Bull and Bear Market Analysis of the Nikkei 225 Futures and TOPIX Futures *

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Abstract

This paper analyzes bull and bear markets of the Nikkei 225 Futures and TOPIX Futures using the MS-ARMA-GARCH model. Empirical analysis exhibited statistically significant bull and bear market regimes in the Nikkei 225 Futures and TOPIX Futures. It was made clear that the bear market regime has higher volatility than the bull market regime. In conclusion, it can be said that the MS-ARMA-GARCH model and MS-GARCH model are valid methods of analysis for the Nikkei 225 Futures and TOPIX Futures.

1 Introduction

If it is possible to predict upward and downward trends in the stock market, investors would be able to hold long positions during upward trends and short positions during downward trends. However, it is difficult to define trends in general, and a variety of analysis methods have been developed. For basic trend analysis many investors use moving averages – such as the 25-day, 75-day, 13-week or 26-week moving averages – or the difference from the moving average. However, these methods are not practical, and most investors use them merely for reference. It is difficult to accurately identify the turning point between upward and downward trends using these indicators alone.

If trends do exist, it should be possible to observe upward and downward trends – so-called bull and bear markets – using a time series model. One common trend analysis model is the Markov-switching model. When using the Markov-switching model for trend analysis, market trends are first separated into the two regimes: bull and bear. The mean of the change rate of stock prices has two states, negative or positive. If the positive value continues, this indicates an upward trend (bull market), and if the negative value continues, this indicates a downward trend (bear market). The assumption is that these two states follow the transition of a Markov process. In general, a model of changing volatility[1)] is used for time series analysis of asset prices. Studies using the Markov-

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switching model, a model of changing volatility, include Hamilton and Susmel (1994) and Cai (1994), which use the Markov-switching ARCH (Autoregressive conditional heteroscedasticity) model, and Gray (1996), Klaassen (2002), and Haas *et al.* (2004), which use the Markov-switching GARCH (Generalized ARCH) model (hereinafter, the MS-GARCH model)[2]. In the Japanese stock market, Satoyoshi (2004) uses the Markov-switching model on the TOPIX, and Satoyoshi and Mitsui (2011b, 2012) use the same on the Nikkei Average.

In recent years, the stock market has seen violent fluctuations due to hedge funds and future driven market prices by institutional investors. Therefore, this paper analyzes bull and bear markets of the Nikkei 225 Futures and TOPIX Futures, instead of stock price indexes such as the Nikkei Average and TOPIX. Empirical analysis was conducted using daily data of the Nikkei 225 Futures and TOPIX Futures from April 1, 2000, to September 30, 2013. The MS-ARMA-GARCH (Markov-switching Autoregressive moving average GARCH) model was used. Empirical analysis showed statistically significant bull and bear regimes in the Nikkei 225 Futures and TOPIX Futures. In other words, we were able to identify bull markets, with high expected returns and low volatility, and bear markets, with low expected returns and high volatility. It was also made clear that the MS-ARMA-GARH model and MS-GARCH model are valid models for the analysis of bull and bear markets in the Nikkei 225 Futures and TOPIX Futures.

The brief descriptions of the following chapters are as follows: Chapter 2 explains the MSARMA- GARCH model and the comparison models. Chapter 3 shows the empirical results on the Nikkei 225 Futures and TOPIX Futures data. Chapter 4 concludes the study and considers future issues.

2　Methodology

2.1　MS-ARMA (1,1)-GARCH (1,1) Model

In this chapter, we explain briefly the MS-ARMA-GARCH model based on the MS-GARCH model by Klaassen (2002) and Haas *et al.* (2004). In addition, since many empirical studies state that performance is not improved so much even if the order of the ARMA and volatility fluctuation processes are increased, we use the MS-ARMA(1,1)-GARCH(1,1) model in this study.

When R_t is the rate of return on asset price at time t, the process of R_t and volatility σ_t^2 can be expressed as follows:

$$R_t = \mu(s_t) + \phi(s_t) R_{t-1} + \epsilon_t(s_t) + \psi(s_t)\,\epsilon_{t-1}(s_t), \tag{2.1}$$

$$\epsilon_t(s_t) = \sigma_t(s_t) z_t,\ z_t \sim i.i.d.,\ E[z_t]=0,\ Var[z_t]=1, \tag{2.2}$$

$$\sigma_t^2(s_t) = \omega(s_t) + \alpha(s_t)\,\epsilon_{t-1}^2(s_t) + \beta(s_t)\sigma_{t-1}^2(s_t), \tag{2.3}$$

1)　Volatility is defined based on the variance or standard deviation of the return on asset, and is used as the index of the risk of risky assets in finance theory.

2)　Engle (1982) proposed the ARCH model that formulates the volatility at each time as the linear function of the square of the past unexpected shock. In addition, Bollerslev (1986) added the past volatility values to the explanatory variables, and extended the GARCH model to a more general model.

$$\epsilon_{t-1}(s_t) = E[\epsilon_{t-1}(s_{t-1})|s_t, I_{t-1}],$$
$$\sigma^2_{t-1}(s_t) = E[\sigma^2_{t-1}(s_{t-1})|s_t, I_{t-1}].$$

Here, $\mu(s_t)$ represents a constant term and z_t depicts an error term, and it is assumed that there is no autocorrelation in the rate of return. *i.i.d.* means independent and identically distributed. $E[\cdot]$, $Var[\cdot]$ and $E[\cdot|\cdot]$ represent the expected value, variance and conditional expectation. Volatility σ_t^2 is the conditional variance of ϵ_t with the information set $I_{t-1} = \{R_{t-1}, R_{t-2}, \cdots\}$ until time $t-1$ and the state variable s_t at time t being the conditions, that is, $\sigma_t^2 = Var[\epsilon_t|I_{t-1}, s_t]$. It is assumed that the constant term $\mu(s_t)$ and volatility $\sigma_t(s_t)$ follow random variable s_t and switch simultaneously. In addition, to ensure positivity of the volatility, it is assumed that $\omega(s_t)$, $\alpha(s_t)$, $\beta(s_t) \geq 0$. In the Markov-switching model, the unobserval random variable s_t follows a Markov process, and can be defined with the following transition probabilities.

$$p_{i|j} = Pr[s_{t+1} = i|s_t = j],\ i, j = 0, 1. \tag{2.4}$$

Here, $Pr[s_{t+1} = i|s_t = j]$ expresses the probability of the transition from state j to state i. So the probability of moving from state j in one period to state i in the next only depends on the previous state, as shown below[3)].

$$Pr[s_{t+1} = i|s_t = j, s_{t-1}, s_{t-2}, \cdots] = p_{i|j} = Pr[s_{t+1} = i|s_t = j]. \tag{2.5}$$

Here, because the system has to be in one of the 2 states we have that:

$$\sum_{i=0}^{1} p_{i|j} = 1,\ j = 0, 1. \tag{2.6}$$

Then the matrix of transition probabilities $\mathbf{P}$ for $s_t = 2$ is:

$$\mathbf{P} = \begin{pmatrix} p_{0|0} & p_{0|1} \\ p_{1|0} & p_{1|1} \end{pmatrix}. \tag{2.7}$$

Here, $0 \leq p_{0|0}, p_{1|1} \leq 1$.

