

# Life beyond 100: An Estimation of the Centenarian Population in Japan

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## Introduction

According to the population census conducted in October 2010, the total population of Japan is 128,057,352 (62,327,737 males and 65,729,615 females), a 0.2% rise compared to five years before. However, the natural population growth for Japanese people living in Japan, calculated by deducting the number of deaths from the number of births, turned negative in 2005 for the first time since the records began in 1899, and has remained negative ever since, with the exception of year 2006.

At the time the baby boom generation was born, the total fertility rate (TFR, the number of children a woman gives birth to during her life) in Japan was around 4, but as exemplified by the expression the “1.57 shock” created by the Japanese media in 1999, in the following decades it fell sharply. In 2005 the Japanese TFR reached the bottom at 1.26 children per woman, but has since rebounded to 1.43 in 2013. However, along with a dramatic fall in fertility, Japan also recorded a significant decline in mortality and achieved gains in terms of longevity. In 2013, the average life span in Japan was 80.2 for men and 86.6 for women, the highest in the world. As a result of the declines in fertility and mortality, Japan’s population has been undergoing rapid structural changes, and this island country is now widely known as the society with the most profound population transformations in the world: fertility reduction, population aging and population decrease.

According to a projection by the National Institute of Population and Social Security Research, Japan’s population is expected to shrink to 115.22 million in 2030 and further sink to 86.74 million in 2060. It is certain that this steadfast reduction in the population size will have a tremendous impact on Japan’s society and economy, and that the country’s overall power and security, as well as other areas of life, will be affected. However, if we turn our attention to different age groups within the shrinking Japanese population, we can notice that among them there is one that continues growth even after 2005, when the reduction in population size became apparent, at an average rate of 12%. That is the population of centenarians.

An increase in the population of 100 years and over is, in a sense, a realization of the human dream for longevity. Indeed, we now live in an age of longevity, in which reaching old age has become quite an ordinary thing. However, life span has not increased gradually through the long history of the human kind. Rather, our life span started expanding modestly only recently, around 250 years ago, when modernization began to occur in some of the developed countries. Until then, human society did not have effective solutions for preventing and curing diseases, and passing safely

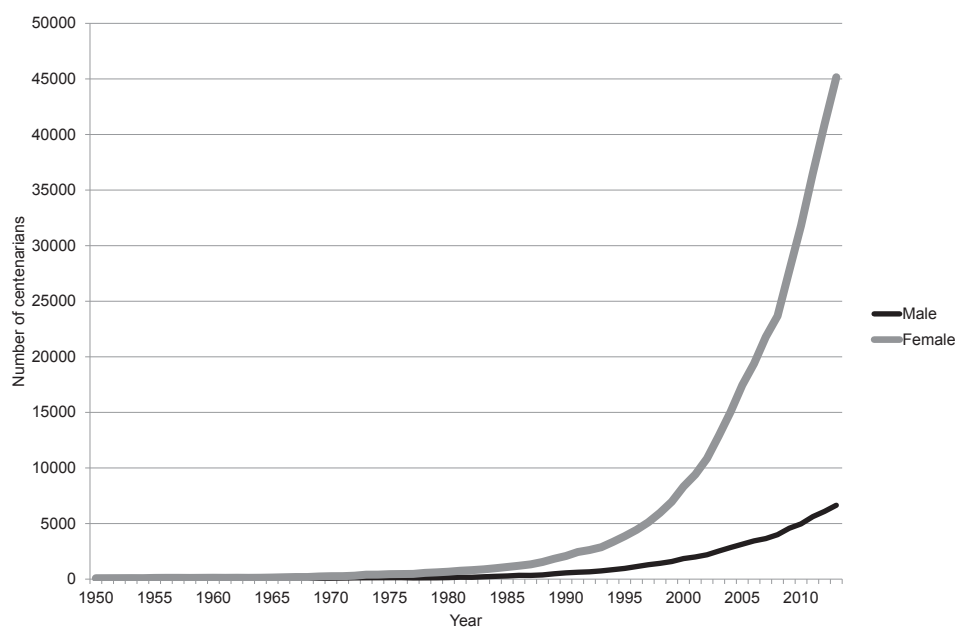
through childhood and early youth and becoming a mature adult was considered good fortune. Furthermore, among those who managed to reach adulthood, a large portion would not live long enough to experience old age, so life in old age was a rarity. Long life was thus, largely, a dream, something to be found in the words of well-wishers.

In Japan, this dream is now increasingly becoming a reality for many. With the lowest and yet the fastest declining mortality since early 1980s (Oeppen and Vaupel, 2002), Japan is seeing a progressive rise in the number of centenarians. Fortunately, high quality demographic data (Human Mortality database: [www.mortality.org](http://www.mortality.org)) and a large population provide a solid basis to analyze and forecast the changes in the numbers and other aspects of centenarians in Japan between 1950 and 2050.

### The Centenarian Population in Japan

In 1950, there were only 30 male centenarians in Japan and this number remained almost unchanged until 1964, when it reached 25. The situation was similar for female centenarians. Although the number of female centenarians rose from 81 in 1950 to 108 in 1955, it grew only marginally from 117 in 1956 to 123 in 1964. Thus, prior to 1964, centenarians in Japan were rare and there was no sign of them increasing.

Starting from 1965, the number of centenarians rose almost exponentially for both sexes, reaching about 6648 males and 45149 females in 2013, as shown in Figure 1. The change in the



Source: Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at [www.mortality.org](http://www.mortality.org) or [www.humanmortality.de](http://www.humanmortality.de) (data downloaded on Monday, December 01, 2014).

Figure 1. Trend in centenarians in Japan, 1950-2013

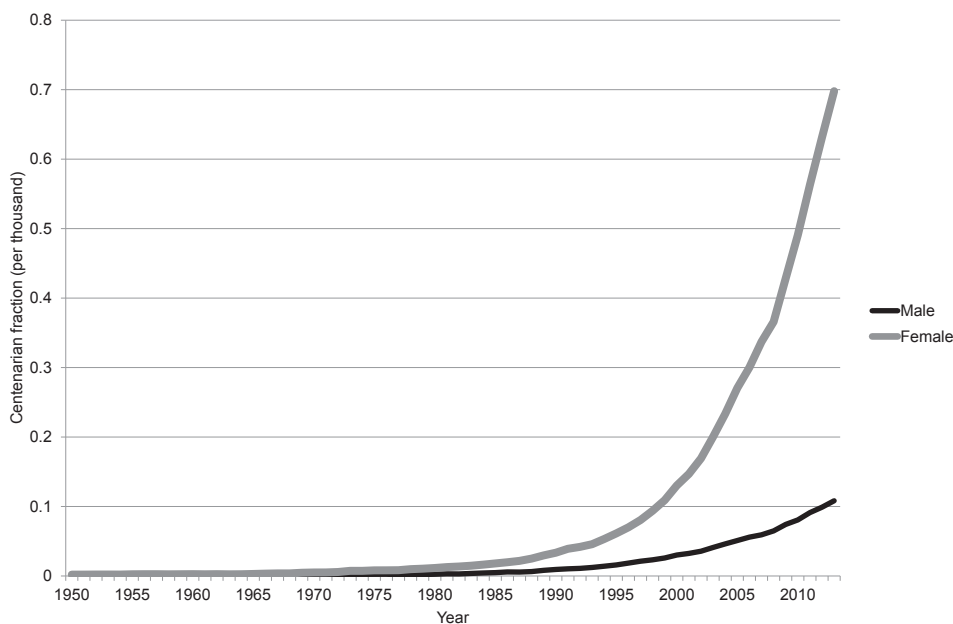


Figure 2. Proportion of centenarians in the population of Japan, 1950-2013

gender and age structure of Japanese centenarians, however, was not as remarkable as that in the numbers. The ratio of those aged 105 and older to all centenarians remained about 5% for both male and female in this period.

And the ratio of female to male centenarians grew from 2.7 in 1950 to 6.8 in 2013. As shown in Figure 2, centenarians as a proportion of the population amplified almost exponentially, implying that centenarians increased much faster than the total population. As illustrated in Figure 3, between 1990 and 2013 the rate of increase for the total population was basically zero while the rate of increase for centenarians was significantly positive. In fact, the average annual increase rate for centenarians over this period was 12% for males and 14% for females. If the rate were to remain at this level, the number of centenarians would continue to double every six years.

