

# Multiple Agents and Countervailing Incentives

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## Abstract

This paper examines optimal industrial structures in a model in which a government (the principal) procures two complementary products or facilities from two firms (the agents) under asymmetric information. We consider two different industrial structures. One is a decentralized industry in which each of the two firms supplies one of the two products. The other is an integrated industry in which a unified firm supplies both products. We extend the literature on optimal organizations with multiple agents under asymmetric information to a setting in which each firm's cost comprises not only a variable cost but also a fixed cost, both of which depend on its private information. Equilibrium contracts are characterized by comparing with the case of complete information. We show that when a difference in the amount of fixed costs with respect to each firm's type is sufficiently large, countervailing incentives may arise in this multi-agent contract setting. We also show that contrary to the literature, that the government chooses a decentralized industrial structure provided that the difference in fixed costs with respect to productivity types is sufficiently small.

## 1 Introduction

This paper examines optimal industrial structures in a procurement contract model in which a government (the principal) procures two complementary products from two firms (the agents). Unlike the literature on optimal organizations with multiple agents under asymmetric information, we examine a setting in which each firm's cost comprises not only a variable cost but also a fixed cost, both of which depend on its private information. We assume that there are two productivity types for each firm. One type has a high marginal cost and a low fixed cost. The other has a low marginal cost and a high fixed cost. Two industry structures are considered. One industrial structure is a decentralized industry in which each of the two firms produces one of the two complementary products. The other is an integrated industry in which a unified firm produces both products. We show that if a difference in the amount of fixed costs with respect to each firm's type is sufficiently

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small, then the government chooses the decentralized industry provided that the level of output when one firm is of low marginal cost type and the other firm is of high marginal cost type is sufficiently small. In contrast, we demonstrate that if the difference in fixed costs with respect to productivity types is sufficiently large, then the government chooses the integrated industry provided that a difference in the output levels between decentralization and integration when both firms are of low marginal cost type is sufficiently small.

This paper is related to two strands of the literature on contract theory. The first is the literature on optimal organizations with multiple agents under asymmetric information. Dana (1993) analyzes optimal industry structures in which the government procures two substitutes. The optimal industry structure, he concludes, depends on whether marginal costs of the two products are sufficiently positively correlated or not. Baron and Besanko (1992, 1999), Gilbert and Riordan (1995), and Severinov (2008) analyze optimal industry structures in which marginal costs of the two products are independently determined. These papers show that the optimal industry structure depends on the degree of complementarity or substitutability. However, these papers do not consider fixed costs that depend on agents' types. In this paper, we extend the literature to a more general cost structure in which not only a variable cost but also a fixed cost are considered.

The second strand of research related to this paper is concerned with the issues of countervailing incentives under asymmetric information (Laffont and Tirole, 1993; Laffont and Martimort, 2002). Lewis and Sappington (1989) assume one-dimensional uncertainty regarding marginal costs and fixed costs and analyze countervailing incentives. Maggi and Rodriguez-Clare (1995) further examine the issues on countervailing incentives. Jullien (2000) explores the effects of type-dependent participation constraints on optimal contracts. However, these papers consider only a single agent. In contrast, we consider multiple agents.

Here, we examine optimal organizations in a model in which the cost function of each firm comprises not only a variable cost but also a fixed cost, both of which depend on its private information. We show that when the difference in the amount of fixed costs with respect to productivity type is sufficiently large, countervailing incentives can result because the set of binding incentive compatibility and participation constraints depends on the value of this difference. Thus, the results in this paper suggest that it is of critical importance to consider type-dependent participation constraints when we examine optimal industrial structures in contract models.

The paper is organized as follows. In Section 2, we present a model and basic assumptions. In Section 3, we characterize optimal contracts under decentralization. We examine the conditions under which countervailing incentives arise. In Section 4, we characterize optimal contracts under integration. We demonstrate the possibilities of countervailing incentives for the consolidated industry. In Section 5, we compare these two industrial structures and discuss implications for optimal industrial structures. Section 6 concludes.