This study considers the condition $s_t = 0$ a bull market and $s_t = 1$ a bear market. Therefore, $p_{0|1}$ is the transition probability from bull to bear market, and $p_{1|0}$ is the transition probability from bear to bull market. Moreover, $p_{0|0}$ and $p_{1|1}$ represent the transition probabilities of a maintained bull market and maintained bear market, respectively. The restriction $\mu(0) > \mu(1)$ is imposed[4)].

In our empirical analysis, the distribution of the error term is assumed to follow the standard

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3) This can also be expressed as: $p_{ji} = Pr[s_{t+1} = i|s_t = j]$.

4) $\mu(0) > 0$, $\mu(1) < 0$ does not always hold depending on the asset data.

normal distribution as shown below[5].

$$z_t \sim i.i.d.N\,(0,\,1). \tag{2.8}$$

Therefore, the estimated parameters are $\{\mu(0),\ \mu(1),\ \phi(0),\ \phi(1),\ \psi(0),\ \psi(1),\ \omega(0),\ \omega(1),\ \alpha(0),\ \alpha(1),\ \beta(0),\ \beta(1),\ p_{0|0},\ p_{1|1}\}$. Henneke *et al.* (2011) conducted the Bayesian estimation with the Markov chain Monte Carlo method for the estimation of MS-ARMA-GARCH model, but model parameters can be estimated with the maximum likelihood method. In this study, parameters are estimated with the maximum likelihood method for simplicity.

2.2 Model for Comparison

In order to identify the validity of the MS-ARMA(1,1)-GARCH(1,1) model, empirical analysis was conducted on the following models with different formularizations, which are included in the MS-ARMA(1,1)-GARCH(1,1) model.

(*i*) MS-ARMA(1,1)-GARCH(1,1)-c model: specifying as Eqs. (2.2), (2.3), (2.7), (2.8) and

$$R_t = \mu(s_t) + \phi R_{t-1} + \epsilon_t(s_t) + \psi\epsilon_{t-1}. \tag{2.9}$$

In the MS-ARMA(1,1)-GARCH(1,1)model, the ARMA coefficients ϕ and ψ are the fixed parameters.

(*ii*) MS-ARMA(1,0)-GARCH(1,1) model: specifying as Eqs. (2.2), (2.3), (2.7), (2.8) and

$$R_t = \mu(s_t) + \phi(s_t) R_{t-1} + \epsilon_t(s_t). \tag{2.10}$$

(*iii*) MS-ARMA(1,0)-GARCH(1,1)-c model: specifying as Eqs. (2.2), (2.3), (2.7), (2.8) and

$$R_t = \mu(s_t) + \phi R_{t-1} + \epsilon_t(s_t). \tag{2.11}$$

In the MS-ARMA(1,0)-GARCH(1,1) model, the ARMA coefficient ϕ is a fixed parameter.

(*iv*) MS-GARCH(1,1) model: specifying as Eqs. (2.2), (2.3), (2.7), (2.8) and

$$R_t = \mu(s_t) + \epsilon_t(s_t). \tag{2.12}$$

(*v*) MS model: specifying as Eqs. (2.2), (2.3), (2.7), (2.8) and

$$R_t = \mu(s_t) + \epsilon_t(s_t). \tag{2.13}$$

5) It is known that the distribution of stock returns follows a distribution with a thicker tail compared to a normal distribution. Therefore, it is necessary to analyze the error term using fat-tailed distributions, such as *t*-distribution and GED (Generalized Error Distribution).

(*vi*) ARMA(1,1)-GARCH(1,1) model: in which the processes of return R_t and volatility σ_t^2 are as follows.

$$R_t = \mu + \phi R_{t-1} + \epsilon_t + \psi\epsilon_{t-1}, \tag{2.14}$$
$$\epsilon_t = \sigma_t z_t,\ z_t \sim i.i.d.N(0,1), \tag{2.15}$$
$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \tag{2.16}$$

(*vii*) ARMA(1,0)-GARCH(1,1) model: specifying as Eqs. (2.15), (2.16) and

$$R_t = \mu + \phi R_{t-1} + \epsilon_t. \tag{2.17}$$

(*viii*) GARCH(1,1) model: specifying as Eqs. (2.15), (2.16) and

$$R_t = \mu + \epsilon_t. \tag{2.18}$$

2.3 Estimation Method

Let θ denote the set of unknown parameters. When the error term has a normal distribution, $\theta = \{\mu(0),\ \mu(1),\ \phi(0),\ \phi(1),\ \psi(0),\ \psi(1),\ \omega(0),\ \omega(1),\ \alpha(0),\ \alpha(1),\ \beta(0),\ \beta(1),\ p_{0|0},\ p_{1|1}\}$. Let $L(\theta)$ denote the likelihood function. The likelihood function $L(\theta)$ becomes as follows:

$$\begin{aligned} L(\theta) &= f(R_1, R_2, \cdots, R_T) = \prod_{t=1}^{T} f(R_t|I_{t-1};\theta) \\ &= \prod_{t=1}^{T}\sum_{j=0}^{1}(R_t|s_t=j, I_{t-1})\Pr[s_t=j, |I_{t-1}]. \end{aligned} \tag{2.19}$$

Then, the log-likelihood function $\ln L$ can be expressed as

$$\begin{aligned} \ln L &= \sum_{t=1}^{T}\ln\left\{\sum_{j=0}^{1}(R_t|s_t=j, I_{t-1};\theta)\Pr[s_t=j, |I_{t-1};\theta]\right\} \\ &= \sum_{t=1}^{T}\ln\left\{\mathrm{i}'\left(\eta_t \odot \hat{\xi}_{t|t-1}\right)\right\}, \end{aligned} \tag{2.20}$$

where

$$i = \begin{pmatrix}1\\1\end{pmatrix},\ \eta_t = \begin{pmatrix} f(R_t|s_t=0, I_{t-1};\theta) \\ f(R_t|s_t=1, I_{t-1};\theta)\end{pmatrix},\ \hat{\xi}_{t|t-1} = \begin{pmatrix}\Pr[s_t=0, |I_{t-1};\theta] \\ \Pr[s_t=1, |I_{t-1};\theta]\end{pmatrix}.$$

Here, the symbol $\odot$ denotes element-by-element multiplication. $\hat{\xi}_{t|t-1}$ in equation (2.20) is obtain with the filtering method proposed by Hamilton (1989) (Hmilton Filter)[6] .We can express

$$\hat{\xi}_{t|t-1} = (\mathrm{P} \otimes \mathrm{Q})\, \hat{\xi}_{t-1|t-1}, \tag{2.21}$$

$$\hat{\xi}_{t|t} = \frac{\left(\eta_t \odot \hat{\xi}_{t|t-1}\right)}{i'\left(\eta_t \odot \hat{\xi}_{t|t-1}\right)}. \tag{2.22}$$

Here, the symbol $\otimes$ expresses the Kronecker product. By repeating the above equations (2.21) and (2.22) alternately, $\hat{\xi}_{t-1|t-1}$ is calculated for $t = 1, 2, \ldots, T$, and it is substituted into equation (2.20)[7] .

