

Note

Trend Analysis of Nikkei 225 Futures Price

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1 Introduction

It is difficult to explicitly define a trend in stock investment. In general, investors use the 25-day moving average line and the 26-week moving average line for trend analysis. However, these are not so practical, and most investors use them only for reference. If there is a trend, it may be possible to identify an upward (bull regime) or downward (bear regime) trend with a statistical model. A representative analytical model is the Markov-switching model. In most cases of the trend analysis of asset prices with the Markov-switching model, trends are divided into bull and bear phases. The mean of the change rate of a stock price is positive or negative, and if the mean remains positive, it is called an upward (bull market) trend, while if the mean remains negative, it is called a downward (bear market) trend. It is assumed that these two states follow the Markov process. However, Maheu *et al.* (2012) considered that investors further classify market trends, including short-term ones, and proposed a 4-state Markov-switching model, in which trends are classified into bear market state, bear market rally, bull market state, and bull market correction phases. This research outlines the 4-state Markov-switching model, and empirically studies Nikkei 225 futures. In trend analysis, stock price indices, such as the Nikkei Stock Average and TOPIX, are used¹⁾. The data of Nikkei 225 futures is used, because nowadays the stock market often fluctuates wildly as the prices of futures change due to hedge funds.

This note is composed of the following sections. Section 2 describes the 4-state Markov-switching model in which trends are classified into bear, rally, bull, and correction phases. Section 3 mentions the data of Nikkei 225 futures used in this study and the results of the empirical research. Lastly Section 4 summarizes this study.

2 Analytical Model

Let R_t describe the rate of return on asset price at time t , 4-state Markov-switching model by Maheu *et al.* (2012) can be represented as follows:

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¹⁾ Shibata (2012) analyzed the trends of TOPIX, while Satoyoshi and Mitsui (2011), Satoyoshi and Mitsui (2012), and Mitsui (2012) analyzed the trends of the Nikkei Stock Average.

$$R_t = \mu + \sigma z_t, \quad z_t \sim i.i.d., E[z_t] = 0, V[z_t] = 1, \quad (2.1)$$

$$\mu = \mu_1 S_{1t} + \mu_2 S_{2t} + \mu_3 S_{3t} + \mu_4 S_{4t}, \quad (2.2)$$

$$\sigma = \sigma_1 S_{1t} + \sigma_2 S_{2t} + \sigma_3 S_{3t} + \sigma_4 S_{4t}, \quad (2.3)$$

$$p_{ij} = \Pr[S_t = j | S_{t-1} = i], \quad i, j = 1, 2, 3, 4, \quad (2.4)$$

$$\sum_{j=1}^4 p_{ij} = 1, \quad i = 1, 2, 3, 4, \quad (2.5)$$

where the constant term μ is the mean, z_t is the error term, and the assumption is that there is no autocorrelation in returns. *i. i. d.* expresses an independently and identically distributed. S_t is a state variable that follows a first-order Markov process. $(S_{1t}, S_{2t}, S_{3t}, S_{4t})$ is $(1, 0, 0, 0)$ when $S_t = 1$, $(0, 1, 0, 0)$ when $S_t = 2$, $(0, 0, 1, 0)$ when $S_t = 3$ and $(0, 0, 0, 1)$ when $S_t = 4$. Also, the assumption is that mean μ and volatility σ follow S_t and switch simultaneously. The following restrictions apply to μ .

$$\mu_1 < 0 \text{ (bear market state)}, \quad (2.6)$$

$$\mu_2 > 0 \text{ (bear market rally)}, \quad (2.7)$$

$$\mu_3 < 0 \text{ (bull market correction)}, \quad (2.8)$$

$$\mu_4 > 0 \text{ (bull market state)}. \quad (2.9)$$

A bear market rally is a temporary rise during a long-term downward trend. A bull market correction is a temporary fall during a long-term upward trend. The transition matrix of S_t is

$$P = \begin{pmatrix} p_{11} & p_{12} & 0 & p_{14} \\ p_{21} & p_{22} & 0 & p_{24} \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & p_{43} & p_{44} \end{pmatrix}, \quad (2.10)$$

