Collusion, Countervailing Incentives, and Private Information

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Abstract

This paper studies optimal contracts and collusion in a principal-supervisor-agent model with private information. Unlike the existing literature on collusion and organizations under asymmetric information, we examine the possibility of collusion between the supervisor and the agent in a setting in which the agent's cost is composed of not only a variable cost but also a fixed one, both of which depend on private information. We show that when a difference in the amount of fixed costs with respect to the agent's type is sufficiently large, countervailing incentives may arise. We characterize optimal collusion-proof contracts under the conditions that the supervisor can collude with the agent and that countervailing incentives will prevail.

Keywords: Collusion; Contract; Countervailing incentives; Private information.

JEL: D86, L22, L51

1 Introduction

This paper examines optimal contracts and the possibility of collusion between a supervisor and an agent in a three-layer hierarchy model with private information. Unlike the literature on collusion under asymmetric information, we examine collusion-proof contracts in a setting in which the agent's cost is composed of not only a variable cost but also a fixed one, both of which depend on private information. We assume that the agent has two types. One type has a high marginal cost and a low fixed cost. The other has a low marginal cost and a high fixed cost. When the supervisor receives a signal about the agent's type, he can either transmit his information to the principal truthfully or conceal what was received. Then the possibility of collusion between the supervisor and the agent arrises. We show that when a difference in the amount of fixed costs with respect to the agent's type is sufficiently large, countervailing incentives may result. This implies that the efficient agent produces a higher quality product than the first best quality level and that the inefficient agent obtains an information rent. We derive optimal collusion-proof contracts when countervailing incentives exist.

This paper is related to two strands of the literature on contract theory. The first strand deals

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with collusion under asymmetric information. Since the pioneering work of Tirole (1986), much work has been done in examining collusion under asymmetric information. Laffont (1990) examines optimal contracts in a principal-supervisor-agent model (see also Laffont and Tirole (1993)). Laffont and Martimort (1997) consider collusion-proof contracts and characterize optimal contracts with collusion-proofness. However, these papers do not consider fixed costs that depend on the agents' types. We consider a more general cost function that includes fixed costs.

The second strand is concerned with the problems of countervailing incentives under asymmetric information. Lewis and Sappington (1989) assume one-dimensional uncertainty regarding marginal costs and fixed costs when examining the possibility of countervailing incentives. Maggi and Rodriguez-Clare (1995) further examine the issue on countervailing incentives. Jullien (2000) explores the effects of type-dependent participation constraints on optimal contracts. However, these papers do not address the question of collusion under asymmetric information (see also Laffont and Martimort (2002)).

In this paper, we study optimal contracts and collusion in a three-layer hierarchy model in which the cost function of the agent includes fixed costs that depend on its type. We show that when the difference in the magnitude of fixed costs with respect to productivity types is sufficiently large, countervailing incentives may arise because the set of binding incentive compatibility constraints and participation constraints depends on the value of the difference. Thus, it is of critical importance to consider type dependent participation constraints when examining optimal contracts and collusion under asymmetric information.

The remainder of the paper is organized as follows. In Section 2, we present a principal-supervisor-agent model and note basic assumptions. In Section 3, as a benchmark, we characterize the optimal contract without collusion. In Section 4, we examine optimal contracts when the supervisor and the agent can collude. In Section 5, we extend the basic supervision technology to a more general form and examine optimal collusion-proof contracts. Section 6 concludes the paper.

2 The Model

We consider a principal-supervisor-agent hierarchy model. Suppose that a government (the principal) contracts with a firm (the agent) implementing a public project. The quantity or quality of the project is denoted as q (which henceforth will be used to refer to quality). The project yields social benefit $S(q)$. For all $q > 0$, we assume that $S(q)$ is twice continuously differentiable, strictly increasing and concave.

The cost function is given by

$$C(q, \theta) = \theta q + F(\theta), \tag{1}$$

where $\theta > 0$ is the constant marginal cost and $F(\theta)$ is the fixed cost. The parameter θ is the relevant private information of the firm. We assume that parameter θ takes either θ_1 or θ_2 with θ_1

$<\theta_2$ and that $F(\theta_1)>F(\theta_2)$ Thus we consider the case in which a higher marginal cost is associated with a lower fixed cost and vice versa. This inverse relationship may arise because in general a higher fixed cost guarantees a lower marginal cost and vice versa. Let $\alpha = \Pr(\theta = \theta_1)$, $0 < \alpha < 1$. Let t denote monetary transfers from the government to the firm, $t \geqq 0$.