For estimation of the parameters, maximum likelihood estimation is conducted using the statistical and time series analysis software ***PcGive***[8]

3 Data Sources and Empirical Results

3.1 Data Sources

This study uses the Nikkei 225 Futures near maturity contracts listed on the Osaka Securities Exchange[9] . Contracts from 9:00 to 15:15 are used, whereas night sessions from 16:00 to 3:00 of the following day are not[10] . The TOPIX Futures near maturity contracts listed on the Tokyo Stock Exchange are also used[11] . For the TOPIX Futures also, those from 9:00 to 15:15 are used while evening sessions from 16:30 to 23:30 are not. The data was obtained from Nikkei NEEDS-FinancialQuest. The sample period is from April 1, 2000 to September 30, 2013 (see Figs 1 and 2)[12] . The rates of return are calculated as the rate of change [%] of the future contracts at the closing price (see Figs 3 and 4). The sample period is from May 1, 2000 to September 30, 2013, with a total of 3,376 samples.

As the summary statistics of data, mean, standard deviation, skewness, kurtosis, maximum, minimum, and normality test statistic[13] are tabulated in Table 1. Since kurtosis of the rate of return of the Nikkei 225 Futures exceeds 3 and the normality test was significant, it is obvious that the distribution of the rate of return of the Nikkei 225 futures has thicker tails than the normal distribution. The histogram and density function of the rate of return are shown in Fig. 5. In this

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6) For details, refer to Kim and Nelson (1999).

7) For details, refer to Satoyoshi and Mitsui (2011a).

8) For further information on using *PcGive* for Markov-switching estimation, refer to Doornik and Hendry (2013b).

9) See the Osaka Securities Exchange website <http://www.ose.or.jp/derivative/225futures> for further information on the Nikkei 225 Futures.

10) This study also excludes Nikkei 225 Futures tradings at CME (Chicago Mercantile Exchange) and SGX-DT (Singapore Exchange Derivatives Trading Limited).

11) See the Tokyo Stock Exchange website <http://www.tse.or.jp/rules/topixf/topixf7.html> for further information on the TOPIX Futures.

12) In this study, diagrams were produced with *PcGive* (software for statistical time-series analysis). For the details of *PcGive*, refer to Doornik and Hendry (2013a).

13) The method proposed by Jarque and Bera (1987), in which skewness and kurtosis are used, was used for testing the normality of the distribution of the rate of return. For further information, see Jarque and Bera (1987).

Figure 1: Daily Closing Prices on the Nikkei 225 Futures（1/4/2000 – 9/30/2013）

Figure 2: Daily Closing Prices on the TOPIX Futures（1/4/2000 – 9/30/2013）

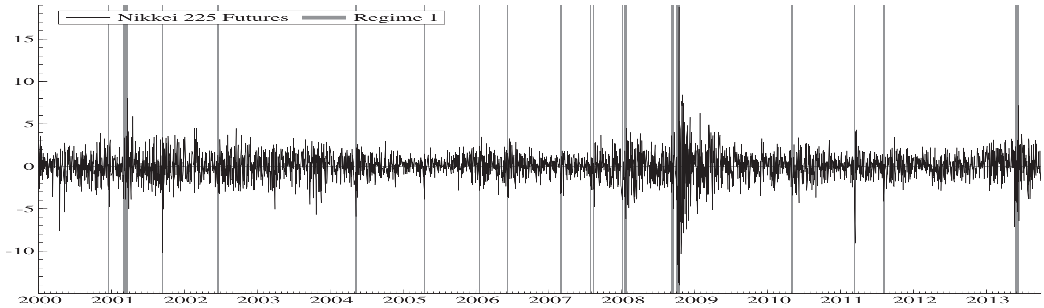

Figure 3: Daily Return Sries on the Nikkei 225 Futures（1/5/2000 – 9/30/2013）

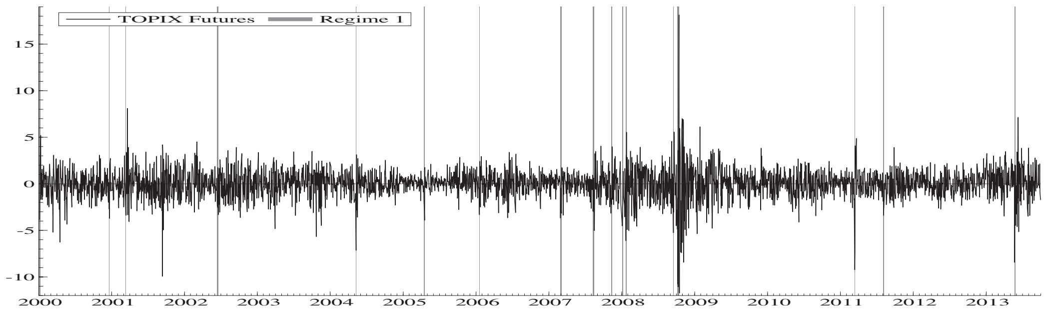

Figure 4: Daily Return Sries on the TOPIX Futures（1/5/2000 – 9/30/2013）

figure, the density and normal distributions are superimposed. According to Table 1, N (s = 1.643) follows the normal distribution N (−0.008, 1.643) with a mean of −0.008 and a variance of 1.643^2.

Since kurtosis of the rate of return of the TOPIX exceeds 3 and the normality test was significant, it is obvious that the distribution of the rate of return of the TOPIX futures also has thicker tails than the normal distribution. The histogram and density function of the rate of return are shown in Fig. 6. According to Table 1, N (s = 1.563) follows the normal distribution N (−0.0011, 1.563) with a mean of −0.0011 and a variance of 1.563^2.

