Nikkei 225 Long-term Trend Analysis

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1. Introduction

For stock investment, if upward and downward trends are clearly identified, by taking a long position during upward trends and a short position on downward trends, it is possible to make a profit. However, it is difficult to define trends in general, and so a variety of trend analysis methods have been developed. Many investors use moving averages and moving average divergence for basic trend analysis, such as 25/75-day moving averages, or 13/26-week moving averages. However, even using these indicators it is difficult to pinpoint the turning point between upward and downward trends for stock prices.

The Markov-switching model is a representative trend analysis model using time series analysis. For asset price analysis, it is standard to use a model incorporating volatility. For experimental studies of Japanese stock markets using the Markov-switching model in contrast to volatility models, there is the study by Satoyoshi (2004) on the TOPIX. There are also studies by Satoyoshi and Mitsui (2011b, 2013) on the Nikkei 225. This paper uses a Markov-switching GARCH model (MS-GARCH) combining the Markov-switching model and a representative volatility model, the GARCH (Generalized Autoregressive conditional heteroscedasticity) model, to analyze long-term bear and bull trends in the Nikkei 225.

The brief descriptions of the following chapters are as follows: Chapter 2 explains the MS-GARCH model. Chapter 3 shows the empirical results on the Nikkei 225 data. Chapter 4 concludes the study and considers future issues.

2. Model

A brief explanation of MS-GARCH will now be provided [1]. Assuming GARCH's order selection as GARCH (1,1), the MS-GARCH (1,1) model will be considered. When R_t is the rate of return on asset price at time t, the process of R_t and volatility σ_t^2 can be expressed as follows:

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1) For details, see Gray (1996), Klaassen (2002), and Haas *et al.* (2004).

$$R_t = \mu(s_t) + \epsilon_t(s_t), \tag{2.1}$$
$$\epsilon_t(s_t) = \sigma_t(s_t)z_t,\ z_t \sim i.i.d., E[z_t] = 0, Var[z_t] = 1, \tag{2.2}$$
$$\sigma_t^2(s_t) = \omega(s_t) + \alpha(s_t)\epsilon_{t-1}^2(s_t) + \beta(s_t)\sigma_{t-1}^2(s_t), \tag{2.3}$$
$$\sigma_{t-1}^2(s_t) = E[\sigma_{t-1}^2(s_{t-1})|s_t, I_{t-1}]. \tag{2.4}$$

Here, $\mu(s_t)$ is the constant term and $\epsilon_t(s_t)$ is the variance term, and we will assume there is no autocorrelation in the returns. *i.i.d.* indicates that it is independent and identically distributed from the past. $E[\cdot]$ is the expected value, $Var[\cdot]$ is the variance, and $E[\cdot|\cdot]$ is the conditional expected value. I_{t-1} is the information set $I_{t-1} = \{R_{t-1}, R_{t-2}, \cdots\}$ up until $t-1$. Also, it is assumed that the constant term $\mu(s_t)$ and the volatility $\sigma_t(s_t)$ will both switch simultaneously in accordance with the stochastic variable s_t. To ensure the non-negativity of the volatility, it is assumed that $\omega(s_t)$, $\alpha(s_t)$, $\beta(s_t) > 0$. The stochastic variable s_t which is not observed in the Markov-switching model follows the Markov process, and is defined by the following transition probability.

$$p_{i|j} = Pr[s_{t+1} = i \mid s_t = j],\ i, j = 0, 1. \tag{2.5}$$

Here, $Pr[s_{t+1} = i \mid s_t = j]$ indicates the probability of transition from state j to state i [2].