Furthermore, I have calculated the proportion of Japanese centenarians in the total world centenarian population based on UN population projections, the results of which can be seen in Figure 4. When the UN started the work on its population projections in 1990, Japanese centenarians accounted for 3% of the total world population of centenarians. Since that year, that proportion has risen, reaching 12% in 2010, and is expected to continually grow to 21% in 2047. With every fifth centenarian in the world being Japanese, the East Asian island country will, of course, lead the world in the number of centenarians, until 2084 (when 10% of the entire world population will be over 100).

Japanese centenarian population has, thus, been showing a distinct growth pattern, but just how high will it rise in the future? In the background of this remarkable growth is a steep decline in the mortality rate. The UN and the Japanese government have carried out population projections that encompass centenarians, but, in terms of the methodology employed, they have not created

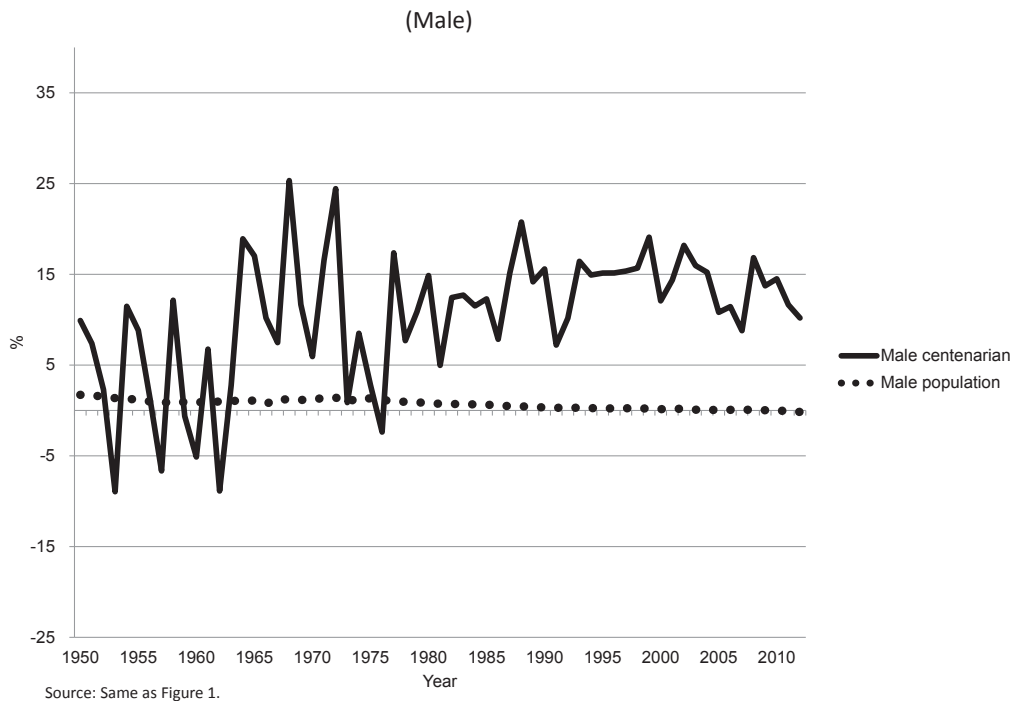


Figure 3a. Annual growth rates of the total and centenarian population of Japan, 1950-2012

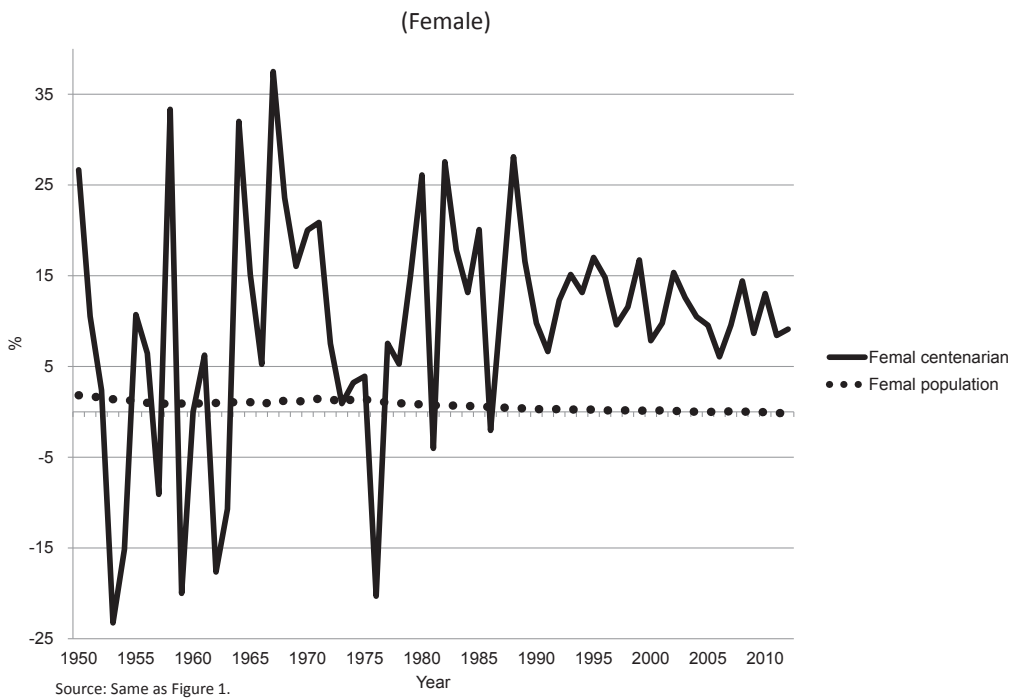
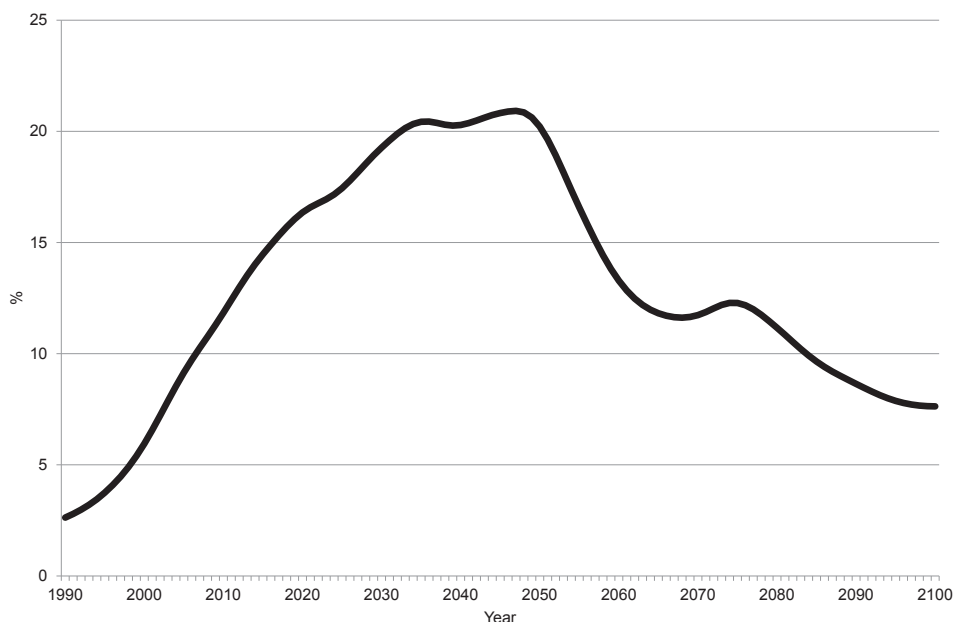


Figure 3b. Annual growth rates of the total and centenarian population of Japan, 1950-2012



Source: United Nations, World Population Prospects: The 2012 Revision, 2013.

Figure 4. Proportion of Japanese centenarians in the world centenarian population, 1950-2100

models specifically designed for the estimation of centenarian populations, and have treated centenarians as an open-end age group, which is why they have not been able to produce a detailed estimation. In this paper, upon reviewing past studies, I shall present a model for the estimation of the centenarian population.

### Model for the estimation of centenarians

Since births did not increase at a rate higher than 10% per year in the second half of the 19<sup>th</sup> century and immigration has never been substantial in Japan, the main demographic source of the remarkable growth of centenarians in Japan can only lie in mortality decline. In Japan in the years around 1950, mortality at younger ages caused by infectious diseases declined faster than at older ages, but following this period, reductions of deaths at older ages due to chronic diseases become dominant (Lee and Miller, 2001), thus leading to the significant growth of centenarians.