The government can contract with a supervisor (the regulatory agency) to bridge its information gap. The supervisor makes his report r to the government. Let m be a transfer from the government to the supervisor, $m \geqq 0$. Following Laffont and Tirole (1993), we assume that there exists distortion or deadweight loss by funding the public project. Let $\lambda > 0$ denote the cost of public funds.

Consumers have the following utility function:

$$V = S(q) - (1+\lambda)(t+m). \tag{2}$$

The firm's payoff U is given by

$$U = t - \theta q - F(\theta). \tag{3}$$

The supervisor's utility function is given by

$$X = m - m_R \geqq 0, \tag{4}$$

where m_R is his reservation utility.

We assume that all parties are risk neutral.

The benevolent government maximizes social welfare W which is given by

$$W = V + U + X = S(q) - (1+\lambda)\{m_R + \theta q + F(\theta)\} - \lambda U - \lambda X. \tag{5}$$

The government offers a contract (q_i, t_i) to the firm and a contract (r_i, m_i) to the supervisor, $i = 1, 2$. When designing optimal contracts, the government solves its payoff maximization problem subject to incentive compatibility constraints and participation constraints. An incentive compatibility constraint (ICC) guarantees that the firm prefers the contract that is designed for it. A participation constraint (PC) guarantees that the firm accepts the designated contract.

The sequence of events in the contracting game proceeds as follows.

At t = 1, nature determines a firm's productivity type θ. Only the firm discovers it. The supervisor learns a signal.

At t = 2, the government offers contracts to the supervisor and the firm. Then the supervisor can sign a side contract with the firm.

At t = 3, the supervisor makes his report to the government and the firm undertakes the public project.

At t = 4, the government provides transfers to the firm and the supervisor.

3 Characterization of optimal contracts without collusion

In this section, as a benchmark, we consider an optimal contract when there is no collusion between the supervisor and the agent. First, under full information, the government maximizes the following expected social welfare:

$$W = \alpha\,[S(q_1) - (1+\lambda)\,\{m_R + \theta_1 q_1 + F(\theta_1)\} - \lambda U_1 - \lambda X_1] \\ + (1-\alpha)\,[S(q_2) - (1+\lambda)\,\{m_R + \theta_2 q_2 + F(\theta_2)\} - \lambda U_2 - \lambda X_2], \tag{6}$$

where $U_i = t_i - e_i q_i - F(\theta_i)$ and $X_i = m_i - m_R$, $i = 1, 2$.

Then, the optimal contract satisfies

$$S_q(q^{FB}) = (1+\lambda)\,\theta_1 \tag{7}$$

and

$$S_q(q_1^{FB}) = (1+\lambda)\,\theta_2, \tag{8}$$

where S_q denotes $\dfrac{dS(\cdot)}{dq}$ and q_i^{FB} the first best quality for $\theta = \theta_i$, $i = 1, 2$.

Next, we examine optimal contracts under asymmetric information. The government's problem in this case is to maximize the expected welfare subject to the following incentive compatibility constraints (ICCs):

$$U_1 = t_1 - \theta_1 q_1 - F(\theta_1) \geqq t_2 - \theta_1 q_2 - F(\theta_1)$$

and

$$U_2 = t_2 - \theta_2 q_2 - F(\theta_2) \geqq t_1 - \theta_2 q_1 - F(\theta_2)$$

and the participation constraints (PCs):

$$t_1 - \theta_1 q_1 - F(\theta_1) \geqq 0$$

and

$$t_2 - \theta_1 q_2 - F(\theta_1) \geqq 0.$$

These ICCs and PCs can be rewritten as follows:

$$U_1 \geqq U_2 + (\theta_2 - \theta_1) q_2 + \{F(\theta_2) - \mathrm{F}(\theta_1)\}, \tag{9}$$

$$U_2 \geqq U_1 - (\theta_2 - \theta_1) q_1 - \{F(\theta_2) - F(\theta_1)\}, \tag{10}$$

$$U_1 \geqq 0, \tag{11}$$

and

$$U_2 \geqq 0. \tag{12}$$

The following proposition characterizes the optimal contract with positive rents for the firm.