Table 1: Summary Statistics for the Nikkei 225 Futures and the TOPIX Daily Returns Rt (%)

Sample Period: Jan. 5, 2000 – Sep. 30, 2013
Sample Size: 3,376

	Mean	Std Dev.	Skewness	Kurtosis	Max.	Min.	Normality test
Nikkei 225 Futures	−0.008	1.643	−0.316	14.360	18.812	−14.003	3847.5 **
TOPIX Futures	−0.0011	1.563	−0.191	13.412	18.130	−11.726	3543.2 **

** denotes statistical significance at the 1% level.

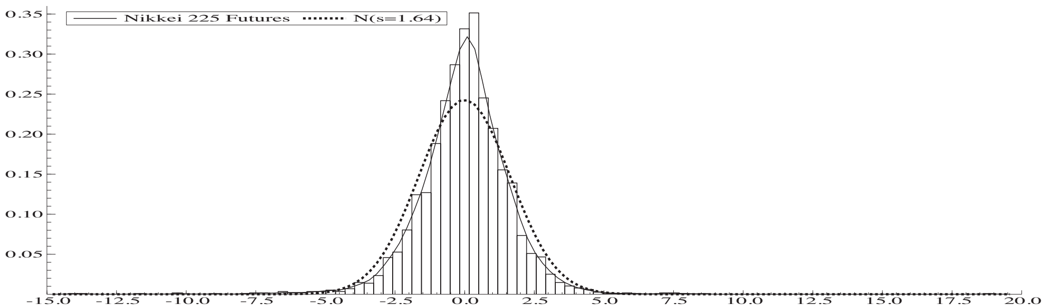

Figure 5: Histogram and Estimated Density with Normal Approximation on the Nikkei 225Futures

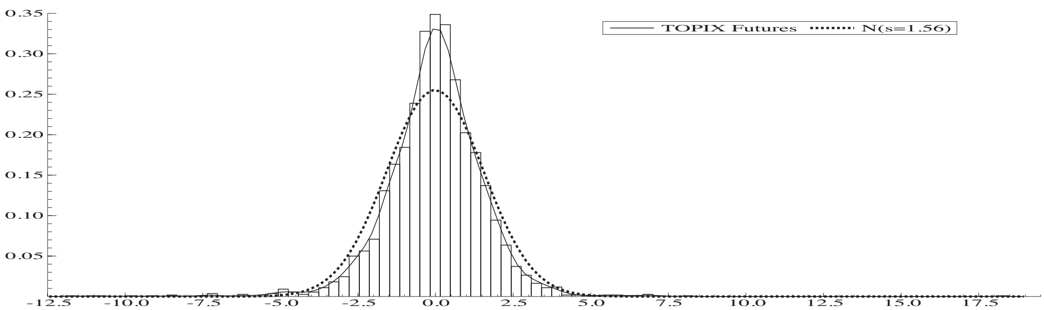

Figure 6: Histogram and Estimated Density with Normal Approximation on the TOPIX Futures

3.2 Empirical Results

Figure 2 shows the estimation results of the Nikkei 225 Futures. Let us consider the MS-ARMA(1,1)-GARCH(1,1) model. The estimated values of μ (0) and μ (1) are 0.141 and −1.110, respectively, which are statistically significant results. $\mu(0)$, which expresses a bull market, is a positive value and $\mu(1)$, which expresses a bear market, is a negative value. This confirmed that when the state variable $s_t = 0$, the Nikkei 225 Futures is a bull market, and when $s_t = 1$, the Nikkei 225 Futures is a bear market. The estimated values of $\omega(0)$ and $\omega(1)$ are 0.135 and 2.253, respectively, which are statistically significant results. Because $\omega(0) < \omega(1)$, we can see that a bear market has higher volatility value than a bull market.

The estimated values of transition probabilities $p_{0|0}$ and $p_{1|1}$ are 0.982 and 0.775, respectively, which are statistically significant results. $p_{0|0}$ is extremely close to 1, implying that if it switches to a bull market, that state will be maintained for a long period. Because $p_{0|0} > p_{1|1}$, we can see that a bear market does not last as long as a bull market. Also, the mean μ and volatility σ switch simultaneously following the state variable st, so when it switches to a low volatility state it remains in that state for a long period, whereas when it switches to a high volatility state it does not stay there for long.

$Q(20)$ and $Q^2(20)$ represent the Ljung-Box Q-statistic with standardized residuals ($\hat{\epsilon}\hat{\sigma}^{-1}$) up to 20th order, and with the squared standardized residuals up to 20th order, respectively. They follow the asymptotically χ^2 distribution with degree of freedom equal to 20. Statistically significant estimation values were not produced with the MS-ARMA(1,1)-GARCH(1,1) model. Regarding $Q(20)$ and $Q^2(20)$, a null hypothesis cannot be rejected with a 10% significance level. Therefore, we can see that the MS-ARMA(1,1)-GARCH(1,1) model captures the autocorrelation of Nikkei 225 Futures volatility.

Figure 3 shows the estimation results of the TOPIX Futures. Regarding the MS-ARMA(1,1)-GARCH(1,1) model, the estimated values of $\mu(0)$ and $\mu(1)$ are 0.140 and -1.387, and the estimated values of $\omega(0)$ and $\omega(1)$ are 0.159 and 2.834, respectively. Because $\mu(0) > \mu(1)$ and $\omega(0) < \omega(1)$, we can see that the TOPIX Futures also has a bull market with high mean, low volatility, and a bear market with low mean, high volatility. The estimated values of transition probabilities $p_{0|0}$ and $p_{1|1}$ are 0.982 and 0.627, respectively, which are statistically significant results. Because $p_{0|0} > p_{1|1}$, we see that in the TOPIX Futures also, a bear market does not last as long as a bull market. Likewise, when it switches to a low volatility state it remains in that state for a long period, whereas a high volatility state does not last so long. The values of $Q(20)$ and $Q^2(20)$ are not statistically significant, so the MS-ARMA(1,1)-GARCH(1,1) model captures the autocorrelation of the TOPIX Futures volatility.

Next, let us compare all models. When comparing the log-likelihood function value, the MS-ARMA(1,1)-GARCH(1,1) model yields the highest value with the Nikkei 225 Futures data as well as the TOPIX Futures data. When comparing the information criteria[14], with the Nikkei 225 Future

14) In general, the degree of order for the MS-ARCH-GARCH model is determined based on the two information criteria: Akaike's Information Criterion (AIC) and Schwart's Information Criterion (SBIC). When parameters are estimated with the maximum-likelihood method, AIC and SBIC can be expressed by the following equations:

$$AIC = (-2 \ln L + 2n)/T,$$
$$SBIC = (-2 \ln L + n \ln T)/T,$$

where ln L represents the log likelihood calculated from estimated parameters, and n represents the number of data the MS-ARMA(1,1)-GARCH(1,1)-c model yields the lowest value by the AIC standard, and the MS-GARCH(1,1) model yields the lowest by the SBIC standard. With the TOPIX Future data, the MS-ARMA(1,1)-GARCH(1,1) model yields the lowest, while the MS-GARCH(1,1) model yields the lowest by the SBIC standard. This indicates that the MS-ARMA(1,1)-GARCH(1,1) model and MS-GARCH(1,1) model is valid for trend analysis of stock market index futures trading.