Reduced mortality would result in greater survivorship of centenarians and lengthening of their lives. As improved quality of life and extension of life are universal objectives, we may expect mortality to continue to decline in the future. Consequently, we may anticipate the growth of centenarians to continue in Japan. But at what rates and what levels of certainty? This is a problem of forecasting. Notably, in the case of forecasting centenarians for a period up to 100 years, we are concerned only with mortality, not fertility or migration. However, this is no easy task.

To forecast centenarians, we cannot close the life table by making an open age group such as

100+; we have to deal with age-specific death rates at ages older than 100 years. But the quality of this data is highly questionable. So before talking about forecasting, the first problem is how to reasonably estimate the historical mortality of centenarians. Measuring mortality at very old ages is difficult due to the quickly declining number of survivors at all ages and the large numbers required to obtain accurate measures. Supposing there were  $n$  identical individuals at a certain age subject to death probability  $q$ , then according to the binomial distribution, the observed probability of death would have a mean of  $q$ , but associated with a standard error  $\sqrt{q(1-q)/n}$ . Taking Japanese males in 2013 as an example, the relative error  $\sqrt{(1-q)/(nq)}$  is 5% at age 100 and 22% at age 105.

Direct measuring of centenarian mortality cannot be reliable, so we have to use models to estimate it. The most widely used model in this regards is Gompertz's law (1825), which claims that the force of mortality rises with age exponentially or at a constant rate. This law, however, is well recognized to work well up to age 80 or so, older than which the rate of increasing mortality declines. To model the rate of decline, Coale and Guo (1989) took the simplest way, namely, they assumed the rate of increasing mortality declined linearly. In other words, the logged death rate is described as a quadratic function of age. The constant and linear terms of this function compose the Gompertz curve, while the square term, with a negative coefficient, describes the deceleration. If the modeling errors were identically and independently distributed (i.i.d.) across age, this negative coefficient could be well estimated through many simple ways using observed death rates at ages 80 years and over. But unfortunately these errors are not i.i.d., so Coale and Guo assumed "an arbitrary high value" for the death rate at age 110 as 0.66 plus the death rate at age 80. To avoid crossover mortality change between males and females, Coale and Kisker (1990) modified the assumed values of death rate at age 110 as 1 for males and 0.8 for females. Using these assumptions, of course, the coefficient of the square term is determined. In dealing with mortality at ages 80 years and older, the Coale-Kisker model is perhaps the most widely used one, because it avoids using mortality data at ages 80 years and older. The main problem with using the Coale-Kisker model for our task, besides the arbitrary assumption which, as criticized by Wilmoth (1995), cannot be expected to hold well everywhere and every time, is eluding to infer the change of mortality at ages older than 110 years.

In fact, assuming a value of mortality at age 110, one can establish many models (see Thatcher, Kannisto, and Vaupel, 1998) to modify the constant rate of mortality rise in Gompertz's law, for example, a logistic model in which the increase of mortality is ultimately bounded. But the logistic model may not be appropriate, because the evidence only shows deceleration in mortality rise, and does not clearly suggest whether it will level off or drop to zero. In fact, we do not even know whether mortality would rise to infinity; we only know it would not do so along the exponential way as the Gompertz model suggests. Before obtaining evidence to inductively distinguish the case, potential progress seems to require deductive rationale to infer mortality at extremely old ages. The frailty approach initiated by Vaupel, Manton, and Stallard (1979), which we introduce below, provides perhaps the first deductive model to serve the purpose.

### The frailty-mortality model and its estimation

The frailty-mortality model admits that individuals are different with respect to mortality, even though they share factors such as age, gender and so on. It uses frailty, which is defined as positive, to distinguish individuals. For two individuals, the model defines the ratio of their frailty as the ratio of their mortality. For the value of frailty, the model makes two assumptions. First, it is a lifetime constant. Second, it obeys a gamma distribution with mean 1 and variance  $\sigma^2$  at the starting age, which, in this paper, is 80 years. Under the two assumptions, the frailty-mortality model can be expressed in two ways (Vaupel, Manton, and Stallard, 1979). For a certain cohort born in year  $c$ , the model denotes its force of mortality at age  $x$  as  $\bar{\mu}(x, c)$  and the force of mortality of individuals whose frailty is 1 at age 80 in this cohort as  $\mu(x, c)$ . Then, one way is to describe  $\bar{\mu}(x, c)$  using  $\mu(x, c)$  as

$$\bar{\mu}(x, c) = \frac{\mu(x, c)}{1 + \sigma(c)^2 \int_{80}^x \mu(y, c) dy}, \quad x \geq 80. \quad (1)$$

When  $\mu(x, c)$ , which is also called the baseline individual force of mortality, is identified, the model values of  $\bar{\mu}(x, c)$  can be given by (1). It can be seen that  $\bar{\mu}(x, c)$  rises with age slower than  $\mu(x, c)$  does. And this deceleration is explained deductively by the fact that the frailer individuals tend to die earlier and survivors would be more robust.

Another way is to describe  $\mu(x, c)$  using  $\bar{\mu}(x, c)$  as

$$\mu(x, c) = \bar{\mu}(x, c) \exp\left[\sigma(c)^2 \int_{80}^x \bar{\mu}(y, c) dy\right] = \bar{\mu}(x, c) \bar{\sigma}(x, c)^{-\sigma(c)^2}, \quad x \geq 80. \quad (2)$$

When frailty variance  $\sigma^2$  is estimated, (2) can be used to identify  $\mu(x, c)$ .

To describe the three possible scenarios of  $\bar{\mu}(x, c)$  rising to infinity, leveling off, and dropping to zero, Li and Vaupel (2004a) proposed a parametric model for  $\mu(x, c)$  as

$$\mu(x, c) = \bar{\mu}(80, c) \exp[r(c)(x - 80)^{P(c)}], \quad (3)$$

where  $P(c)$  is a positive number and is named the individual mortality power. Inserting (3) into (1), it can be shown that  $\bar{\mu}(x, c)$  rises to infinity when  $P(c) > 1$ , approaches a positive constant if  $P(c) = 1$ , and drops to zero provided  $P(c) < 1$ . Although (3) is sufficient to yield infinity or zero  $\bar{\mu}(\infty, c)$  using  $P(c) > 1$  or  $P(c) < 1$ , the exponential  $\mu(x, c)$  is sufficient and necessary for the scenario of  $\bar{\mu}(x,$

$c$ ) approaching a positive constant. For a cohort born over a long period, through which disturbances on observed mortality would be largely removed, the method for estimating parameters  $P(c)$ ,  $r(c)$ , and  $\sigma(c)$  has been described elsewhere (Li and Vaupel, 2004a).

For cohorts born in a single year, which we have to deal with, Li and Vaupel (2004b) suggested that  $P(c)$  and  $\sigma(c)$  be assumed constant over the cohort for the following reasons. Since  $\sigma^2$  describes the variance of heterogeneity with respect to mortality, which should reflect the social heterogeneity and should not change significantly when the society is in a stable state, we may assume that the value of  $\sigma$  does not change over the cohort born in a single year. Note that  $P$  and its significance test determine the scenario that  $\bar{\mu}(x, c)$  follows, and that it is unlikely for the scenario to change over cohort. We may thus assume that the value of  $P$  does not change over the cohort born in a single year. In this recent paper, Li and Vaupel have produced empirical evidence in support of assuming constant  $P(c)$  and  $\sigma(c)$ . Accordingly, we ignore the argument  $c$  in  $P(c)$  and  $\sigma(c)$  thereafter.

Li and Vaupel applied their method to Japan (2004b). Following the suggestion of Thatcher, Kannisto, and Vaupel (1998), the values of  $\bar{\mu}(x, c)$  for  $80 \leq x \leq 98$  are used for estimating to avoid possible misreporting at ages 99 and 100 years and large disturbance at older ages. The value of  $P$  is concluded as 1 for both males and females, because the differences between the estimated values of  $P$  and 1 are statistically insignificant. The values of  $\sigma^2$  are estimated as 0.083 for males and 0.171 for females. Using the estimated values of  $\sigma$ ,  $\mu(x, c)$  can be identified by (2). Having the identified  $\mu(x, c)$ , the values of  $r(c)$  are estimated by (3) and shown in Figure 5.