Proposition 1 *The optimal contract has the following features:*

(i)

$$S_q(q_1^{SB}) = (1+\lambda)\,\theta_1 \tag{13}$$

and

$$S_q(q_2^{\mathrm{SB}}) = (1+\lambda)\,\theta_2 + \lambda \frac{\alpha}{1-\alpha}(\theta_2 - \theta_1),\; q_2^{SB} < q_2^{FB}, \tag{14}$$

where superscript SB denotes the second best. Note that q_1^{SB} satisfies $q_1^{SB} \leqq \dfrac{F(\theta_1) - F(\theta_2)}{\theta_2 - \theta_1}$.

(ii)

$$S_q(q_1^{\mathrm{C}}) = (1+\lambda)\,\theta_1 - \lambda \frac{\alpha}{1-\alpha}(\theta_2 - \theta_1),\; q_1^{C} < q_1^{FB} \tag{15}$$

and

$$S_q(q_2^{\mathrm{SB}}) = (1+\lambda)\,\theta_2, \tag{16}$$

where superscript C denotes the countervailing incentives. Note that q_1^C satisfies $q_1^C \leqq \frac{F(\theta_1)-F(\theta_2)}{\theta_2-\theta_1}$.

Proof : See the Appendix.

This proposition says that in regime 1 ((i) above), θ_1-agent obtains a positive rent and there exists a downward distortion for θ_2.

It also shows that in regime 2 ((ii) above), countervailing incentives exist and thus we have an upward distortion for θ_1. The intuition behind this result is as follows. When a difference in fixed costs with respect to the firm's productivity types, $F(\theta_1)-F(\theta_2)$ is sufficiently large, countervailing incentives may arise, and, thus, there is an upward distortion for the efficient type θ_1.

4 Optimal contracts when collusion is possible

Now we analyze a three-level hierarchy in which there exist a principal, a supervisor, and an agent. Recall that X is the utility of the supervisor, that is, $X = m - m_R \geqq 0$, where m is a transfer from the government.

Suppose that the supervisor observes a signal ω with probability β that the firm is of type θ_1 and otherwise $\omega = ø$ with probability $1-\beta$. The supervisor can hide his information and report that the signal is empty. If $\theta = \theta_2$ the supervisor observes no information. If the supervisor reveals it to the government, the government can learn the signal. We assume that a signal is hard information.

To begin with, we consider the case of regime 1 (case (i) in Proposition 1). Let us examine optimal contracts when the government has full information and then analyze the case of asymmetric information. Suppose that $\omega = r = \theta$. Recall that r denotes the supervisor's report to the government. Then, the government is informed. Under full information, social welfare W^F is

$$W^F = \alpha\,[S(q_1^{FB}) - (1+\lambda)\,\{m_R + \theta_1 q^{FB} + F(\theta_1)\}\,] \\ + (1-\alpha)\,[S(q_1^{FB}) - (1+\lambda)\,\{m_R + \theta_2 q^{FB} + F(\theta_2)\}\,].$$

Next, suppose that $\omega = ø$. Then, the government is uninformed. Under the asymmetric information, social welfare W^N is

$$W^N = \alpha\,[S(q_1^{FB}) - (1+\lambda)\,\{m_R + \theta_1 q_1^{FB} + F(\theta_1)\} - \lambda\,\{F(\theta_1) - F(\theta_2) + (\theta_2 - \theta_1)q^{SB}\}\,] \\ + (1-\alpha)\,[S(q_2^{SB}) - (1+\lambda)\,\theta_2 q^{SB}].$$

Thus, the expected social welfare is

$$\beta W^F + (1-\beta)\,W^N. \tag{17}$$

Suppose that $r = ø$. Then, the government is uninformed. Note that we have

$$\Pr(\theta = \theta_1 \mid \omega = \text{ø}) = \frac{(1-\beta)\alpha}{1-\beta\alpha}.$$

Then, the first order conditions for the maximization of the expected social welfare (17) are

$$S_q(q^S) = (1+\lambda)\,\theta_1 \tag{18}$$

and

$$\begin{aligned} S_q(q_2^{S}) &= (1+\lambda)\,\theta_2 + \lambda \cdot \frac{(1-\beta)\alpha}{1-\alpha}\,(\theta_2 - \theta_1) \\ &> S_q(q_2^{SB}), \end{aligned} \tag{19}$$

where the superscript S denotes the case in which we deal with the supervisor. It follows from equation (19) that a downward distortion exists for the inefficient agent θ_2.