## 2 The Model

We consider a two-product industry in which one or two firms produce products  $A$  and  $B$ . Output (quantity or quality) of product  $A$  or  $B$  is denoted  $q^A$  or  $q^B$  respectively. The two products are supposed to be complementary. A government procures these products and supplies a final product (a public good). Let  $V(q^A, q^B)$  denote social benefit that consumers obtain from the final product, where

$$V(q^A, q^B) = S(q), \text{ with } q = \min \{q^A, q^B\}.$$

For all  $q > 0$ ,  $S(q)$  is twice continuously differentiable, increasing and strictly concave.

The cost function of each product is given by

$$C(q, \theta) = \theta q + F(\theta),$$

where  $\theta$  is the marginal cost and  $F(\theta)$  is the fixed cost. We assume that parameter  $\theta$  takes either  $\theta_1$  or  $\theta_2$  with  $\theta_1 < \theta_2$ . We also assume  $F(\theta_1) > F(\theta_2)$ . A high marginal cost is associated with a low fixed cost and vice versa. For instance, this inverse relationship can arise because a high fixed cost guarantees a low marginal cost and vice versa in constructing facilities such as highways and bridges.

We assume the marginal cost  $\theta$  is private information for each firm. For firm  $A$ , let  $\theta_i$  denote its type,  $i=1, 2$ . Similarly, for firm  $B$ , let  $\theta_j$  denote its type  $j=1, 2$ . Let  $p_{ij}$  denote a joint probability between  $\theta_i$  and  $\theta_j$ . For simplicity, we assume the probability distributions over  $\theta_i$  and  $\theta_j$  are independent. Let  $p = \Pr(\theta_i = \theta_1) = \Pr(\theta_j = \theta_1)$ ,  $0 < p < 1$ . Hence, we have

$$\begin{aligned} p_{11} &= p^2, \\ p_{12} &= p_{21} = p(1 - p), \\ \text{and } p_{22} &= (1 - p)^2. \end{aligned}$$

In this paper, we consider a non-benevolent government. Thus the government's payoff is defined as social benefit minus a monetary transfer to the firm<sup>1)</sup>. The firm's payoff is defined as a monetary transfer from the government minus a realized production cost. When designing optimal contracts, the government solves its payoff maximization problem subject to incentive compatibility constraints and participation constraints. An incentive compatibility constraint (ICC) guarantees that each firm prefers the contract that is designed for it. A participation constraint (PC) guarantees that each firm

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<sup>1)</sup> For simplicity, we assume that firms' informational rents are not included in the government's objective function. This assumption is a special case in the Baron and Myerson (1982) regulation model in that the weight of the agent's payoff in the principal's objective function is zero.

accepts the designated contract.

We consider two different industry structures. One is a decentralized industry in which each of the two firms produces product  $A$  or  $B$ . The other is a horizontally integrated industry in which the consolidated firm produces both products  $A$  and  $B$ .

Let  $q_{ij}$  be the quantity when firm  $A$ 's type is  $\theta_i$  and firm  $B$ 's type  $\theta_j$ . Under the decentralized industry, the government's expected payoff  $\Pi^D$ , *ex post* payoff  $u^A$  of the firm producing product  $A$ , and *ex post* payoff  $u^B$  of the firm producing product  $B$  are given by, respectively,

$$\begin{aligned}\Pi^D &= \sum_{ij} p_{ij} [S(q_{ij}) - \tau_{ij}^A - \tau_{ij}^B], \\ u^A &= \tau_{ij}^A - \theta_i q_{ij} - F(\theta_i), \\ \text{and } u^B &= \tau_{ij}^B - \theta_j q_{ij} - F(\theta_j), \quad i, j = 1, 2,\end{aligned}$$

where monetary transfers from the government to the firms are denoted  $\tau_{ij}^A$  and  $\tau_{ij}^B$ .