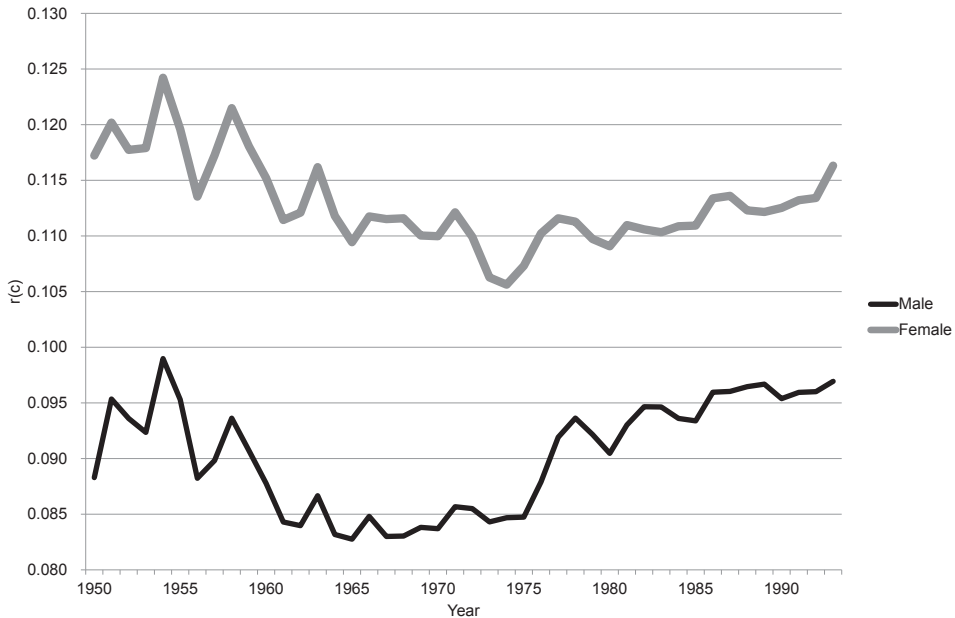


Figure 5. Estimated Value of r(c)



### Mortality and population forecasts

In order to forecast centenarians, we need to forecast mortality not only for centenarians but also at younger ages. And to forecast mortality with associated uncertainty, we chose the widely adopted Lee-Carter method (Lee and Carter, 1992) for its solid basis, simplicity, and successful performance (Lee and Miller, 2001). The age- and sex-specific death rates at ages younger than 80 years for years between 2014 and 2050 can be readily forecasted using the Lee-Carter method.

Using the Coale-Kisker model to extend the data of period death rates to age 110, the Lee-Carter forecast could be done up to 110 years of age. If the focus were not centenarians, the Coale-Kisker assumption of mortality at age 110 would not matter much. And if the forecasting horizon were only a few years, ending at age 110 would not matter much either. Unfortunately, we do not have these two conditions. Moreover, we cannot first use the frailty-mortality model to extend the data to an age much larger than 110 years and then apply the Lee-Carter method, because by doing so a lot of period data would be lost as the frailty-mortality model is strictly built on a cohort basis. A reasonable strategy, thus, seems to use the Lee-Carter method to forecast mortality up to age 80, and then extend the forecasted mortality up to the oldest possible age by the frailty-mortality model. For Japan, the mortality data are available from 1950 through 2012. At 2013, the oldest age at which the mortality could be estimated by the frailty-mortality model, using mortality of the cohort aged 80 in 1950, is 143 years. And in principle, this oldest-age mortality is used to make the first year forecast of the oldest population. Thus, we set the oldest age in our forecast as 143 years, due to the limit of data, not the model. Should the historical data be available in years earlier than 1950, the oldest age would be over 143.

The Lee-Carter forecasted  $\bar{\mu}(80, c)$ , however, are for cohorts who are aged 80 in years later than 2013. The  $r(c)$  of these cohorts are unknown. Moreover, the  $r(c)$  for cohorts who are aged 80 at years later than 2000, with unknown  $\bar{\mu}(x, c)$  at ages 92 years and younger, are also not estimated. Thus, given the estimated constant values of  $P$  and  $\sigma$ , to extend the forecasted mortality to ages older than 80 years requires forecasting the  $r(c)$  for cohorts aged 80 at years later than 2000 using the estimated values shown in Figure 5. Regarding these values of  $r(c)$  as samples of a random variable, it is natural to use a time-series model to describe this variable and forecast its future change. How to model it could be discussed from the standpoint of the Lee-Carter method.

Let the over-time average of  $\log [\bar{\mu}(x, t)]$  be  $a(x)$ . The Lee-Carter method first models  $\log [\bar{\mu}(x, t)]$  as

$$\log [\bar{\mu}(x, t)] \approx a(x) + b(x)k(t), \quad (4)$$

where  $b(x)$  is a row (age) and  $k(c)$  a column (time) vector. The reason for doing so is that the errors in (4) are usually small, because  $\log [\bar{\mu}(x, t)] - a(x)$  as row vectors are similar over time as is well-known in model life table approaches. By doing so, the task of forecasting an age-specific

vector  $\log [\bar{\mu}(x, t)]$  is transferred into forecasting a scalar  $k(t)$ . In terms of forecasting  $k(t)$ , in most applications to date, it has been found that a random walk with drift (RWD) fits very well, although it is not always the best model overall. Using the RWD to fit historical  $k(t)$ , parameters can be estimated and stochastic trajectories of future  $k(t)$  can be generated as forecasts. Inserting these stochastic trajectories of  $k(t)$  into (4) obtains the stochastic forecast of  $\log [\bar{\mu}(x, t)]$ .

Li and Vaupel (2004b) showed that (4) also stands for cohort version, in which the  $a(x)$  should be the over-cohort average and argument  $t$  should be  $c$ . This is because the change of  $\log [\mu(x, t)] - a(x)$  across  $x$  describes mainly the process of biological maturing and aging, which should be similar between cohorts.

Making  $\log [\mu(x, c) / \mu(x, c-1)]$  from (3) and the cohort version of (4), we have

$$b(x)[k(c) - k(c-1)] \approx \log \left[ \frac{\mu(80, c)}{\mu(80, c-1)} \right] + (x+80)^p [r(c) - r(c-1)]. \quad (5)$$

According to (5), when  $\mu(x, c)$  declines over  $c$  by different rates at different ages,  $k(c)$  decreases with  $c$ ,  $b(x)$  should be proportional to  $(x-80)^p$ , and  $r(c)$  should decrease with  $c$  in the way similar to that of  $k(c)$ . A special case is  $\mu(x, c)$  declines over  $c$  by the same rate at all ages. In this special case,  $b(x)$  is constant across  $x$ ,  $r(c)$  must be constant over  $c$ , and the decline in  $\mu(x, c)$  is described by the decrease of  $\mu(80, c)$ .

In applying the Lee-Carter method,  $k(c)$  often declines linearly with random fluctuations, as RWD. Accordingly,  $r(c)$  should also change randomly, but with or without a linearly declining trend. The random fluctuations in  $k(c)$  or  $r(c)$  are results of uncertainty in historical mortality change, and should be modeled if the forecast aims to cover future uncertainty. As to the persistent trend, when  $r(c)$  declines linearly, the drift term should differ significantly from zero and so the RWD should be adopted. However, for the special case that  $r(c)$  keeps constant, the drift term should not be statistically significant, and the random walk (RW) is preferred. Thus, we ought to model  $r(c)$  as RW or RWD:

$$r(c) = r(c-1) - d + s \cdot e(c), \quad e(c) \sim N(0, 1), \quad E[e(c)e(c-1)] = 0. \quad (6)$$

In (6),  $d$  is the drift term that describes the linearly declining trend and  $s$  represents the standard error of the random fluctuations. Let the number of cohorts with  $r(c)$  estimated be  $C$ , then  $d$  and  $s$  can be estimated as

$$d = \frac{1}{C} \sum_{c=1}^C [r(c-1) - r(c)], \quad (7)$$

$$s = \sqrt{\frac{1}{C-1} \sum_{c=1}^C [r(c-1) - r(c) - d]^2} \quad \text{for RWD,} \quad (8)$$

$$s = \sqrt{\frac{1}{C} \sum_{c=1}^C [r(c-1) - r(c)]^2} \quad \text{for RW.}$$

Whether or not there should be a drift term can be checked by the t-test, for which the statistic is

$$t(C-1) = \frac{|\sum_{c=1}^C r(c)|}{\sqrt{\sum_{c=1}^C [r(c-1) - r(c) - d]^2}}. \quad (9)$$

When the sampled t (C-1) is larger than the critical value at confidence level  $\alpha$ , the null hypothesis  $d=0$  is rejected, and by doing so the chance of error is less than  $\alpha$ .