Next, we consider the case in which the supervisor and the firm can collude. Suppose that $\theta = \theta_1$ and $\omega = \theta_1$ Then, the firm obtains an information rent, $[(\theta_2 - \theta_1) q_2^{S} + F(\theta_1) - F(\theta_2)]$, if the supervisor conceals the information of $\omega = \theta_1$. We assume that there exists a transaction cost between the supervisor and the firm. Let ρ represent the transaction cost. To induce truth-telling behavior of the supervisor, the following collusion proofness constraint must be satisfied.

$$m^* \geqq [(\theta_2 - \theta_1) q_2^{S} + F(\theta_1) - F(\theta_2)] \cdot \left(\frac{1}{1+\rho}\right), \tag{20}$$

where m^* is the payment received by the supervisor if he reports that the firm is the lower marginal cost (efficient) type θ_1. We assume $\rho > \lambda$; otherwise, collusion occurs trivially.

The expected social cost is $\lambda \cdot \alpha \cdot \beta \cdot [(\theta_2 - \theta_1) q_2^{S} + F(\theta_1) - F(\theta_2)] \cdot \left(\frac{1}{1+\rho}\right)$ with probability $\alpha \cdot \beta$. Thus, the expected social welfare is

$$\begin{aligned} &\alpha\beta[S(q_1^{FB}) - (1+\lambda)\,\theta_1 q_1^{FB}] \\ &+ (1-\alpha\beta)\Big[\frac{(1-\beta)\alpha}{1-\beta\alpha}\,[S(q_1^{S}) - (1+\lambda)\,\theta_1 q_1^{S} - \lambda\,\{(\theta_2 - \theta_1) q_2^{S} + F(\theta_1) - F(\theta_2)\}] \\ &+ \frac{(1-\beta)\alpha}{1-\beta\alpha}\,\{S(q_2^{SB}) - (1+\lambda)\,\theta_2\, q_2^{SB}\}\Big] \\ &- \lambda \cdot \alpha \cdot \beta \cdot [(\theta_2 - \theta_1) q_2^{S} + F(\theta_1) - F(\theta_2)] \cdot \left(\frac{1}{1+\rho}\right). \end{aligned} \tag{21}$$

Then we obtain the following result.

Proposition 2 *The government sets up incentives for the supervisor to avoid collusion between the supervisor and the firm by distorting the level of product quality downward.*

Proof : See the Appendix.

Thus far we have considered collusion-proof contracts for the case of regime 1 in Proposition 1. Next, we analyze optimal contracts and the possibility of collusion for the case in which countervailing incentives can emerge at equilibrium. Thus we consider regime 2 (case (ii) in Proposition 1). We assume that the supervisor observes a signal ω with probability ζ that the firm is of type θ_2 and otherwise $\omega = ø$ with probability $1-\zeta$.

Suppose that $\omega = r = \theta$. Then, the government is informed. Thus, we have, with probability ζ,

$$W^F = \alpha\,[S(q_1^{FB}) - (1+\lambda)\,\{\theta_1 q_1^{FB} + F(\theta_1)\}] + (1-\alpha)\,[S(q_2^{FB}) - (1+\lambda)\,\{\theta_2 q_2^{FB} + F(\theta_2)\}].$$

Suppose that $\omega = ø$ Then, the government is uninformed. Thus, we have, with probability $1-\zeta$,

$$W^U = \alpha\,[S(q_1^{C}) - (1+\lambda)\,\{\theta_1 q_1^{C} + F(\theta_1)\}] \\ + (1-\alpha)\,[S(q_2^{FB}) - (1+\lambda)\,\theta_2 q_2^{FB} - \lambda\,\{F(\theta_1) - F(\theta_2) - (\theta_2-\theta_1) q_1^{C}\}].$$

Hence, the expected social welfare is

$$\zeta W^F + (1-\zeta) W^U. \tag{22}$$

Suppose that $r = ø$. Then, the government is uninformed. Note that we would have

$$\Pr(\theta = \theta_2 \mid \omega = ø) = \frac{\zeta(1-\alpha)}{1-\zeta(1-\alpha)}.$$

Now, we analyze the case in which the supervisor and the firm can collude when there are countervailing incentives.

The expected social cost is $\lambda \cdot (1-\alpha) \cdot \zeta \cdot [(F(\theta_1) - F(\theta_2) - (\theta_2-\theta_1) q_1^{C}] \cdot \left(\frac{1}{1+\rho}\right)$ with probability $(1-\alpha) \cdot \zeta$.