However, the probability of transition from state j of this period to state i of the next period is only dependent on the state of this period as shown below:

$$Pr[s_{t+1} = i \mid s_t = j, s_{t-1}, s_{t-2}, \dots] = p_{i|j} = Pr[s_{t+1} = i \mid s_t = j]. \tag{2.6}$$

Here,

$$\sum_{i=0}^{1} p_{i|j} = 1, j = 0, 1. \tag{2.7}$$

At this time, the transition matrix $\mathbf{P}$ for s_t is

$$\mathbf{P} = \begin{pmatrix} p_{0|0} & p_{0|1} \\ p_{1|0} & p_{1|1} \end{pmatrix}. \tag{2.8}$$

However, $0 \leq p_{0|0}, p_{1|1} \leq 1$. For this study, $s_t = 0$ will be considered a bull market, and $s_t = 1$ will be considered a bear market. Therefore, $p_{1|0}$ indicates the transition probability from a bull market to a bear market, and $p_{0|1}$ indicates the transition probability from a bear market to a bull market. Also, $p_{0|0}$, $p_{1|1}$ indicate the transition probability that a bull market will be maintained or that a bear

2) It may also be written as $p_{ji} = Pr\ [s_{t+1} = i \mid s_t = j]$

Table 1 : Summary Statistics for the Nikkei 225 Monthly Returns R_t (%)

Sample Period: Jun. 2000 – Oct. 2013

Sample Size: 773

	Mean	Std Dev.	Skewness	Kurtosis	Max.	Min.	Normality test
Nikkei 225	0.569	5.909	− 0.486	4.649	23.002	− 27.216	49.991**

** denotes statistical significance at the 1% level.

market will be maintained, respectively. The constraint of $\mu(0) > \mu(1)$ will also be placed. The distribution of the variance term when carrying out experimental analysis will be assumed to follow the standard normal distribution below [3].

$$z_t \sim i.i.d.N(0,1). \tag{2.9}$$

For estimation of the parameters, maximum likelihood estimation is conducted using the statistical and time series analysis software *PcGive* [4]

3. Data and Empirical Results

3.1 Data

For this paper, the monthly Nikkei 225 was used as data, with data acquired from Nikkei NEEDS Financial Quest. The sample period used was from May 1949 to October 2013 (refer to the solid line on Figure 1) . The return was calculated using the ratio of change (%) of each closing price (refer to the solid line on Figure 2) . The Nikkei 225 returns sample period was from June 1949 to October 2013, with 773 samples. As summary statistics of the data, the mean, standard deviation, skewness, kurtosis, maximum value, minimum value and normality are given in Table 1. A histogram and density function for the Nikkei 225 returns are given in Figure 3. Here, the density function and normal approximation are drawn overlapping. N (s = 5.909) demonstrates that the normal approximation from Table 1 follows the normal distribution N (0.569, 5.9092) with an average of 0.569, and the variance is 5.9092.

3.2 Empirical Results

Table 2 shows the estimation results for the MS-GARCH (1,1) model. Estimated values for $\mu(0)$ and $\mu(1)$ were 1.664 and − 2.854, respectively, which are statistically significant estimation results. $\mu(0)$ which represents bull markets was positive, and $\mu(1)$ which represented bear markets was

3) It is known that the distribution of stock return flares more at the bottom than a standard deviation. Therefore the variance term must be analyzed using a distribution with a broad base, such as t distribution or Generalized Error Distribution. This point will be further examined in the future.

4) For further information on using *PcGive* for Markov-switching estimation, refer to Doornik and Hendry (2013).