Table 2: Empirical Results of the Models for the Nikkei 225 Futures

	MS-ARMA(1,1)-GARCH(1,1)	MS-ARMA(1,1)-GARCH(1,1)-c	MS-ARMA(1,0)-GARCH(1,1)	MS-ARMA(1,0)-GARCH(1,1)-c
μ (0)	0.141 *	0.134 *	0.074 *	−0.007 *
	(0.071)	(0.066)	(0.024)	(0.023)
μ (1)	−1.110 *	−1.182 *	−1.090 *	−1.072 *
	(0.262)	(0.279)	(0.280)	(0.289)
ϕ (0)	0.314 *	−	−0.024	−
	(0.181)	−	(0.019)	−
ϕ (1)	0.230 *	−	−0.170	−
	(0.108)	−	(0.116)	−
ϕ	−	0.385 *	−	−0.030
	−	(0.172)	−	(0.019)
ψ (0)	−0.341 *	−	−	−
	(0.179)	−	−	−
ψ (1)	−0.208 *	−	−	−
	(0.117)	−	−	−
ψ	−	−0.418 *	−	−
	−	(0.171)		
ω (0)	0.135 *	0.136 *	0.149 *	0.151 *
	(0.024)	(0.024)	(0.023)	(0.022)
ω (1)	2.253	2.379	0.000	0.000
	(0.350)	(0.273)	0.810	(0.882)
α (0)	0.045 *	0.046 *	0.057 *	0.060 *
	(0.012)	(0.012)	(0.010)	(0.009)
α (1)	0.115 *	0.110 *	0.142 *	0.293 *
	(0.099)	(0.089)	(0.183)	(0.187)
β (0)	0.936 *	0.935 *	0.923 *	0.921 *
	(0.014)	(0.013)	(0.011)	(0.011)
β (1)	0.873 *	0.878 *	0.851 *	0.541 *
	(0.055)	(0.051)	(0.064)	(0.166)
$p_{0\|0}$	0.982 *	0.983 *	0.982 *	0.982 *
	(0.007)	(0.007)	(0.008)	(0.008)
$p_{1\|1}$	0.775 *	0.802 *	0.581 *	0.541 *
	(0.135)	(0.112)	(0.186)	(0.166)
$\ln L$	−5892.20	−5893.54	−5894.21	−5895.03
AIC	3.4999	3.4995	3.4999	3.4998
SBIC	3.5254	3.5214	3.5218	3.5198
$Q(20)$	9.744	9.609	8.841	8.307
$Q^2(20)$	50.374	46.113	50.553	49.694

* denotes statistical significance at the 5% level.

estimated parameters, and T denotes the number of samples.

	MS-GARCH(1,1)	MS	ARMA(1,1)-GARCH(1,1)	ARMA(1,0)-GARCH(1,1)	GARCH(1,1)
μ (0)	0.067 *	0.029	–	–	–
	(0.023)	(0.024)	–	–	–
μ (1)	−1.10 *	−0.275	–	–	–
	(0.307)	(0.169)	–	–	–
μ	–	–	0.042	0.042	0.041
	–	–	(0.022)	(0.022)	(0.022)
ϕ	–	–	0.208	−0.022	–
	–	–	(0.190)	(0.017)	–
ψ	–	–	−0.227	–	–
	–	–	(0.183)	–	–
ω (0)	0.155 *	1.274 *	–	–	–
	(0.022)	(0.024)	–	–	–
ω (1)	2.788 *	3.281 *	–	–	–
	(0.389)	(0.189)	–	–	–
ω	–	–	0.045 *	0.045 *	0.046 *
	–	–	(0.013)	(0.013)	(0.013)
α (0)	0.061 *	–	–	–	–
	(0.009)	–	–	–	–
α (1)	0.121 *	–	–	–	–
	(0.151)	–	–	–	–
α	–	–	0.101 *	0.101 *	0.101 *
	–	–	(0.014)	(0.014)	(0.014)
β (0)	0.919 *	–	–	–	–
	(0.011)	–	–	–	–
β (1)	0.862 *	–	–	–	–
	(0.060)	–	–	–	–
β	–	–	0.884 *	0.884 *	0.883 *
	–	–	(0.014)	(0.014)	(0.014)
$p_{0\|0}$	0.984 *	0.992 *	–	–	–
	(0.007)	(0.002)	–	–	–
$p_{1\|1}$	0.531 *	0.938 *	–	–	–
	(0.162)	(0.016)	–	–	–
ln$L$	−5899.58	−6047.99	−5955.99	−5956.39	−5959.16
AIC	3.5009	3.5864	3.5320	3.5316	3.5327
SBIC	3.5191	3.5974	3.5429	3.5407	3.5399
Q(20)	11.733	19.00	5.109	8.922	9.156
Q^2(20)	46.992	431.65	23.938	24.150	24.491

* denotes statistical significance at the 5% level.

Table 3: Empirical Results of the Models for the TOPIX Futures

	MS-ARMA(1,1)-GARCH(1,1)	MS-ARMA(1,1)-GARCH(1,1)-c	MS-ARMA(1,0)-GARCH(1,1)	MS-ARMA(1,0)-GARCH(1,1)-c
μ (0)	0.140 *	0.093 *	0.051 *	0.051 *
	(0.069)	(0.045)	(0.020)	(0.021)
μ (1)	−1.387 *	−1.193 *	−3.063 *	−1.233 *
	(0.391)	(0.485)	(0.283)	(0.488)
ϕ (0)	0.629 *	−	0.004	−
	(0.107)	−	(0.018)	−
ϕ (1)	0.312 *	−	−1.215 *	−
	(0.133)	−	(0.134)	−
ϕ	−	0.472 *	−	0.004
	−	(0.119)	−	(0.018)
ψ (0)	−0.631 *	−	−	−
	(0.110)	−	−	−
ψ (1)	−0.326 *	−	−	−
	(0.132)	−	−	−
ψ	−	−0.477 *	−	−
	−	(0.123)		
ω (0)	0.159 *	0.164 *	0.182 *	0.175 *
	(0.024)	(0.023)	(0.024)	(0.024)
ω (1)	2.834 *	2.941 *	1.923	0.489
	(0.517)	(0.761)	(0.861)	(0.801)
α (0)	0.052 *	0.058 *	0.075 *	0.070 *
	(0.012)	(0.012)	(0.010)	(0.011)
α (1)	0.125 *	0.106 *	0.000	0.342 *
	(0.148)	(0.122)	(0.077)	(0.120)
β (0)	0.922 *	0.917 *	0.890 *	0.905 *
	(0.014)	(0.014)	(0.013)	(0.014)
β (1)	0.874 *	0.882 *	0.134	0.884 *
	(0.050)	(0.056)	(0.680)	(0.097)
$p_{0\|0}$	0.982 *	0.989 *	0.992 *	0.989 *
	(0.008)	(0.004)	(0.003)	(0.005)
$p_{1\|1}$	0.627 *	0.665 *	0.353 *	0.551 *
	(0.159)	(0.136)	(0.106)	(0.159)
ln$L$	−5727.59	−5732.19	−5732.27	−5735.87
AIC	3.4034	3.4049	3.4050	3.4066
SBIC	3.4288	3.4268	3.4268	3.4265
Q(20)	10.041	9.943	9.279	10.278
Q^2(20)	40.045	32.016	30.203	27.775

* denotes statistical significance at the 5% level.