Then, the expected social welfare is

$$\{1-(1-\alpha) \cdot \zeta\}\, S(q_2^{FB}) - (1+\lambda)\,\theta_2 q_2^{FB}] \\ + (1-\alpha) \cdot \zeta \left[\frac{(1-\alpha)(1-\zeta)}{1-\zeta(1-\alpha)}\,\{S(q_2^{S}) - (1+\lambda)\,\theta_2 q_2^{S} - \lambda\,\{F(\theta_1) - F(\theta_2) - (\theta_2-\theta_1) q_1^{C}\}\} \right. \\ \left. + \frac{\alpha}{1-\zeta(1-\alpha)}\,\{S(q_1^{C}) - (1+\lambda)\,\theta_1 q_1^{C}\}\right]$$

$$- \lambda \cdot (1-\alpha) \cdot \zeta \, [(F(\theta_1) - F(\theta_2) - (\theta_2 - \theta_1) q_1^{C}] \cdot \left(\frac{1}{1+\rho} \right). \tag{23}$$

The following proposition shows the optimal collusion-proof contract for regime 2.

Proposition 3 *The optimal contract is characterized as follows.*

$$S_q(q_1^{C*}) = (1+\lambda)\,\theta_1 - \lambda \frac{(1-\alpha)}{\alpha}(\theta_2 - \theta_1)\left[1 - \zeta \left\{1 - \left(\frac{1}{1+\rho}\right)\right\}\right] \tag{24}$$

and

$$S_q(q_2^{S*}) = (1+\lambda)\,\theta_2. \tag{25}$$

Proof : See the Appendix.

This proposition suggests that countervailing incentives exist and thus, too, upward distortion for the efficient agent. It also shows that if ζ increases, q_1^{C*} will decrease.

5 An Extension

In this section, we extend the supervision technology assumed in the previous sections to a more general supervision technology. Suppose the supervisor observes a signal ω with probability β that the firm is of type θ and otherwise $\omega = ø$ with probability $1-\beta$. The supervisor can hide his information and report that the signal is empty.

There are four states as follows:
State 1: $(\theta=\theta_1\ \omega=\theta_1)$ with probability $\alpha\beta$,
State 2: $(\theta=\theta_1\ \omega=ø)$ with probability $\alpha(1-\beta)$,
State 3: $(\theta=\theta_2\ \omega=\theta_2)$ with probability $(1-\alpha)\beta$,
State 4: $(\theta=\theta_2\ \omega=ø)$ with probability $(1-\alpha)(1-\beta)$,

Then, the expected social welfare W^G is

$$\begin{aligned} & W^G \\ = \ & \alpha\beta[S(q) - (1+\lambda)\{m_R + (\theta_2-\theta_1)q + F(\theta_1) - F(\theta_2)\} - \lambda\{U_{s1} + (m_{s1} - m_R)] \\ & + \alpha(1-\beta)[S(q) - (1+\lambda)\{m_R + (\theta_2-\theta_1)q + F(\theta_1) - F(\theta_2)\} - \lambda\{U_{s2} + (m_{s2} - m_R)] \\ & + (1-\alpha)\beta[S(q) - (1+\lambda)\{m_R + (\theta_2-\theta_1)q + F(\theta_1) - F(\theta_2)\} - \lambda\{U_{s3} + (m_{s3} - m_R)] \\ & + (1-\alpha)(1-\beta)[S(q) - (1+\lambda)\{m_R + (\theta_2-\theta_1)q + F(\theta_1) - F(\theta_2)\} - \lambda\{U_{s4} + (m_{s4} - m_R)]. \end{aligned} \tag{26}$$

Let η denote the transaction cost between the supervisor and the firm. Then collusion-proof constraints are given as follows:

$$(1+\eta)(m_{s1}-m_{s2}) \geqq U_{s2}-U_{s1} \tag{27}$$

and

$$(1+\eta)(m_{s3}-m_{s4}) \geqq U_{s4}-U_{s3}, \tag{28}$$

where si denotes state i, $i = 1, 2, 3, 4$.

Incentive compatibility constraints are given as

$$U_{s2} \geqq U_{s4} + (\theta_2-\theta_1)q + F(\theta_1) - F(\theta_2) \tag{29}$$

and

$$U_{s4} \geqq U_{s2} + (\theta_2-\theta_1)q + F(\theta_1) - F(\theta_2). \tag{30}$$

Participation constraints are,
for the firm

$$U_{si} \geqq 0, \quad i = 1, 2, 3, 4, \tag{31}$$

and for the supervisor

$$m_{si} - m_R \geqq 0, \quad i = 1, 2, 3, 4. \tag{32}$$

The government's problem is to choose (q_{si}, m_{si}, U_{si}) to maximize (26) subject to (27), (28), (29), (30), (31), and (32).