Under the integrated industry, the government's payoff  $\Pi^I$  and *ex post* payoff  $u^I$  of the consolidated firm producing both products  $A$  and  $B$  are given by

$$\begin{aligned}\Pi^I &= \sum_{ij} p_{ij} [S(q_{ij}) - \omega_{ij}], \\ \text{and } u^I &= \omega_{ij} - (\theta_i + \theta_j) q_{ij} - F(\theta_i) - F(\theta_j), \quad i, j = 1, 2,\end{aligned}$$

where a monetary transfer from the government to the firm is denoted  $\omega_{ij}$ .

The sequence of events proceeds as follows: At stage 1, a government decides on an industrial structure: a decentralized industry or an integrated industrial structure. At stage 2, nature determines each firm's productivity type:  $\theta_1$  or  $\theta_2$ . Only the firms observe it. At stage 3, the government offers contracts that firms accept or refuse. Finally at stage 4, when accepting the contracts, the firms produce the products and the government provides monetary transfers to the firms.

### 3 Optimal Contracts under Decentralization

In this section, we examine optimal contracts under decentralization. Under the decentralized industry, each of the two firms supplies its own product to the government.

The government's (the principal's) problem under the decentralized industry, (P-1), can be stated as follows:

$$\text{Maximize } \Pi^D = \sum_{ij} p_{ij} [S(q_{ij}) - \tau_{ij}^A - \tau_{ij}^B] \quad (\text{P-1})$$

$$\begin{aligned}
\text{subject to } & \sum_{j=1}^2 p_{1j} [\tau_{1j}^A - \theta_1 q_{1j} - F(\theta_1)] \geq \sum_{j=1}^2 p_{1j} [\tau_{2j}^A - \theta_1 q_{2j} - F(\theta_1)], \\
& \sum_{j=1}^2 p_{i1} [\tau_{i1}^B - \theta_1 q_{i1} - F(\theta_1)] \geq \sum_{i=1}^2 p_{i1} [\tau_{i2}^B - \theta_1 q_{i2} - F(\theta_1)], \\
& \sum_{j=1}^2 p_{2j} [\tau_{2j}^A - \theta_2 q_{2j} - F(\theta_2)] \geq \sum_{j=1}^2 p_{2j} [\tau_{1j}^A - \theta_2 q_{1j} - F(\theta_2)], \\
& \sum_{i=1}^2 p_{i2} [\tau_{i2}^A - \theta_2 q_{i2} - F(\theta_2)] \geq \sum_{i=1}^2 p_{i2} [\tau_{i1}^B - \theta_2 q_{i1} - F(\theta_2)], \\
& \tau_{1j}^A - \theta_1 q_{1j} - F(\theta_1) \geq 0, \\
& \tau_{2j}^A - \theta_2 q_{2j} - F(\theta_2) \geq 0, \\
& \tau_{i1}^B - \theta_1 q_{i1} - F(\theta_1) \geq 0, \\
& \text{and } \tau_{i2}^B - \theta_2 q_{i2} - F(\theta_2) \geq 0, \quad i = 1, 2 \text{ and } j = 1, 2
\end{aligned}$$

We focus on the cases in which firms earn positive informational rents. For the decentralized industry structure, the following two propositions summarize the results. Proposition 1 shows the second best output is smaller than the first best output if the difference in fixed costs with respect to the firms' types is sufficiently small. Proposition 2 demonstrates the second best output is larger than the first best output because of countervailing incentives if the difference in fixed costs with respect to the firms' types is sufficiently large.

**Proposition 1** *Under decentralization, when  $F(\theta_1) - F(\theta_2)$  is sufficiently small, the optimal contracts are characterized as follows:*

$$\begin{aligned}
S_q(q_{11}^D) &= 2\theta_1, \\
S_q(q_{12}^D) &= S_q(q_{21}^D) = \theta_1 + \theta_2 + \frac{p}{1-p} (\theta_2 - \theta_1), \\
\text{and } S_q(q_{22}^D) &= 2\theta_2 + \frac{2p}{1-p} (\theta_2 - \theta_1).
\end{aligned}$$

Proof: See Appendix A.