	MS-GARCH(1,1)	MS	ARMA(1,1)-GARCH(1,1)	ARMA(1,0)-GARCH(1,1)	GARCH(1,1)
μ (0)	0.050 *	0.026	–	–	–
	(0.021)	(0.023)	–	–	–
μ (1)	−1.422 *	−0.245	–	–	–
	(0.520)	(0.150)	–	–	–
μ	–	–	0.038	0.037	0.037
	–	–	(0.023)	(0.023)	(0.023)
ϕ	–	–	0.261	0.004	–
	–	–	(0.209)	(0.018)	–
ψ	–	–	−0.257	–	–
	–	–	(0.212)	–	–
ω (0)	0.177 *	1.163 *	–	–	–
	(0.024)	(0.024)	–	–	–
ω (1)	2.591 *	3.049 *	–	–	–
	(0.629)	(0.156)	–	–	–
ω	–	–	0.046 *	0.046 *	0.047 *
	–	–	(0.017)	(0.016)	(0.016)
α (0)	0.071 *	–	–	–	–
	(0.011)	–	–	–	–
α (1)	0.105 *	–	–	–	–
	(0.096)	–	–	–	–
α	–	–	0.111 *	0.111 *	0.112 *
	–	–	(0.016)	(0.018)	(0.018)
β (0)	0.904 *	–	–	–	–
	(0.014)	–	–	–	–
β (1)	0.877 *	–	–	–	–
	(0.108)	–	–	–	–
β	–	–	0.873 *	0.873 *	0.871 *
	–	–	(0.020)	(0.020)	(0.020)
$p_{0\|0}$	0.990 *	0.989 *	–	–	–
	(0.004)	(0.003)	–	–	–
$p_{1\|1}$	0.560 *	0.929 *	–	–	–
	(0.148)	(0.016)	–	–	–
ln$L$	−5739.41	−5870.84	−5804.70	−5805.72	−5808.99
AIC	3.4071	3.4826	3.4434	3.4434	3.4447
SBIC	3.4252	3.4935	3.4543	3.4525	3.4520
Q(20)	50.891	12.707	11.297	11.249	11.552
Q^2(20)	62.586	374.54	11.553	11.783	11.998

* denotes statistical significance at the 5% level.

Figure 4 shows the periods of the Nikkei 225 Futures when s_t = 0 (bull market). The total number of days in a bull market regime was 3,244 days (96.12% of total), and the average number of days in a bull market regime was 108.13 days. Figure 5 shows the periods when s_t = 1 (bear market). The total number of days in a bear market regime was 31 days (3.88% of total) and the average number of days in a bear market regime was 4.52 days. This indicates that it takes time for stock prices to rise, but they fall sharply. Figures 6 and 7 show the bull and bear market regimes of the TOPIX Futures. The TOPIX Futures produced similar results as the Nikkei 225 Futures. The

Table 4: Regime Classification for the Nikkei 225 Futures (Regime 0)

Total: 3,244 days (96.12%) with average duration of 108.13 days

Period	Days	Avg. Prob.
1 / 6 / 2000 − 3 / 10 / 2000	45	0.959
3 / 15 / 2000 − 4 / 14 / 2000	22	0.959
4 / 19 / 2000 − 5 / 10 / 2000	13	0.847
5 / 12 / 2000 − 12 / 14 / 2000	150	0.933
12 / 22 / 2000 − 2 / 27 / 2001	43	0.942
3 / 23 / 2001 − 9 / 11 / 2001	119	0.920
9 / 14 / 2001 − 6 / 12 / 2002	180	0.953
6 / 24 / 2002 − 3 / 10 / 2003	175	0.937
3 / 12 / 2003 − 3 / 28 / 2003	12	0.781
4 / 1 / 2003 − 9 / 19 / 2003	120	0.967
9 / 24 / 2003 − 10 / 22 / 2003	20	0.879
10 / 24 / 2003 − 4 / 30 / 2004	127	0.957
5 / 14 / 2004 − 4 / 13 / 2005	227	0.978
4 / 19 / 2005 − 10 / 5 / 2005	115	0.984
10 / 7 / 2005 − 1 / 16 / 2006	65	0.964
1 / 19 / 2006 − 4 / 21 / 2006	66	0.964
4 / 25 / 2006 − 6 / 6 / 2006	28	0.851
6 / 12 / 2006 − 2 / 27 / 2007	177	0.971
3 / 6 / 2007 − 7 / 26 / 2007	98	0.957
8 / 2 / 2007 − 8 / 9 / 2007	6	0.705
8 / 20 / 2007 − 12 / 28 / 2007	90	0.924
1 / 9 / 2008 − 1 / 9 / 2008	1	0.505
1 / 28 / 2008 − 9 / 4 / 2008	153	0.95
9 / 19 / 2008 − 9 / 29 / 2008	6	0.721
10 / 17 / 2008 − 11 / 26 / 2009	270	0.950
11 / 30 / 2009 − 4 / 27 / 2010	101	0.955
5 / 10 / 2010 − 3 / 9 / 2011	207	0.943
3 / 16 / 2011 − 8 / 2 / 2011	94	0.967
8 / 10 / 2011 − 5 / 22 / 2013	437	0.963
6 / 11 / 2013 − 9 / 30 / 2013	77	0.940

Table 5: Regime Classification for the Nikkei 225 Futures (Regime 1)