The following proposition shows the optimal collusion-proof contract when countervailing incentives exist.

Proposition 4 *The optimal collusion-proof contract is characterized as follows.*

$$Sq(q_1^{C**}) = (1+\lambda)\,\theta_1 - \lambda \cdot \frac{1-\alpha}{\alpha}\,(\theta_2-\theta_1)\left[1-\beta\left(1-\frac{1}{1+\eta}\right)\right] \tag{33}$$

and

$$S_q(q_2^{S**}) = (1+\lambda)\,\theta_2. \tag{34}$$

Proof : See the Appendix.

Proposition 4 shows that if β increases, then $q_1{}^*$ decreases.

6 Conclusion

In this paper, we have analyzed optimal contracts and the possibility of collusion in a three level hierarchy model with adverse selection. We have characterized optimal contracts when the costs of production is composed of a variable cost and a fixed one, both of which depend on the asymmetric information parameter. We have shown that the difference in fixed costs with respect to productivity types affects which of ICCs and PCs are binding and, thus, the firm's information rents. Thus, the optimal contract exhibits different regimes and countervailing incentives may exist. We have also characterized optimal collusion-proof contracts when the supervisor can collude with the firm and obtain positive information rents.

Appendix

Proof of Proposition 1.

Let the Lagrangian for the maximization problem of (6) be

$$\begin{aligned}\mathcal{L} = \ & \alpha\,[S(q_1) - (1+\lambda)\,\{m_R+\theta_1 q_1+F(\theta_1)\} - \lambda U_1 - \lambda X_1] \\ & + (1-\alpha)\,[S(q_2) - (1+\lambda)\,\{m_R+\theta_2 q_2+F(\theta_2)\} - \lambda U_2 - \lambda X_2] \\ & + \lambda_{I1}[U_1 - \{U_2 + (\theta_2-\theta_1)q_2 + (F(\theta_2)-F(\theta_1))\}] \\ & + \lambda_{I2}[U_2 - \{U_1 - (\theta_2-\theta_1)q_1 - (F(\theta_2)-F(\theta_1))\}] \\ & + \lambda_{P1}\cdot U_1 \\ & + \lambda_{P2}\cdot U_2,\end{aligned}$$

where $\lambda_{I1} \geqq 0$, $\lambda_{I2} \geqq 0$, $\lambda_{P1} \geqq 0$, and $\lambda_{P2} \geqq 0$ are Lagrange multipliers. There are 16 possible cases to be examined.

First, observe that $\lambda_{I1} > 0$ and $\lambda_{I2} > 0$ cannot simultaneously hold. Thus, we can eliminate the following four cases:

1. $\lambda_{I1} > 0$, $\lambda_{I2} > 0$, $\lambda_{P1} > 0$, and $\lambda_{P2} > 0$.
2. $\lambda_{I1} > 0$, $\lambda_{I2} > 0$, $\lambda_{P1} > 0$, and $\lambda_{P2} = 0$.
3. $\lambda_{I1} > 0$, $\lambda_{I2} > 0$, $\lambda_{P1} = 0$, and $\lambda_{P2} > 0$.
4. $\lambda_{I1} > 0$, $\lambda_{I2} > 0$, $\lambda_{P1} = 0$, and $\lambda_{P2} = 0$.

Next, we find that incentive constraints (9) and (10) cannot be simultaneously slack.

Thus, we can eliminate the following three cases:

5. $\lambda_{I1} = 0$, $\lambda_{I2} = 0$, $\lambda_{P1} > 0$, and $\lambda_{P2} = 0$.
6. $\lambda_{I1} = 0$, $\lambda_{I2} = 0$, $\lambda_{P1} = 0$, and $\lambda_{P2} > 0$.

7. $\lambda_{I1} = 0$, $\lambda_{I2} = 0$, $\lambda_{P1} = 0$, and $\lambda_{P2} = 0$.

Suppose only $\lambda_{P1} = 0$ holds.
If $\lambda_{I1} = 0$, $\lambda_{I2} > 0$, and $\lambda_{P2} = 0$, then we can decrease t_1.
If $\lambda_{I1} = 0$, $\lambda_{I2} > 0$, and $\lambda_{P2} > 0$, then we can decrease t_1.
Thus, these two cases can be eliminated.