where p_{14} and p_{24} represent the transition probabilities from bear and bull phases, while p_{41} and p_{31} denote the transition probabilities from bull to bear phases. Even if there is a rally, a temporary rise in price, during a bear phase, the bear phase is a downward trend from the long-term viewpoint. Likewise, a bull phase is an upward trend from the long-term viewpoint. Namely, the conditions $E[R_t | S_t = 1, 2] < 0$, $E[R_t | S_t = 3, 4] > 0$ need to be satisfied. Therefore, when the steady-state probability of a Markov process is defined as follows:

$$\pi = \begin{bmatrix} \Pr[S_t = 1] \\ \Pr[S_t = 2] \\ \Pr[S_t = 3] \\ \Pr[S_t = 4] \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix}, \quad (2.11)$$

it is necessary to satisfy the following conditions:

$$E[R_t|S_t = 1, 2] = \frac{\pi_1}{\pi_1 + \pi_2}\mu_1 + \frac{\pi_2}{\pi_1 + \pi_2}\mu_2 < 0, \quad (2.12)$$

$$E[R_t|S_t = 3, 4] = \frac{\pi_3}{\pi_3 + \pi_4}\mu_3 + \frac{\pi_4}{\pi_3 + \pi_4}\mu_4 > 0. \quad (2.13)$$

The steady-state probability π is obtained from

$$\pi = (A'A)^{-1}A'e, \quad (2.14)$$

where $A' = [P' - I, \iota]$, $e' = [0, 0, 0, 0, 1]$, $\iota = [1, 1, 1, 1]'$.

Maheu *et al.* (2012) conducted the Bayesian estimation with the Markov chain Monte Carlo method, but model parameters can be estimated with the maximum likelihood method. In this study, parameters are estimated with the maximum likelihood method for simplicity. In the empirical analysis of this study, in addition to the restriction of Eqs. (2.6)–(2.10), (2.12), and (2.13), the following condition is added for the diagonal elements of the transition matrix of Eq. (2.10):

$$p_{11}, p_{22}, p_{33}, p_{44} > 0.9. \quad (2.15)$$

Then, it is assumed that a phase continues for a certain period of time. In order to carry out the maximum likelihood method so as to satisfy the condition expressed by Eq. (2.15), p_{11} , p_{12} , and p_{14} are formulated as follows:

$$p_{11} = \frac{0.9 + \exp(x_1)}{1 + \exp(x_1)}, \quad (2.16)$$

$$p_{12} = \frac{\exp(x_2)}{1 + \exp(x_2)}(1 - p_{11}), \quad (2.17)$$

$$p_{14} = 1 - p_{11} - p_{12}, \quad (2.18)$$

and x_1 and x_2 are estimated, instead of p_{11} and p_{12} . This method can be applied to other transitional probabilities. For the distribution of z_t , the following standard normal distribution is assumed.

$$z_t \sim i.i.d.N(0, 1). \quad (2.19)$$

3 Data and Empirical Results

3.1 Data

In this research, the Nikkei 225 futures price²⁾ handled at Osaka Stock Exchange was used, and its data was taken from Nikkei NEEDS-Financial Quest. The sample period is from January 4, 1996 to September 30, 2013 (see Fig. 1)³⁾, and the number of samples is 4,222. The rate of return was calculated as the rate of change in the closing price of Nikkei 225 futures [%] (see Fig. 2). As the summary statistics of data, mean, standard deviation, skewness, kurtosis, maximum, minimum, and normality test statistic⁴⁾ are tabulated in Table 1. Since kurtosis exceeds 3 and the normality test was significant, it is obvious that the distribution of the rate of return of Nikkei 225 futures has thicker tails than the normal distribution. The histogram and density function of the rate of return are shown in Fig. 3. In this figure, the density and normal distributions are superimposed. According to Table 1, $N(s = 1.639)$ follows the normal distribution $N(-0.008, 1.639)$ with a mean of -0.008 and a variance of 1.639.



Figure 1 : Daily Closing Prices (1 / 4/1996 – 9/30/2013)

²⁾ The Nikkei 225 futures traded from 9:00 to 15:15 were studied. The Nikkei 225 futures traded during the night session from 16:00 to 3:00 on the following day were not studied. The Nikkei 225 futures traded at the Chicago Mercantile Exchange (CME) and Singapore Exchange Derivatives Trading Limited (SGX-DT) were excluded.