Total: 131 days (3.88%) with average duration of 4.52 days

Period	Days	Avg. Prob.
3 / 13 / 2000 − 3 / 14 / 2000	2	0.720
4 / 17 / 2000 − 4 / 18 / 2000	2	0.908
5 / 11 / 2000 − 5 / 11 / 2000	1	0.686
12 / 15 / 2000 − 12 / 21 / 2000	5	0.696
2 / 28 / 2001 − 3 / 22 / 2001	16	0.744
9 / 12 / 2001 − 9 / 13 / 2001	2	0.805
6 / 13 / 2002 − 6 / 21 / 2002	7	0.694
3 / 11 / 2003 − 3 / 11 / 2003	1	0.505
3 / 31 / 2003 − 3 / 31 / 2003	1	0.571
9 / 22 / 2003 − 9 / 22 / 2003	1	0.707
10 / 23 / 2003 − 10 / 23 / 2003	1	0.873
5 / 6 / 2004 − 5 / 13 / 2004	6	0.675
4 / 14 / 2005 − 4 / 18 / 2005	3	0.804
10 / 6 / 2005 − 10 / 6 / 2005	1	0.580
1 / 17 / 2006 − 1 / 18 / 2006	2	0.619
4 / 24 / 2006 − 4 / 24 / 2006	1	0.566
6 / 7 / 2006 − 6 / 9 / 2006	3	0.573
2 / 28 / 2007 − 3 / 5 / 2007	4	0.984
7 / 27 / 2007 − 8 / 1 / 2007	4	0.715
8 / 10 / 2007 − 8 / 17 / 2007	6	0.843
1 / 4 / 2008 − 1 / 8 / 2008	3	0.590
1 / 10 / 2008 − 1 / 25 / 2008	11	0.733
9 / 5 / 2008 − 9 / 18 / 2008	9	0.656
9 / 30 / 2008 − 10 / 16 / 2008	12	0.845
11 / 27 / 2009 − 11 / 27 / 2009	1	0.905
4 / 28 / 2010 − 5 / 7 / 2010	4	0.722
3 / 10 / 2011 − 3 / 15 / 2011	4	0.848
8 / 3 / 2011 − 8 / 9 / 2011	5	0.744
5 / 23 / 2013 − 6 / 10 / 2013	13	0.792

Table 6: Regime Classification for the TOPIX Futures (Regime 0)

Total: 3,311 days (98.13%) with average duration of 127.35 days

Period	Days	Avg. Prob.
1 / 12 / 2000 − 3 / 10 / 2000	42	0.968
3 / 14 / 2000 − 4 / 14 / 2000	23	0.925
4 / 18 / 2000 − 12 / 18 / 2000	167	0.965
12 / 22 / 2000 − 3 / 9 / 2001	51	0.943
3 / 15 / 2001 − 3 / 19 / 2001	3	0.641
3 / 22 / 2001 − 9 / 11 / 2001	120	0.955
9 / 13 / 2001 − 6 / 13 / 2002	182	0.971
6 / 20 / 2002 − 3 / 28 / 2003	190	0.956
4 / 1 / 2003 − 10 / 22 / 2003	141	0.971
10 / 24 / 2003 − 5 / 7 / 2004	129	0.973
5 / 12 / 2004 − 4 / 14 / 2005	230	0.985
4 / 19 / 2005 − 1 / 16 / 2006	181	0.983
1 / 19 / 2006 − 6 / 7 / 2006	96	0.955
6 / 9 / 2006 − 2 / 27 / 2007	178	0.974
3 / 6 / 2007 − 8 / 9 / 2007	108	0.973
8 / 20 / 2007 − 11 / 8 / 2007	56	0.955
11 / 14 / 2007 − 12 / 28 / 2007	31	0.966
1 / 8 / 2008 − 1 / 18 / 2008	8	0.665
1 / 25 / 2008 − 9 / 4 / 2008	154	0.962
9 / 8 / 2008 − 9 / 12 / 2008	5	0.754
9 / 19 / 2008 − 10 / 3 / 2008	10	0.846
10 / 15 / 2008 − 11 / 26 / 2009	272	0.962
11 / 30 / 2009 − 3 / 11 / 2011	314	0.974
3 / 16 / 2011 − 8 / 4 / 2011	95	0.976
8 / 9 / 2011 − 5 / 22 / 2013	438	0.973
5 / 28 / 2013 − 9 / 30 / 2013	87	0.946

Table 7: Regime Classification for the TOPIX Futures (Regime 1)

Total: 64 days (1.89%) with average duration of 2.21 days

Period	Days	Avg. Prob.
1 / 6 / 2000 − 1 / 11 / 2000	3	0.744
3 / 13 / 2000 − 3 / 13 / 2000	1	0.974
4 / 17 / 2000 − 4 / 17 / 2000	1	0.972
12 / 19 / 2000 − 12 / 21 / 2000	3	0.787
3 / 12 / 2001 − 3 / 14 / 2001	3	0.713
3 / 21 / 2001 − 3 / 21 / 2001	1	0.986
9 / 12 / 2001 − 9 / 12 / 2001	1	1
6 / 14 / 2002 − 6 / 19 / 2002	4	0.701
3 / 31 / 2003 − 3 / 31 / 2003	1	0.678
10 / 23 / 2003 − 10 / 23 / 2003	1	0.975
5 / 10 / 2004 − 5 / 11 / 2004	2	0.836
4 / 15 / 2005 − 4 / 18 / 2005	2	0.764
1 / 17 / 2006 − 1 / 18 / 2006	2	0.778
6 / 8 / 2006 − 6 / 8 / 2006	1	0.526
2 / 28 / 2007 − 3 / 5 / 2007	4	0.959
8 / 10 / 2007 − 8 / 17 / 2007	6	0.821
11 / 9 / 2007 − 11 / 13 / 2007	3	0.556
1 / 4 / 2008 − 1 / 7 / 2008	2	0.606
1 / 21 / 2008 − 1 / 24 / 2008	4	0.671
9 / 5 / 2008 − 9 / 5 / 2008	1	0.76
9 / 16 / 2008 − 9 / 18 / 2008	3	0.619
10 / 6 / 2008 − 10 / 14 / 2008	6	0.787
11 / 27 / 2009 − 11 / 27 / 2009	1	0.981
3 / 14 / 2011 − 3 / 15 / 2011	2	1
8 / 5 / 2011 − 8 / 8 / 2011	2	0.636
5 / 23 / 2013 − 5 / 27 / 2013	3	0.681