Suppose only $\lambda_{P2} = 0$ holds.
If $\lambda_{I1} > 0$, $\lambda_{I2} = 0$, and $\lambda_{P1} > 0$, then we can decrease t_2.
If $\lambda_{I1} = 0$, $\lambda_{I2} > 0$, and $\lambda_{P1} = 0$, then we can decrease t_2.
Thus, these two cases can be eliminated.

Therefore, only the following five cases are relevant for the first-order conditions of the maximization problem.
(I) $\lambda_{I1} > 0$, $\lambda_{I2} = 0$, $\lambda_{P1} = 0$, and $\lambda_{P2} > 0$.
(II) $\lambda_{I1} > 0$, $\lambda_{I2} = 0$, $\lambda_{P1} > 0$, and $\lambda_{P2} > 0$.
(III) $\lambda_{I1} = 0$, $\lambda_{I2} = 0$, $\lambda_{P1} > 0$, and $\lambda_{P2} > 0$.
(IV) $\lambda_{I1} = 0$, $\lambda_{I2} > 0$, $\lambda_{P1} > 0$, and $\lambda_{P2} > 0$.
(V) $\lambda_{I1} = 0$, $\lambda_{I2} > 0$, $\lambda_{P1} > 0$, and $\lambda_{P2} = 0$.

Consider the case (I) in which (9) and (12) are binding and thus we have

$$U_1 = (\theta_2 - \theta_1) q_2 + \{F(\theta_2) - F(\theta_1)\},$$

$$U_2 = 0,$$

$$t_1 = \theta_1 q_1 + (\theta_2 - \theta_1) q_2 + F(\theta_2),$$

and

$$t_2 = \theta_2 q_2 + F(\theta_2).$$

It is easily shown that the transfers t_1 and t_2 satisfy (10) and (11). The first order conditions with respect to q_i are

$$S_q(q_1^{SB}) = (1 + \lambda)\, \theta_1$$

and

$$S_q(q_2^{SB}) = (1+\lambda)\,\theta_2 + \lambda\,\frac{\alpha}{1-\alpha}\,(\theta_2 - \theta_1).$$

Hence, we have

$$q_1^{SB} = q_1^{FB} \text{ and } q_2^{SB} < q_2^{FB}.$$

This case occurs when $F(\theta_1) - F(\theta_2)$ satisfies $\frac{F(\theta_1)-F(\theta_2)}{\theta_2 - \theta_1} \leqq q_2^{SB}$.

Thus, the optimal contract entails lower quality than the first level for the high marginal cost type θ_2.

Next, consider case (II). Then, we have

$$S_q(q_1^{SB}) = (1+\lambda)\,\theta_1$$

and

$$q_2^{SB} = \frac{F(\theta_1)-F(\theta_2)}{\theta_2 - \theta_1}.$$

In this case, $F(\theta_1) - F(\theta_2)$ satisfies $q_2^{SB} \leqq \frac{F(\theta_1)-F(\theta_2)}{\theta_2 - \theta_1} \leqq q_2^{FB}$.

Consider case (III). Then, we have

$$q_1^{SB} = q_1^{FB} \text{ and } q_2^{SB} = q_2^{FB}.$$

In this case, $F(\theta_1) - F(\theta_2)$ satisfies $q_2^{FB} \leqq \frac{F(\theta_1)-F(\theta_2)}{\theta_2 - \theta_1} \leqq q_1^{FB}$.

Consider case (IV). Then, we have

$$q_1^{SB} = \frac{F(\theta_1)-F(\theta_2)}{\theta_2 - \theta_1}$$

and

$$S_q(q_2^{SB}) = (1+\lambda)\,\theta_2.$$

In this case, $F(\theta_1) - F(\theta_2)$ satisfies $F(\theta_1) - F(\theta_2) q_1^{FB} \leqq \frac{F(\theta_1)-F(\theta_2)}{\theta_2 - \theta_1} \leqq q_1^{C}$.

Finally, consider case (V). Then, (10) and (11) are binding. Thus, we have

$$S_q(q_1{}^C) = (1+\lambda)\,\theta_1 - \lambda\,\frac{1-\alpha}{\alpha}\,(\theta_2-\theta_1),\quad q_1{}^C > q_2{}^{SB}$$

and

$$S_q(q_2{}^{SB}) = (1+\lambda)\,\theta_2.$$

Note that we have

$$U_1 = 0$$

and

$$U_2 = F(\theta_1) - F(\theta_2) - (\theta_2-\theta_1)q_1.$$

Here $\frac{F(\theta_1)-F(\theta_2)}{\theta_2-\theta_1}$ satisfies $q_1{}^C \leqq \frac{F(\theta_1)-F(\theta_2)}{\theta_2-\theta_1}$. Q.E.D.