**Proposition 2** *Under decentralization, when  $F(\theta_1) - F(\theta_2)$  is sufficiently large, the optimal contracts are characterized as follows:*

$$\begin{aligned}
S_q(q_{11}^{D,c}) &= 2\theta_1 - \frac{2p}{1-p} (\theta_2 - \theta_1) \\
S_q(q_{12}^{D,c}) &= S_q(q_{21}^{D,c}) = \theta_1 + \theta_2 - \frac{p}{1-p} (\theta_2 - \theta_1) \\
\text{and } S_q(q_{22}^{D,c}) &= 2\theta_2.
\end{aligned}$$

Proof: See Appendix A.

Proposition 1 shows that the agent with lower marginal costs obtains positive informational rents whereas Proposition 2 proves that the agent with high marginal costs earns positive informational rents.

#### 4 Optimal Contracts under Integration

In this section, we examine optimal contracts under integration.

The government's (the principal's) problem (P-2) is as follows:

$$\text{Maximize } \Pi^I = \sum_{ij} p_{ij} [S(q_{ij}) - \omega_{ij}] \quad (\text{P-2})$$

subject to

$$\begin{aligned} \omega_{1j} - (\theta_1 + \theta_j)q_{1j} - F(\theta_1) - F(\theta_j) &\geq \omega_{2j} - (\theta_1 + \theta_j)q_{2j} - F(\theta_1) - F(\theta_j), \\ \omega_{i1} - (\theta_i + \theta_1)q_{i1} - F(\theta_i) - F(\theta_1) &\geq \omega_{i2} - (\theta_i + \theta_1)q_{i2} - F(\theta_i) - F(\theta_1), \\ \omega_{2j} - (\theta_2 + \theta_j)q_{2j} - F(\theta_2) - F(\theta_j) &\geq \omega_{1j} - (\theta_2 + \theta_j)q_{1j} - F(\theta_2) - F(\theta_j), \\ \omega_{i2} - (\theta_i + \theta_2)q_{i2} - F(\theta_i) - F(\theta_2) &\geq \omega_{i1} - (\theta_i + \theta_2)q_{i1} - F(\theta_i) - F(\theta_2), \\ \omega_{1j} - (\theta_1 + \theta_j)q_{1j} - F(\theta_1) - F(\theta_j) &\geq 0, \\ \text{and } \omega_{2i} - (\theta_2 + \theta_i)q_{2i} - F(\theta_2) - F(\theta_i) &\geq 0, \quad i=1, 2 \text{ and } j=1, 2. \end{aligned}$$

We have the following propositions. Proposition 3 shows the second best output is smaller than the first best output. Proposition 4 demonstrates the second best output is larger than the first best output because of countervailing incentives.

**Proposition 3** *Under integration, when  $F(\theta_1) - F(\theta_2)$  is sufficiently small, the optimal contracts are characterized as follows:*

$$\begin{aligned} S_q(q_{11}^I) &= 2\theta_1, \\ S_q(q_{12}^I) &= S_q(q_{21}^I) = \theta_1 + \theta_2 + \frac{p}{2(1-p)}(\theta_2 - \theta_1), \\ \text{and } S_q(q_{22}^I) &= 2\theta_2 + \frac{p(2-p)}{(1-p)^2}(\theta_2 - \theta_1). \end{aligned}$$

Proof: See Appendix B.

**Proposition 4** *Under integration, when  $F(\theta_1) - F(\theta_2)$  is sufficiently large, the optimal contracts are characterized as follows:*

$$S_q(q_{11}^{Ic}) = 2\theta_1 - \frac{(1-p^2)}{p^2}(\theta_2 - \theta_1),$$

$$S_q(q_{12}^{I,C}) = S_q(q_{21}^{D,C}) = \theta_1 + \theta_2 - \frac{(1-p)}{2p} (\theta_2 - \theta_1),$$

$$\text{and } S_q(q_{22}^I) = 2\theta_2.$$

Proof: See Appendix B.