³⁾ In this study, diagrams were produced with PcGive (software for statistical time-series analysis). For the details of PcGive, refer to Doornik and Hendry (2013).

⁴⁾ The method proposed by Jarque and Bera (1987), in which skewness and kurtosis are used, was used for testing the normality of the distribution of the rate of return. For further information, see Jarque and Bera (1987).

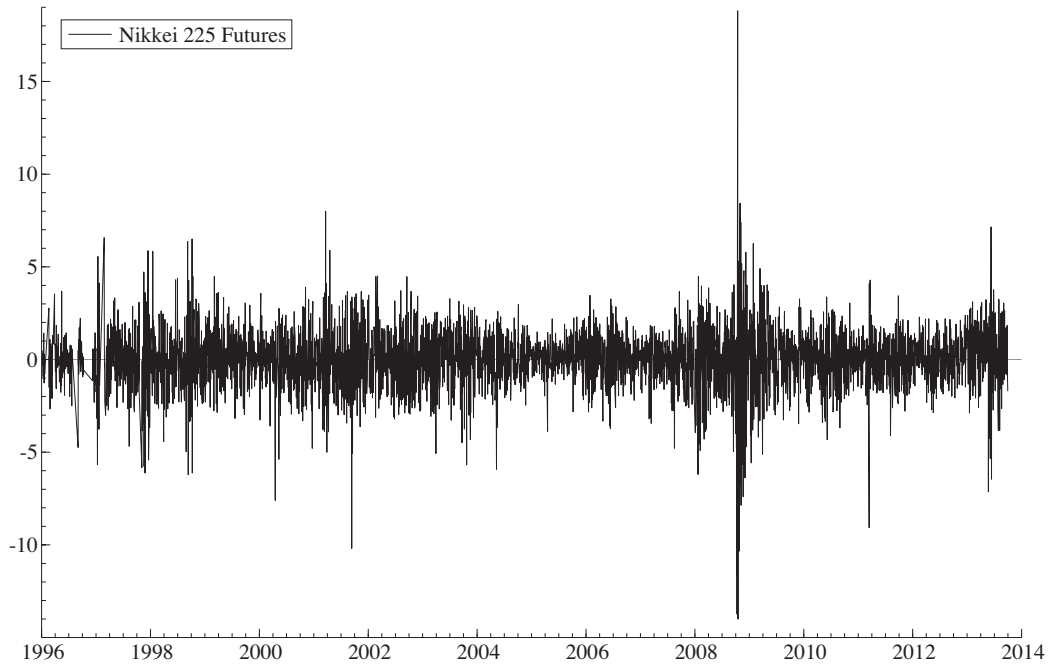


Figure 2 : Daily Return Sries (1 / 5 /1996 – 9 /30 /2013)