Proof of Proposition 2.

From the first order conditions with respect to q_1 and q_2, we have

$$S_q(q_1{}^S) = (1+\lambda)\,\theta_1$$

and

$$S_q(q_2{}^S) = (1+\lambda)\,\theta_2 + \lambda\cdot\frac{\alpha}{1-\alpha}\,(\theta_2-\theta_1)\left[1-\beta+\beta\cdot\left(\frac{1}{1+\rho}\right)\right].$$

Thus, $q_2{}^S < q_2{}^{FB}$. Q.E.D.

Proof of Proposition 3.

From the first order conditions with respect to q_1 and q_2, we have

$$S_q(q_1{}^S) = (1+\lambda)\,\theta_1 - \lambda - \frac{(1-\alpha)}{\alpha}\,(\theta_2-\theta_1)\left[1-\zeta\left\{1-\left(\frac{1}{1+\rho}\right)\right\}\right]$$

and

$$S_q(q_2^S) = (1+\lambda)\,\theta_2.$$

Thus, $q_1^S > q_1^{FB}$. Q.E.D.

Proof of Proposition 4.

Let $\mathfrak{M}$ be the Lagrangian for the maximization problem of (26),

$$\begin{aligned}\mathfrak{M} = {} & W^G + \varphi_1[(1+\eta)(m_{s1}-m_{s2}) - U_{s2} + U_{s1}] \\ & + \varphi_2[(1+\eta)(m_{s3}-m_{s4}) - U_{s4} + U_{s3}] \\ & + \psi_1[U_{s2} - \{U_{s4} + (\theta_2-\theta_1)q + F(\theta_1) - F(\theta_2)\}] \\ & + \psi_2[U_{s4} - \{U_{s2} + (\theta_2-\theta_1)q + F(\theta_1) - F(\theta_2)\}] \\ & + \mu_1[m_{s1}-m_R] + \mu_2[m_{s2}-m_R] + \mu_3[m_{s3}-m_R] + \mu_4[m_{s4}-m_R] \\ & + \nu_1 U_{s1} + \nu_2 U_{s2} + \nu_3 U_{s3} + \nu_4 U_{s4},\end{aligned}$$

where φ_1, φ_2, ψ_1, ψ_2, μ_1, μ_2, μ_3, μ_4, ν_1, ν_2, ν_3, and ν_4 are Lagrange multipliers.

We show that $\varphi_1 = 0$, $\varphi_2 > 0$, $\psi_1 = 0$, $\psi_2 > 0$, $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 = 0$, $\mu_4 > 0$, $\nu_1 > 0$, $\nu_2 > 0$, $\nu_3 > 0$, and $\nu_4 = 0$.

Step 1. First, by neglecting constraints (27) and (29), it can be easily shown that we have

$$m_{s1} = m_R, \quad m_{s2} = m_R, \quad m_{s4} = m_R,$$

and

$$U_{s1} = 0.$$

This yields

$$U_{s2} = 0.$$

Step 2: Because decreasing U_{s4} leads to increasing the expected social welfare, incentive constraint (28) must be binding.

Thus,

$$U_{s4} > 0.$$

Step 3: Because $U_{s4} > 0$ coupled with collusion-proof constraint (30), either $m_{s3} > m_R$ or $U_{s3} > 0$ holds true.

Thus, constraint (30) must be binding.

Hence,

$$(1+\eta)(m_{s3}-m_R) = U_{s4} - U_{s3}.$$

Substituting this into (26), the expected social welfare is found to be strictly decreasing in U_{s3}.
Thus, $U_{s3}^* = 0$.
It can be shown that these solutions satisfy (27) and (29).
Thus, the government's problem becomes

$$\max \quad \beta \cdot W^F + (1-\beta) W^N - (1-\alpha)\beta\lambda \left(\frac{F(\theta_1) - F(\theta_2) - (\theta_2 - \theta_1) q_1}{1+\eta} \right).$$

The first-order conditions with respect to q_1 and q_2 are

$$S_q(q_1^*) = (1+\lambda)\theta_1 - \lambda \cdot \frac{\alpha}{1-\alpha} (\theta_2 - \theta_1) \left[1 - \beta \left(1 - \frac{1}{1+\eta} \right) \right]$$

and

$$S_q(q_2^*) = (1+\lambda)\theta_2.$$

Thus, if β increases, then q_1^* decreases. Q.E.D.

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