Proposition 3 shows that the agent with low marginal costs obtains positive informational rents. In contrast to Proposition 3, Proposition 4 shows that the agent with high marginal costs earns positive informational rents.

## 5 Optimal Industrial Structures

We characterized the optimal contracts under decentralization and those under integration in the previous sections. In this section, we compare the decentralized industry with the integrated industry.

First, we assume  $F(\theta_1) - F(\theta_2)$  is sufficiently small. Thus, we compare regime 1-1 (in Appendix A) with regime 2-1 (in Appendix B). Then, we have the following result.

**Proposition 5** *Suppose that a difference in fixed costs with respect to firms' productivity type,  $F(\theta_1) - F(\theta_2)$ , is sufficiently small. Then, the principal prefers the decentralized industry structure to the integrated one provided that  $q_{12}^I (= q_{21}^I)$  is sufficiently small.*

Proof: The principal's payoff in regime 1-1 is

$$\begin{aligned} \Pi^D = & p_{11} [S(q_{11}^D) - 2\theta_1 q_{11}^D - 2(\theta_2 - \theta_1)(q_{21}^D + q_{12}^D) - 2F(\theta_2)] \\ & + p_{12} [S(q_{12}^D) - (\theta_2 - \theta_1)(q_{12}^D + q_{22}^D) - 2F(\theta_2)] \\ & + p_{21} [S(q_{21}^D) - (\theta_2 - \theta_1)(q_{21}^D + q_{22}^D) - 2F(\theta_2)] \\ & + p_{22} [S(q_{22}^D) - 2\theta_2 q_{22}^D - 2F(\theta_2)]. \end{aligned}$$

The principal's payoff in regime 2-1 is

$$\begin{aligned} \Pi^I = & p_{11} \left[ S(q_{11}^I) - 2\theta_1 q_{11}^I - 2(\theta_2 - \theta_1) \left( \frac{q_{12}^I + q_{21}^I}{2} + q_{12}^I \right) - 2F(\theta_2) \right] \\ & + p_{12} [S(q_{12}^I) - (\theta_1 + \theta_2) q_{12}^I - (\theta_2 - \theta_1) q_{22}^I - 2F(\theta_2)] \\ & + p_{21} [S(q_{21}^I) - (\theta_1 + \theta_2) q_{21}^I - (\theta_2 - \theta_1) q_{22}^I - 2F(\theta_2)] \\ & + p_{22} [S(q_{22}^I) - 2\theta_2 q_{22}^I - 2F(\theta_2)]. \end{aligned}$$

Recall that we have

$$q_{22}^I < q_{22}^D < q_{12}^D = q_{21}^D < q_{12}^I = q_{21}^I < q_{11}^D = q_{11}^I.$$

Because  $S(\cdot)$  is increasing and strictly concave, we have

$$\begin{aligned} S(q_{12}^D) - (\theta_2 - \theta_1)q_{22}^D &< S(q_{12}^I) - (\theta_2 - \theta_1)q_{22}^I, \\ (\theta_2 - \theta_1)q_{12}^D &< (\theta_1 + \theta_2)q_{12}^I, \\ \text{and } S(q_{22}^D) - 2\theta_2q_{22}^D &> S(q_{22}^I) - 2\theta_2q_{22}^I. \end{aligned}$$

Thus, in general,  $\Pi^D - \Pi^I$  can be positive, zero, or negative. However, if  $q_{12}^I (= q_{21}^I)$  is sufficiently small, then  $\Pi^D > \Pi^I$ . Q.E.D.

The literature has dealt with settings without fixed costs. Proposition 5 extends the literature to the setting in which the fixed costs of agents depend on private information.

Next we assume  $F(\theta_1) - F(\theta_2)$  is large. Thus, we compare regime 1-5 with regime 2-4 and obtain the following result.