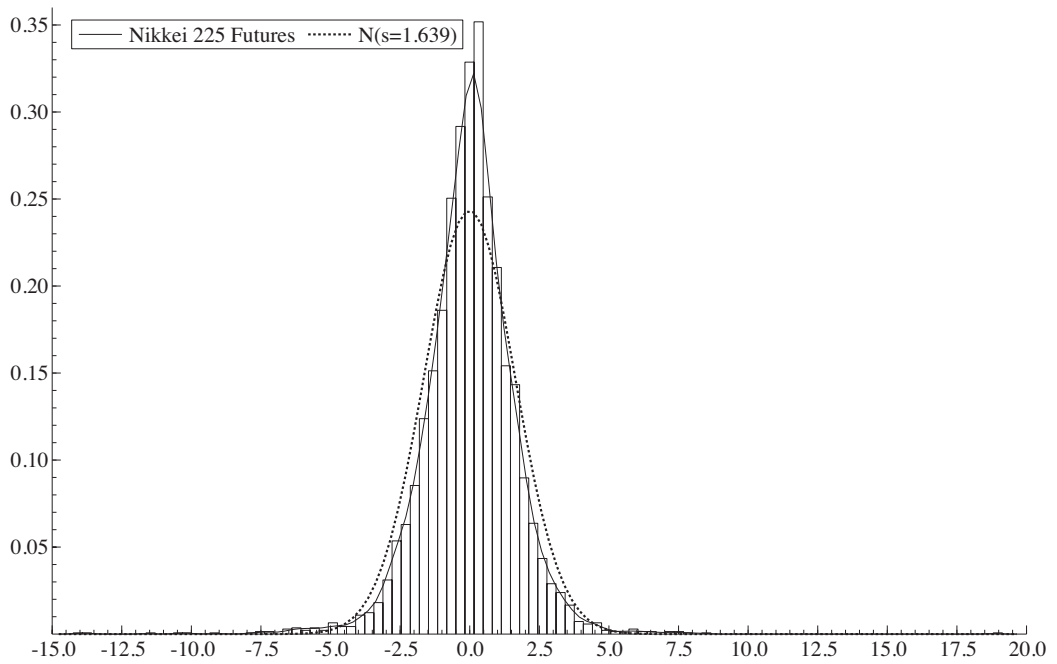


Figure 3 : Histogram and Estimated Density with Normal Approximation (1 / 5 /1996 – 9 /30 /2013)

Table 1: Summary Statistics for the Nikkei 225 Futures Daily Returns R_t (%)

Sample Period: Jan. 5, 1996 – Sep. 30, 2013

Sample Size	Mean	Std Dev.	Skewness	Kurtosis	Max.	Min.	Normality test
4,222	- 0.008	1.639	- 0.244	12.607	18.812	- 14.003	3954.7**

** denotes statistical significance at the 1% level.

3.2 Empirical Results

Table 2 shows the model estimation results. The continuity of transition probabilities indicates that p_{11} is near the lower limit 0.9 while the other estimates p_{22} , p_{33} , and p_{44} are nearly equal to 1, and the rally ($S_t = 2$), correction ($S_t = 3$), and bull ($S_t = 4$) phases are continuous. The transition probabilities from a bear regime (bear and rally phases) to a bull phase are $p_{14} = 0.012$ and $p_{24} = 0.033$, and the bull phase is transitioned from only the rally phase, not the bear phase. The transition probabilities from a bull regime (correction and bull phases) to a bear phase are $p_{31} = 0.031$ and $p_{41} = 0.024$, and a bear phase is transitioned from mainly a correction phase. The significant mean is $\mu_4 = 0.358$ only, but the signs of the other parameters are considered to represent the trend of each phase. Volatility is highest in bear phases and lowest in bull phases, and nearly constant in rally and correction phases, but volatility is a little higher in rally phases. Namely, volatility is higher in a bear state, which is the same as the results of many previous studies.

Table 2: Estimation Results for the 4-state Markov-switching Model

$$\begin{aligned}
 R_t &= \mu + \sigma z_t, \quad z_t \sim i.i.d.N(0, 1), \\
 \mu &= \mu_1 S_{1t} + \mu_2 S_{2t} + \mu_3 S_{3t} + \mu_4 S_{4t}, \\
 \sigma &= \sigma_1 S_{1t} + \sigma_2 S_{2t} + \sigma_3 S_{3t} + \sigma_4 S_{4t}.
 \end{aligned}$$

$$P = \begin{pmatrix} p_{11} & p_{12} & 0 & p_{14} \\ p_{21} & p_{22} & 0 & p_{24} \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & p_{43} & p_{44} \end{pmatrix}.$$

	p_{11}	p_{12}	p_{21}	p_{22}	p_{31}	p_{33}	p_{43}	p_{44}
Estimates	0.912	0.088	0.005	0.962	0.031	0.969	0.044	0.932
Standard Errors	0.093	0.098	0.011	0.015	0.035	0.098	0.076	0.064

	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	$\ln L$
Estimates	- 0.490	0.034	- 0.581	0.358	4.837	1.941	1.295	0.941	- 6385.25
Standard Errors	0.521	0.199	0.369	0.158	0.512	0.285	0.211	0.169	

4 Summary

In this study, the author conducted an empirical analysis by using the 4-state Markov switching model developed by Maheu *et al.* (2012). However, it is necessary to compare and discuss this model and other models in effectiveness to a sufficient degree⁵⁾. For example, it is necessary to discuss the effectiveness of the 4-state Markov switching model by comparing bull and bear turning points, which are determined with another method, and the turning point of the model of this research. With the Markov switching model, it is possible to predict next day's bull and bear trends with the information collected up to the present, and so the feasibility of trend prediction is intriguing.

References

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⁵⁾ For the details of model extension, refer to Mitsui (2013).