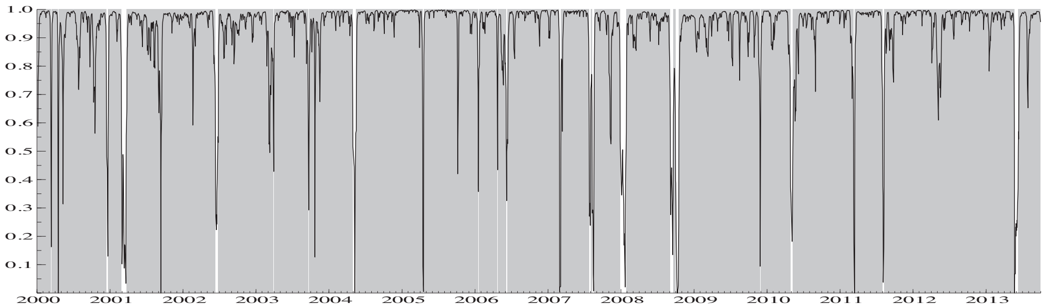

Figure 7: The Smoothed Probability of Bull Regime for the Nikkei 225 Futures

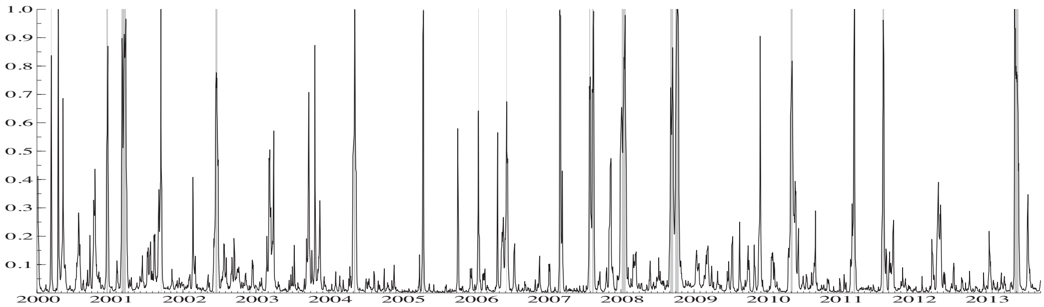

Figure 8: The Smoothed Probability of Bear Regime for the Nikkei 225 Futures

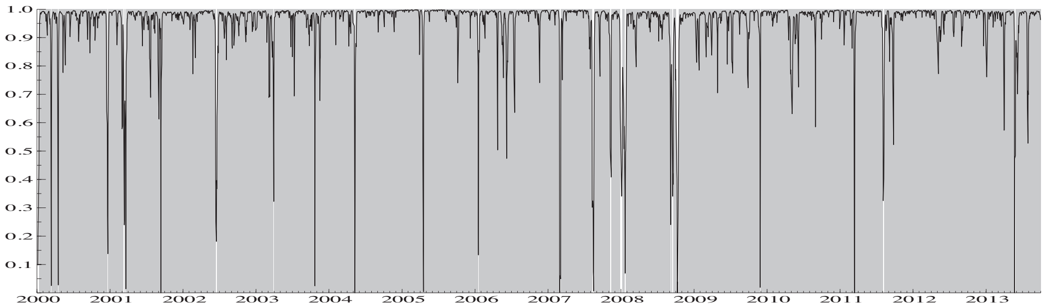

Figure 9: The Smoothed Probability of Bull Regime for the TOPIX Futures

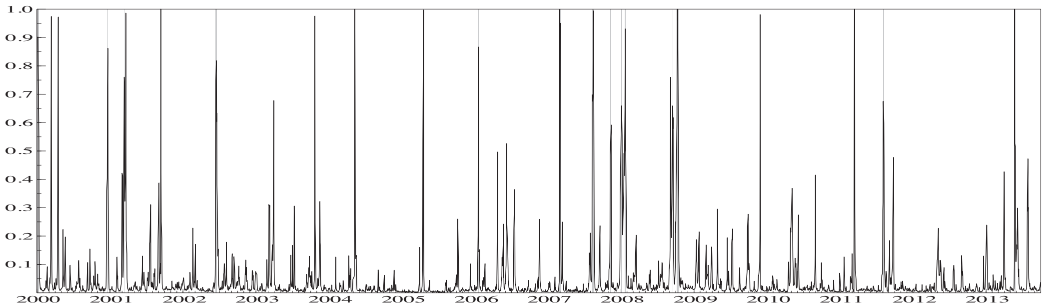

Figure 10: The Smoothed Probability of Bull Regime for the TOPIX Futures

shaded areas in Figs 1 – 4 indicate bear market regimes in the Nikkei 225 Futures and TOPIX Futures. We can see that Figs 1 and 2, which are graphs of the closing prices, particularly capture the sudden fall. The black lines in Figs 7 – 10 indicate the smoothed probabilities[15] of bull and bear markets. The shaded areas in Figs 7 and 9 show bull markets, and in Figs 8 and 10 bear markets.

4 Concluding Remarks

This study conducts trend analysis on the Nikkei 225 Futures and TOPIX Futures, using an MS-ARMA(1,1)-GARCH(1,1) model. Empirical analysis was conducted with a focus on bull and bear markets, using the daily data from the Nikkei 225 Futures and TOPIX Futures. The empirical analysis showed that the model exhibits statistically significant bull and bear market regimes in both the Nikkei 225 Futures and TOPIX Futures. In other words, the model captures bull market regimes with high mean, low volatility and bear market regimes with low mean, high volatility. The analysis also verified that the MS-ARMA-GARCH model and MSGARCH model are valid for bull and bear market analysis of the Nikkei 225 Futures and TOPIX Futures.

Future issues include analysis using the MS-ARMA-EGARCH model. This is because, while this study modelized volatility changes using the GARCH model, Henry (2009) proposed the MS-EGARCH model[16] which comprises the EGARCH (Exponential GARCH) model by Nelson (1991). In addition, Maheu *et al.* (2012) proposes a four-state Markov-switching model that identifies four trends – the bull market, bear market, bear market rally, and bear market correction[17]. Therefore, it is important to analyze trends in detail. Because this study uses daily data, the bear market regime came out quite long in the empirical analysis, and using weekly or monthly data will likely eliminate this problem. It is also necessary to include Nikkei 225 Futures night sessions and TOPIX Futures evening sessions in the data. Finally, the validity of MS-ARMA-GARCH model should be considered by comparison with other methods that analyze bull and bear markets in different ways, such as those proposed by Lunde and Timmermann (2004) and Shibata (2012).

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15) The smoothed probabilities can be computed using the backward iteration suggested by Kim (1993):

$$\xi_{t|T} = \xi_{t|t} \odot \{P'[\xi_{t+1|T}(P \oslash \xi_{t|t})]\},$$
$$\xi_{t|t} = \frac{diag(\eta_t)}{\xi'_{t|t-1}\eta_t},\ \xi_{t+1|t} = P\xi_{t|t},$$

where $\oslash$ is used for element by element division and diag (η_t) creates a diagonal matrix with η_t on the diagonal.

16) Satoyoshi and Mitsui (2011), Mitsui (2012) conducted bull and bear market analysis on the Nikkei Average using the MS-EGARCH model.

17) Satoyoshi and Mitsui (2013) conducted trend analysis on the Nikkei Average using a four-state Markovswitching model, as with Maheu *et al.* (2012). Mitsui (2014) also conducted empirical analysis on Nikkei 225 Futures.

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