How well (6) works is usually evaluated by the F-test, for which the statistic is

$$F(2, C-1) = \frac{\{\sum_{c=1}^C [r(c) - \frac{1}{C} \sum_{c=1}^C r(c)]^2 - \sum_{c=2}^C [r(c-1) - r(c) - d]^2\}}{2 \sum_{c=1}^C [r(c) - \frac{1}{C} \sum_{c=1}^C r(c)]^2 / (C-1)} \quad \text{for RWD,} \quad (10)$$

$$F(1, C-1) = \frac{\{\sum_{c=1}^C [r(c) - \frac{1}{C} \sum_{c=1}^C r(c)]^2 - \sum_{c=2}^C [r(c-1) - r(c)]^2\}}{2 \sum_{c=1}^C [r(c) - \frac{1}{C} \sum_{c=1}^C r(c)]^2 / (C-1)} \quad \text{for RW.}$$

The F-test is about fitting  $r(c)$ ; the null hypothesis is that there is no difference between using the chosen model and using an i.i.d. variable. Similarly, when the sampled F (\*, C-1) is larger than the critical value at confidence level  $\alpha$ , the null hypothesis is rejected with a less than a probability or error.

The values of t (C-1) for the RWD model are 0.09 for males and 0.63 for females, which are smaller than the critical value at the 5% confidence level of about 2. Thus, using RWD implies rejecting the null hypothesis of  $d=0$ , for which the chance of error is large. Therefore, we turn to RW. The values of F for the RW model are 17.6 for males and 18.2 for females, which are larger than the critical value at the 5% confident level of about 4. So we reject the null hypothesis that RW fits  $r(c)$  equally well as i.i.d. does, as the chance of committing an error is smaller than 5%. We choose RW because it is significantly better than i.i.d. In forecasting, uncertainty increases with successive cohorts and over time if using RW, and it would be constant if i.i.d. were adopted. The difference

between using RW and RWD remains in the mean forecast. The mean mortality at infinitive-large age is  $\text{mean}[r(c)] / \sigma^2$  according to (1) and (3). The mean forecast of ultimate mortality would decline over cohort if applying RWD, while it would be constant if using RW. But at any age the mean forecast of mortality still declines when using RW.

Sampling the  $e(c)$  randomly and independently over  $c$  (i.e.,  $c$  being the cohorts aged 80 from 1981 to 2049) and inserting them into (6) generates a random trajectory of  $r(c)$ . Note that the  $k(t)$  in (4) includes mainly over-time fluctuations, while the  $r(c)$  in (6) involves over-cohort random change and considerable binomial noise. We therefore assume that the random changes in  $k(t)$  and  $r(c)$  are independent, and generate a random trajectory of  $k(t)$  in the way similar to but independent from that of  $r(c)$ . Inserting this random trajectory into (4), a random trajectory of  $\bar{\mu}(x, t)$  for  $x \leq 80$  and  $2014 \leq t \leq 2050$  is then yielded. We have so far obtained the values of  $P$  and  $\sigma^2$ , a random trajectory of  $r(c)$  for  $c$  being the cohorts aged 80 from 1981 to 2049, values of  $\bar{\mu}(80, t)$  for  $1981 < t < 2013$ , and a random trajectory of  $\bar{\mu}(80, t)$  for  $2013 \leq t \leq 2050$ . A random trajectory of  $\mu(x, c)$  for  $80 < x \leq 144$  and  $c$  of the cohorts aged 80 from 1981 to 2049 is then derived from (3), and of a trajectory of  $\bar{\mu}(x, c)$  from (1).

Correspondingly, a random trajectory of the period mortality,  $\bar{\mu}(x, t)$ , is obtained for  $x \leq 130$  and  $2014 \leq t \leq 2050$ . Using a large number of random trajectories, which is 1000 in this paper, the stochastic forecast of  $\bar{\mu}(x, t)$  is composed, from which we are able to draw probability distributions of  $\bar{\mu}(x, t)$  for any  $x \leq 144$  and  $2014 \leq t \leq 2050$ . Neglecting migration, the centenarian forecast is then produced by the age-specific population in 2014 and mortality forecast.

## Results and discussion

The forecasts of age-specific death rates for males and females in 2015 and 2050 are shown in Figures 6 and 7. Sharply differing from the Coale-Kisker assumption, the mean forecasts of death rates at age 110 years declined remarkably; and in 2015 they are obviously smaller than 0.66, the lowest possible level of the Coale-Guo assumption. Although our method stands on a deductive basis and utilizes observed mortality at ages 80 years and older, these mean forecasts may not be necessarily close to reality, because future reality is uncertain.

In this paper, 1000 random trajectories are used for both  $k(t)$  and  $r(c)$  to produce stochastic forecasts. For 2015, the 2nd year of forecasting, the uncertainty is forecasted as small, but the largest 95% confidence interval seems to appear at age 95. Because the cohort aged 114 in 2015 was aged 80 in 2000, the earliest one with unknown  $r(c)$ , so the uncertainty of its forecasted  $r(c)$  is larger than that of cohorts younger than 114 at 2015, whose forecast horizons of  $r(c)$  are shorter. But why is the uncertainty smaller for cohorts older than 115 in 2015? This is because the  $r(c)$  are not estimated but forecast, as in Figure 5.

In 2050, however, the forecasted uncertainty is quite large at ages older than 100 years. For these ages there are two sources of uncertainty. One is the uncertainty in the forecasted death rate at age 80, due mainly to over-time random fluctuations modeled by the Lee-Carter method. The