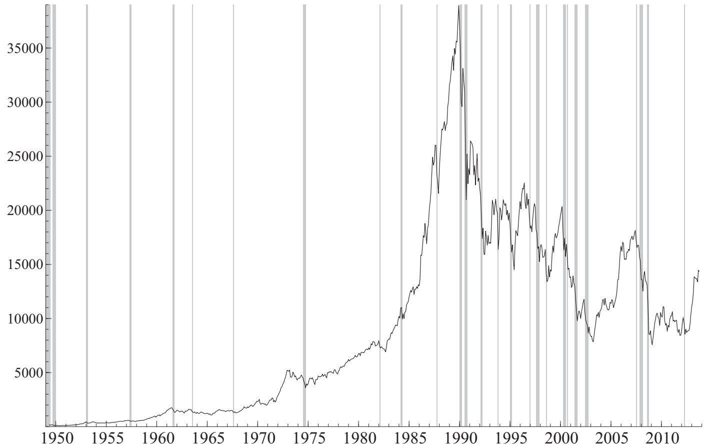

Figure 1 : Monthly Index on the Nikkei 225 and Bear Phases

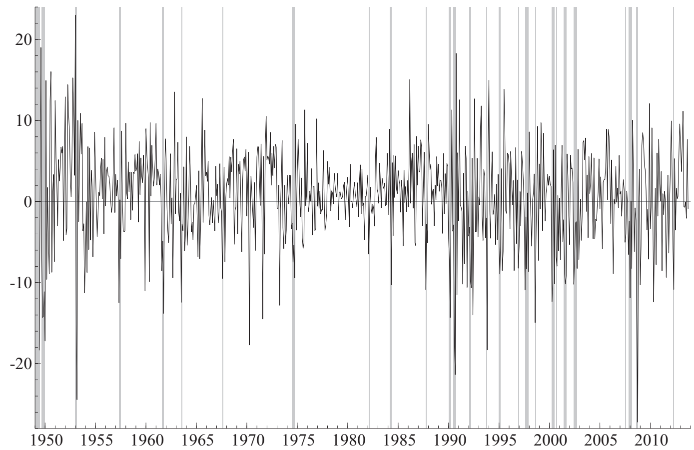

Figure 2 : Monthly Return Series on the Nikkei 225

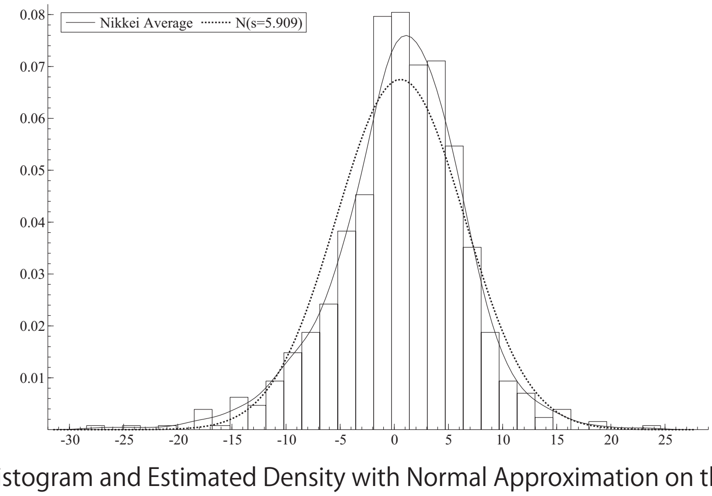

Figure 3 : Histogram and Estimated Density with Normal Approximation on the Nikkei 225

Table 2 : Estimation Results for the MS-GARCH (1,1) Model

$$R_t = \mu(s_t) + \epsilon_t(s_t),\ \epsilon_t(s_t) = \sigma_t(s_t) z_t,\ z_t \sim i.i.d.N(0,1),$$
$$\sigma_t^2(s_t) = \omega(s_t) + \alpha(s_t)\epsilon_{t-1}^2(s_t) + \beta(s_t)\sigma_{t-1}^2(s_t),$$
$$P = \begin{pmatrix} p_{0|0} & p_{0|1} \\ p_{1|0} & p_{1|1} \end{pmatrix}.$$

	μ (0)	μ (1)	ω (0)	ω (1)	α (0)	α (1)	β (0)	β (1)
Estimates	1.664*	− 2.854*	0.456*	2.196*	0.055*	0.211*	0.899*	0.747*
Standard Errors	(0.218)	(1.022)	(0.206)	(1.103)	(0.021)	(0.102)	(0.035)	(0.119)

	$p_{0\|0}$	$p_{1\|1}$	$-\ln L$	$Q(20)$	$Q^2(20)$
Estimates	0.875*	0.613*	− 2386.365	28.626	19.690
Standard Errors	(0.039)	(0.138)			

* denotes statistical significance at the 5 % level.

negative. Therefore it was confirmed that when state variable s_t is $s_t = 0$, is a bull market, and when $s_t = 1$ is a bear market. Estimated values for $\omega(0)$ and $\omega(1)$ were 0.456 and 2.196, respectively, which are statistically significant estimation results. Since $\omega(0) < \omega(1)$, we can see that bear markets have higher volatility values than bull markets. The estimated value of the parameter showing sustention of volatility shock is $\alpha(0) + \beta(0) = 0.954$, $\alpha(1) + \beta(1) = 0.954$, demonstrating that sustention of shock is higher in both bear and bull markets.

