0.000016	-1.817905	0.07765	0.00056	0.00123

References

1. Brass, W. (1971) "On the Scale of Mortality", in Biological Aspect of Demography, ed. W. Brass. London: Taylor and Francis, 86-110.
2. Coale, A. J. and Demeny, P. (1966) Regional Model Life Tables and Stable Populations. Princeton, N. J.: Princeton University Press.
3. Ewbank, D. C., J. C. Gomez de Leon and Stoto, M. A. (1983) "A Reducible Four-Parameter System of Model Life Tables," Population Studies, 37(3), 105-127.
4. Heligman, L. and Pollard, J. H. (1980) "The Age Pattern of Mortality," Journal of the Institute of Actuaries, 107: 49-80.
5. Kaneko, R. (1987) "Development of New Relational Models for Age Pattern of Mortality," Jinko Mondai Kenkyu (Journal of Population Problem) No. 183, 1-22 (in Japanese).
6. Keyfitz, N. (1977) Applied Mathematical Demography, Wiley.
7. Keyfitz, N. (1982) "Choice of Function for Mortality Analysis: Effective Forecasting Depends on a Minimum Parameter Representation," Theoretical Population Biology, 21, 329-352.
8. Keyfitz, N. (1988) "Some Experiments in the Fitting and Projection of Mortality," unpublished ms.
9. Keyfitz, N. (1989) "On Future Mortality," WP-89-59, IIASA.
10. McNown, R. F. and Rogers, A. (1989) "Forecasting Mortality: A Parameterized Time Series Approach," Demography, 26(4), 645-660.
11. Nanjo, Z., Shigematsu, T. and Yoshinaga, K. (1990) "Parameterized Model Schedule of Mortality for Japan - All and Selected Causes of Death," Jinkogakukenkyu (Journal of Population Studies) No. 13, (in Japanese).
12. Sobolev, Leonid, (1989) USSR Census 1989, Novosti Press Agency Publishing House, Moscow.