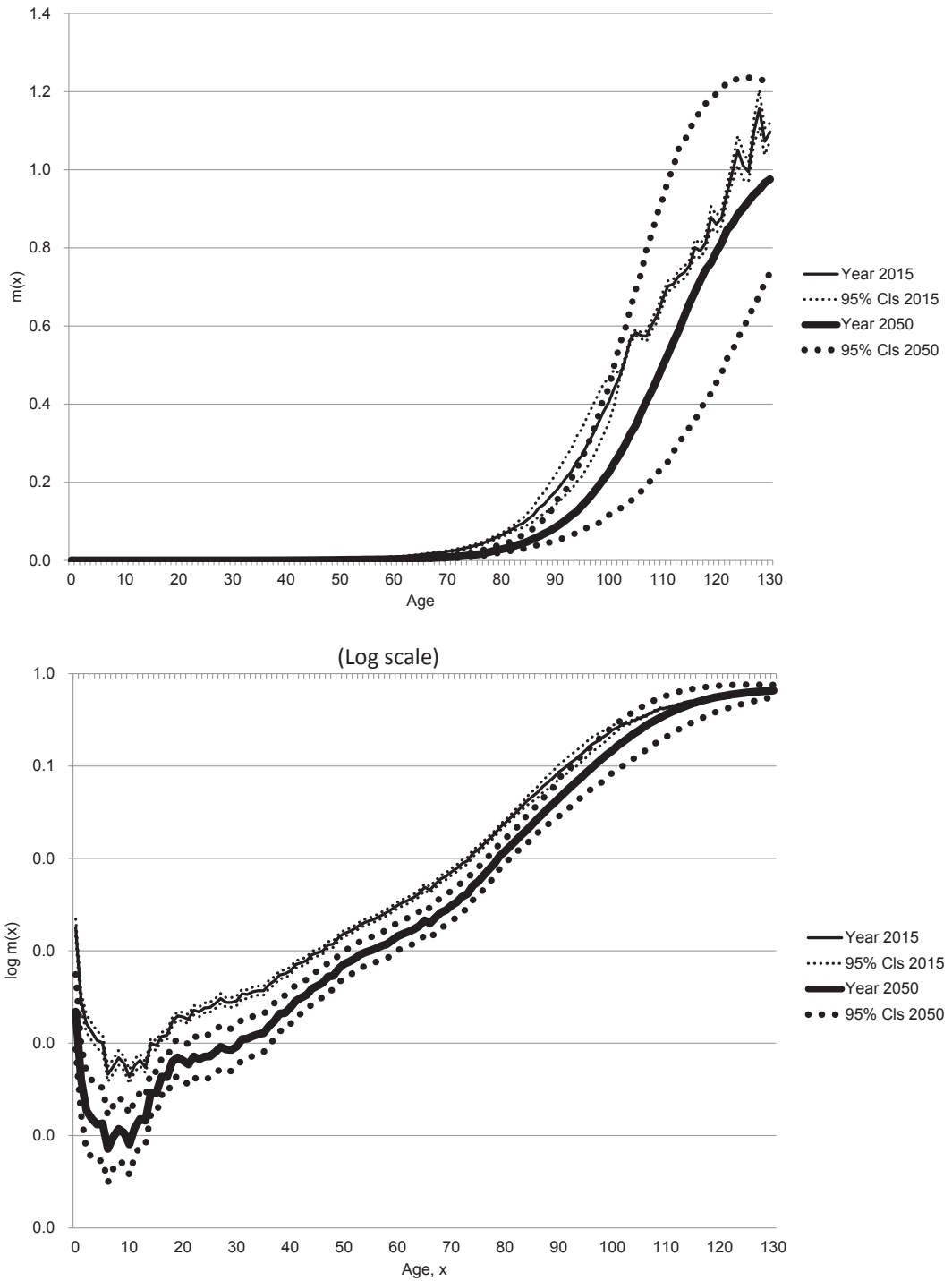


Figure 6. Male mortality rate stochastic forecasts

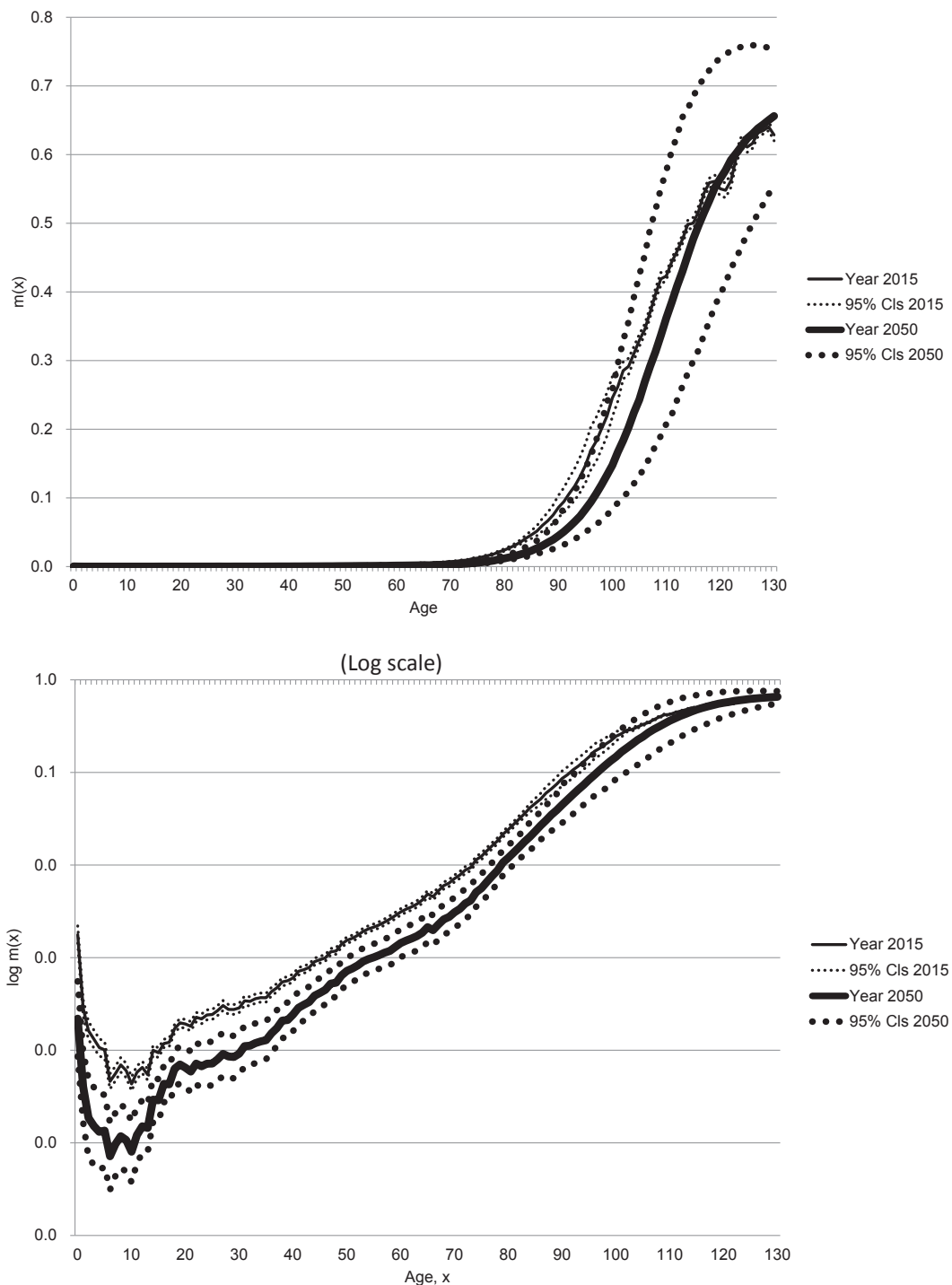


Figure 7. Female mortality rate stochastic forecasts

other is the uncertainty in  $r(c)$ , which comes from over-cohort random changes and binomial noise. Strictly speaking, the strength of binomial noise depends on population size; our results reflect the average effect of populations in the historical period. Nevertheless, these confidence intervals are still informative.

The stochastic forecasts of centenarians are shown in Figures 8 and 9. The 95% confidence interval increases faster than that of mortality, because they are determined by uncertainty in survival rates, which accumulates effects of random changes in mortality. The mean forecasts seem surprising. They suggest 0.29 million male and 1.10 million female centenarians by 2050 in Japan. In fact, however, reaching these numbers the average annual rate of increase is merely 10.7% for male and 8.4% for female centenarians. How large a group in practical terms would the 1.39 million centenarians be relative to the total population? Given that the country's 2010 population of 128 million will be declining, there could be more than 1.1 centenarians for every 100 people in 2050. As an analogy, the proportion of centenarians would be close to twin siblings who hold about 2% population, although centenarians would be of course much less visible in everyday life.

In Figures 8 and 9, we have indicated estimates by the UN and the Japanese government simultaneously. The estimated values are within the 95% standard and, as can be seen, are rather low. Our estimation is, it may be said, a conservative calculation that does not rely on a hypothesis that the centenarian population and yet the difference in the result regarding this population is large, which means that in the future a significant policy blind spot may emerge in terms of the medical and nursing care that centenarians will be needing.

Mortality decline seems to remain the main reason for the continuous increase of centenarians. Figure 10 compares the forecasted number of centenarians to that of mortality being fixed at the level of 2013. Without mortality decline after 2013, female centenarians would probably increase only slightly due to earlier mortality declines, and the number of male centenarians would remain almost constant as effects of WWII would perhaps offset the effects of mortality decline prior to 2013. The ratio of female to male centenarians in 2050 is forecasted as 3.7, down remarkably from that of 6.8 in 2013. This is because at ages younger than 80 years mortality decline is forecasted to be faster for males than for females.

The age structure of centenarians would become much older as shown in Figure 11, reflecting significant effects of mortality decline for centenarians. The ratio of those older than 105 years to all centenarians is forecasted as 12% for males and 21% for females in 2050, compared to about 5% for both sexes before 2014. Super-centenarians, those aged 110 and over, would become visible in 2050, as the ratio of super-centenarians to centenarians is forecasted as 0.6% for males and 2.6% for females.

Life span, defined as the maximum age of living, is an issue of wide interest.

The longest observed life span on November 26, 2014, according to the Gerontology Research Group (<http://supercentenarian-research-foundation.org/TableE.aspx>), was 111 years for males and 116 years for females. Incidentally, both of these individuals are Japanese.