The estimated value for s_t transition probabilities $p_{0|0}$ and $p_{1|1}$ were 0.875 and 0.613 respectively, which are statistically significant estimation results. $p_{0|0}$ is near to 1 and suggests that once switching to bull occurs, that state will continue for a long time. Since $p_{0|0} > p_{1|1}$, we can see that bear markets do not continue longer than bull markets. Since both average μ and volatility σ switch simultaneously according to state variable s_t, we can see that once switched to low volatility that state will continue for a long time, but a high volatility state does not continue for a long time. $Q(20)$ and $Q^2(20)$ indicate the Ljung-Box Q statistic for standardized residual error to $(\hat{\epsilon}\hat{\sigma}^{-1})$ the 20th degree and the square of that. Here, an asymptotical χ^2 distribution with a freedom of 20 is followed. With the values of $Q(20)$ and $Q^2(20)$, with a null hypothesis significance standard of 5% they cannot be dismissed. From this we can see that the MS-GARCH (1,1) model has found the autocorrelation of the Nikkei 225 volatility.

Table 3 demonstrates the phases when $s_t = 0$ (bull market) . The total number of months for bull markets is 670 months (86.68% of the total) , with the result of an average bull market period lasting 14.26 months. Table 4 demonstrates the phases when $s_t = 1$ (bear market) . The total number of months for bear markets is 103 months (13.32% of the total) , with the result of an average bear market period lasting 2.19 months. From these results, it is clear that it takes time for the Nikkei 225 to increase, but declines are far shorter than upwards periods. The shaded portions of Figures 1 and 2 show the Nikkei 225 bear phases. In particular, the graph in Figure 1 shows steep declines clearly.

Table 3 : Regime Classification for the Nikkei 225 (Regime 0)

Total: 670 months (86.68%) with average duration of 14.26 months

Period	Days	Avg. Prob.
1949-08 – 1949-08	1	0.559
1950-02 – 1952-12	35	0.821
1953-04 – 1957-04	49	0.845
1957-08 – 1959-11	28	0.892
1960-01 – 1960-04	4	0.739
1960-06 – 1961-07	14	0.891
1961-11 – 1962-08	10	0.768
1962-10 – 1963-06	9	0.811
1963-09 – 1965-02	18	0.751
1965-04 – 1965-04	1	0.527
1965-06 – 1967-07	26	0.845
1967-10 – 1967-10	1	0.586
1967-12 – 1970-03	28	0.862
1970-05 – 1971-07	15	0.828
1971-09 – 1973-03	19	0.869
1973-05 – 1974-06	14	0.769
1974-11 – 1975-07	9	0.824
1975-09 – 1981-08	72	0.899
1981-10 – 1982-01	4	0.848
1982-04 – 1984-02	23	0.895
1984-06 – 1985-06	13	0.886
1985-08 – 1986-02	7	0.875
1986-04 – 1987-09	18	0.842
1987-12 – 1989-12	25	0.896
1990-05 – 1990-06	2	0.587
1990-11 – 1992-01	15	0.727
1992-05 – 1992-05	1	0.502
1992-07 – 1993-09	15	0.824
1993-12 – 1994-12	13	0.778
1995-04 – 1995-04	1	0.545
1995-06 – 1996-06	13	0.868
1996-08 – 1996-11	4	0.654
1997-02 – 1997-07	6	0.794
1998-01 – 1998-07	7	0.752
1998-10 – 2000-03	18	0.872
2000-08 – 2000-08	1	0.520
2000-11 – 2001-05	7	0.644
2001-10 – 2002-05	8	0.809
2002-11 – 2005-03	29	0.833
2005-05 – 2006-04	12	0.858
2006-06 – 2007-06	13	0.897
2007-09 – 2007-10	2	0.565
2008-04 – 2008-07	4	0.672
2008-11 – 2010-04	18	0.797
2010-06 – 2011-07	14	0.752
2011-09 – 2012-03	7	0.734
2012-06 – 2013-10	17	0.877