However, in stochastic forecasting, life span is uncertain. For each random trajectory in our

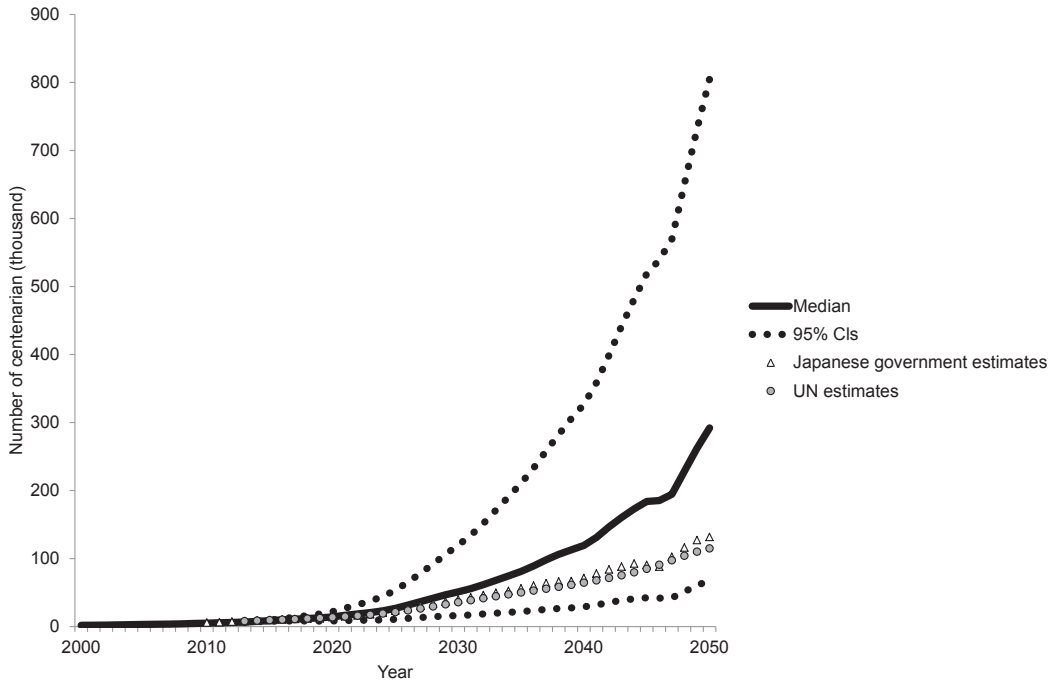


Figure 8. Stochastic forecasts of male centenarian population in Japan, 2000-2050

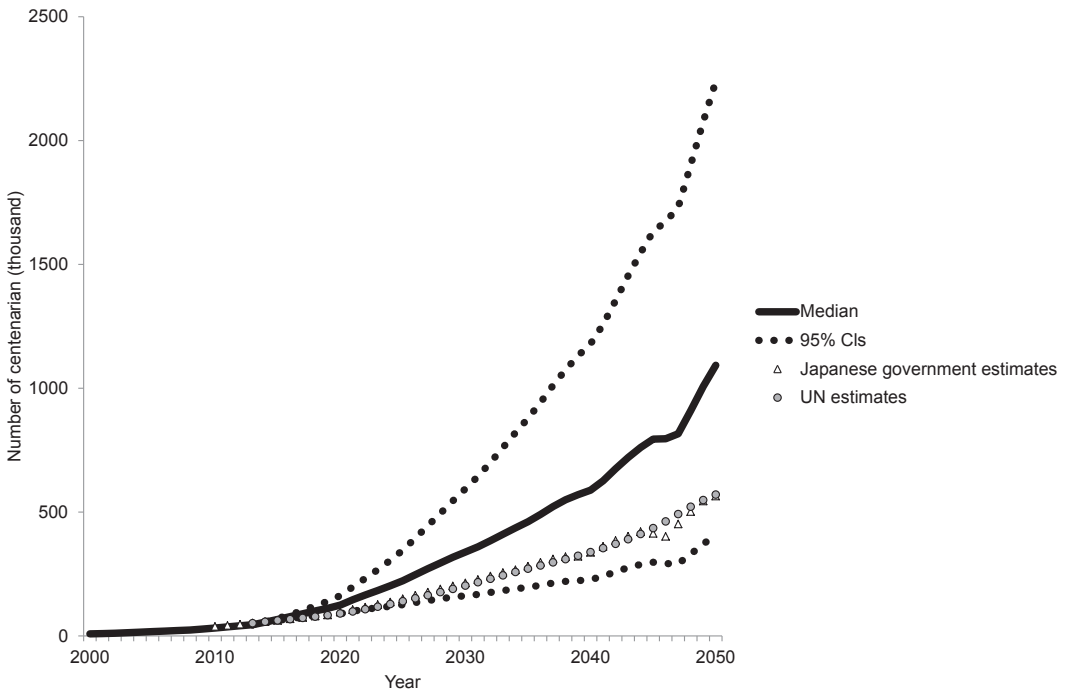


Figure 9. Stochastic forecasts of female centenarian population in Japan, 2000-2050



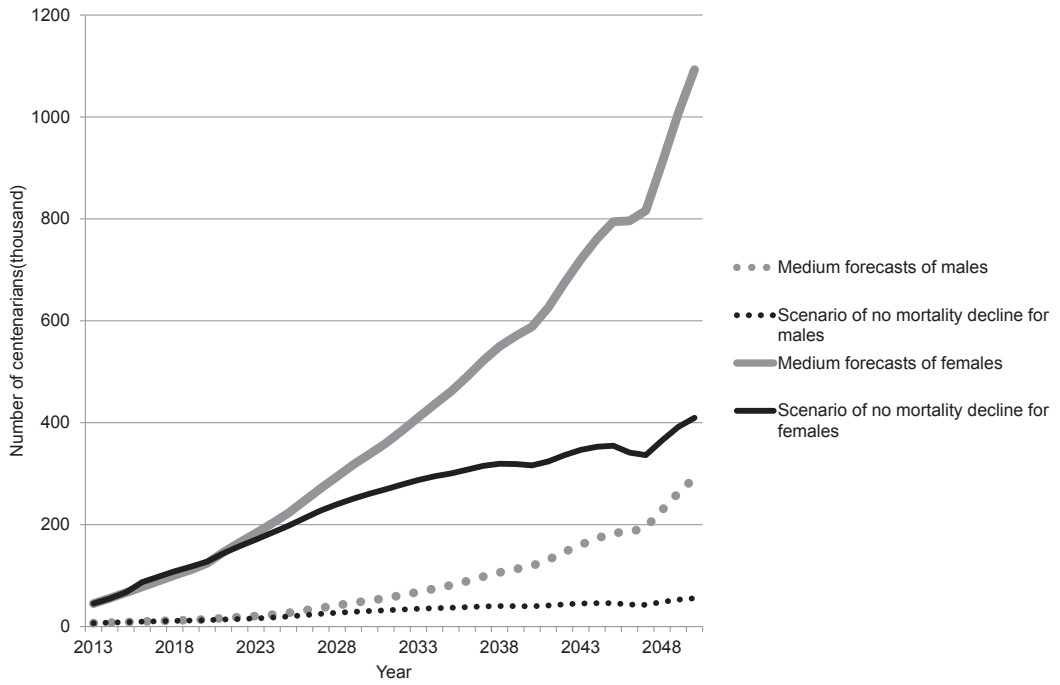


Figure 10. Number of centenarians in Japan by scenario, 2013-2050

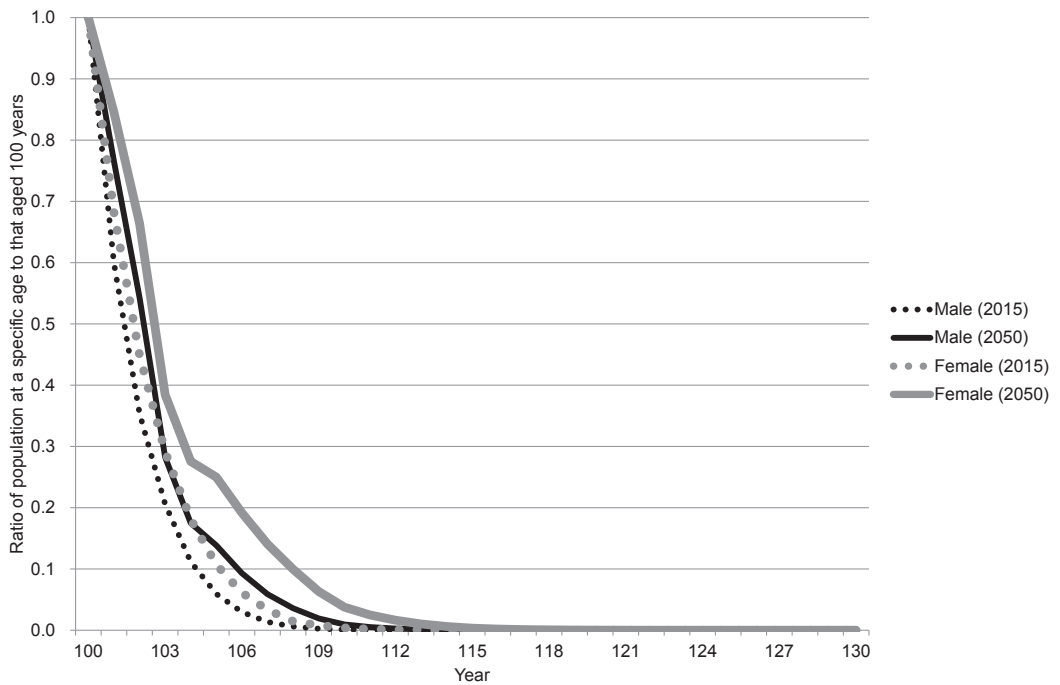


Figure 11. Age structure of centenarians in Japan , 2015 and 2050

stochastic forecast, life span can be defined as the age larger than the age at which the number of population is smaller than 1. The probability distributions of life span in 2050 are shown in Figure 12. It can be seen that the most possible life span, which appears with the largest probability, is 120 years for males and 129 years for females. From the figure we can also see that the most possible ones are not much more possible than others, so it is a better solution to calculate the mean value of life span. We can compute the exact mean value of life span because, as can also be seen in the figure, the maximum modeling age is limited by the data to 144 years.

Consequently, there are no survivors at age 144 in 2050. With this in mind, we have conducted calculations and concluded that the mean value of life span is larger than 121.3 for males and 126.9 for females in 2050. The highest record of human life span was 122 years, achieved by a French woman in 1997. Whether or not life span is a constant biological limit and observations can only approach it is a debatable issue (Weeks, 1999).

It is believed that human life span will dramatically increase in the future thanks to advances in medical and genetic technology, but even if such epochal change is not accomplished, life span will probably continue to increase. That is, life span can be raised continuously, because the consistent efforts for improving living standards will reduce mortality, and because the potential for mortality to decline is infinite at old ages according to the frailty-mortality model.

It might be impossible to dramatically raise the number of people over age 120 in the future without some sort of intervention such as gene manipulation. Nonetheless, judging by our estimation,

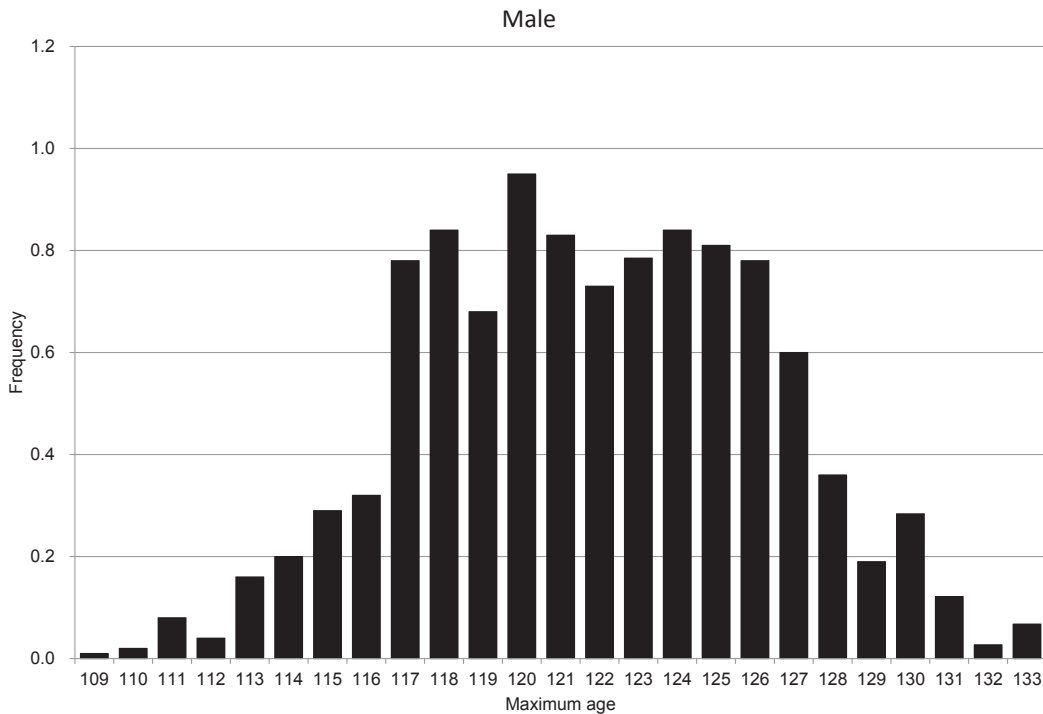


Figure 12a. Japanese life span in 2050

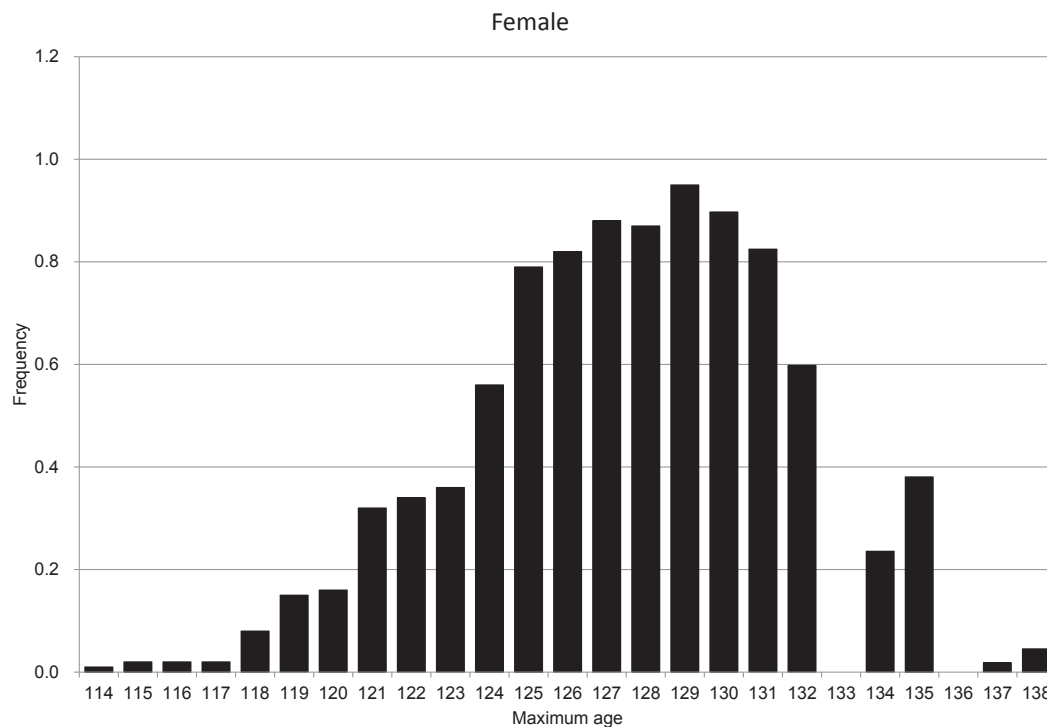


Figure 12b. Japanese life span in 2050

there probably will be no such limit when it comes to increasing the number of humans living to be ninety or 100. There are many things that the state and individuals, after they retire, need to do in the future, such as creating a social security system capable of supporting hundred-year long lives. Japan is undoubtedly now experiencing an age of longevity, an age in which it is an ordinary thing for many individuals to reach old age. It may be said that Japan is the first society to embody the human dream of longevity, which only in the most recent history of human kind began to show signs of becoming a reality. As such, Japan needs to urgently take steps to create social systems that view individuals' lives as lasting a hundred years.

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