Table 4 : Regime Classification for the Nikkei 225 (Regime 1)

Total: 103 months (13.32%) with average duration of 2.19 months

Period	Days	Avg. Prob.
1949-06 – 1949-07	2	0.746
1949-09 – 1950-01	5	0.686
1953-01 – 1953-03	3	0.961
1957-05 – 1957-07	3	0.755
1959-12 – 1959-12	1	0.998
1960-05 – 1960-05	1	0.806
1961-08 – 1961-10	3	0.961
1962-09 – 1962-09	1	0.731
1963-07 – 1963-08	2	0.770
1965-03 – 1965-03	1	0.700
1965-05 – 1965-05	1	0.681
1967-08 – 1967-09	2	0.790
1967-11 – 1967-11	1	0.661
1970-04 – 1970-04	1	1.000
1971-08 – 1971-08	1	0.996
1973-04 – 1973-04	1	0.987
1974-07 – 1974-10	4	0.823
1975-08 – 1975-08	1	0.554
1981-09 – 1981-09	1	0.752
1982-02 – 1982-03	2	0.755
1984-03 – 1984-05	3	0.735
1985-07 – 1985-07	1	0.566
1986-03 – 1986-03	1	0.996
1987-10 – 1987-11	2	0.791
1990-01 – 1990-04	4	0.833
1990-07 – 1990-10	4	0.808
1992-02 – 1992-04	3	0.634
1992-06 – 1992-06	1	0.625
1993-10 – 1993-11	2	0.774
1995-01 – 1995-03	3	0.678
1995-05 – 1995-05	1	0.611
1996-07 – 1996-07	1	0.628
1996-12 – 1997-01	2	0.604
1997-08 – 1997-12	5	0.715
1998-08 – 1998-09	2	0.794
2000-04 – 2000-07	4	0.828
2000-09 – 2000-10	2	0.566
2001-06 – 2001-09	4	0.727
2002-06 – 2002-10	5	0.648
2005-04 – 2005-04	1	0.504
2006-05 – 2006-05	1	0.848
2007-07 – 2007-08	2	0.695
2007-11 – 2008-03	5	0.841
2008-08 – 2008-10	3	0.861
2010-05 – 2010-05	1	0.694
2011-08 – 2011-08	1	0.734
2012-04 – 2012-05	2	0.689

4. Conclusion

In this paper, the MS-GARCH (1,1) model was used to perform long-term trend analysis for the Nikkei 225. Results from performing experimental tests on the monthly Nikkei 225 data focusing on bull and bear markets showed that Nikkei 225 has statistically significant bull and bear markets. In other words, it was possible to determine bull markets with a high expected returns and low volatility, and bear markets with a low expected returns and high volatility. Also, it became clear that although the Nikkei 225 requires long periods to increase, declines occur in much shorter periods than the periods of increase. Although this paper used the GARCH model to formulate volatility, in Satoyoshi and Mitsui (2011a), the MS-EGARCH (Exponential GARCH) model was used to analyze bull and bear markets of the Nikkei average using daily data. Therefore, it seems likely that the MS-EGARCH model can be used for long-term trend analysis. In Maheu *et al.* (2012), trend identification included not only the two phases of bear market and bull market, but instead proposed a 4-state Markov-switching model with the four phases of bear market, bear market rally, bull market, and bear market correction. In Satoyoshi and Mitsui (2013), they follow Maheu *et al.* (2012) and uses a 4-state Markov-switching model for trend analysis of the Nikkei average. Therefore, it is also important to separate trends in detail for long-term trend analysis.

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