**Proposition 6** *Suppose that a difference in fixed costs with respect to firms' productivity type,  $F(\theta_1) - F(\theta_2)$ , is sufficiently large. Then, the principal prefers the integrated industry structure to the decentralized one provided that the difference  $q_{11}^{I,C} - q_{11}^{D,C}$  is sufficiently small.*

Proof: The principal's payoff in regime 1-5 is

$$\begin{aligned} \Pi^D &= p_{11}[S(q_{11}^{D,C}) - 2\theta_1q_{11}^{D,C} - 2F(\theta_1)] \\ &\quad + p_{12}[S(q_{12}^{D,C}) - (\theta_1 + \theta_2)q_{12}^{D,C} + (\theta_2 - \theta_1)q_{11}^{D,C} - 2F(\theta_1)] \\ &\quad + p_{21}[S(q_{21}^{D,C}) - (\theta_1 + \theta_2)q_{21}^{D,C} + (\theta_2 - \theta_1)q_{11}^{D,C} - 2F(\theta_1)] \\ &\quad + p_{22}[S(q_{22}^{D,C}) - 2\theta_2q_{22}^{D,C} + (\theta_2 - \theta_1)(q_{21}^{D,C} + q_{12}^{D,C}) - 2F(\theta_1)]. \end{aligned}$$

The principal's payoff in regime 2-4 is

$$\begin{aligned} \Pi^I &= p_{11}[S(q_{11}^{I,C}) - 2\theta_1q_{11}^{I,C} - 2F(\theta_1)] \\ &\quad + p_{12}[S(q_{12}^{I,C}) - (\theta_1 + \theta_2)q_{12}^{I,C} + (\theta_2 - \theta_1)q_{11}^{I,C} - 2F(\theta_1)] \\ &\quad + p_{21}[S(q_{21}^{I,C}) - (\theta_1 + \theta_2)q_{21}^{I,C} + (\theta_2 - \theta_1)q_{11}^{I,C} - 2F(\theta_1)] \\ &\quad + p_{22}\left[S(q_{22}^{I,C}) - 2\theta_2q_{22}^{I,C} + (\theta_2 - \theta_1)\left(\frac{q_{21}^{I,C} + q_{12}^{I,C}}{2} + q_{11}^{I,C}\right) - 2F(\theta_1)\right]. \end{aligned}$$

We note that

$$q_{11}^{FB} < q_{11}^{D,C} < q_{11}^{I,C}, \quad q_{12}^{D,C} = q_{21}^{D,C} > q_{12}^{I,C} = q_{21}^{I,C}, \quad \text{and } q_{22}^{D,C} = q_{22}^{I,C}.$$

Because  $S(\cdot)$  is increasing and strictly concave, we have



$$\begin{aligned}
& S(q_{11}^{D,C}) - 2\theta_1 q_{11}^{D,C} > S(q_{11}^{I,C}) - 2\theta_1 q_{11}^{I,C}, \\
& S(q_{21}^{D,C}) - (\theta_1 + \theta_2) q_{21}^{D,C} - (\theta_1 - \theta_2) q_{11}^{D,C} < S(q_{21}^{I,C}) - (\theta_1 + \theta_2) q_{21}^{I,C} - (\theta_1 - \theta_2) q_{11}^{I,C} \\
& \text{and } q_{21}^{D,C} + q_{12}^{D,C} < q_{21}^{I,C} + q_{11}^{I,C}.
\end{aligned}$$

Thus, in general,  $\Pi^D - \Pi^I$  can be positive, zero, or negative. However, if  $q_{11}^{I,C} - q_{11}^{D,C}$  ( $>0$ ) is sufficiently small, then  $\Pi^D < \Pi^I$ . Q.E.D.

Proposition 6 contributes the literature in that it provides the conditions under which countervailing incentives occur in the multiple agent setting and integration dominates decentralization.

## 6 Conclusion

We have examined optimal industrial structures in a contracting model of one principal and multiple agents in which a government (the principal) procures complementary products from two firms (the agents). We have characterized optimal contracts under decentralization and integration. We have shown that under each of the two industry structures, there exist cases in which countervailing incentives arises at equilibrium. We have compared the government's payoffs under the two different industry structures: a decentralized industry and a horizontally integrated industry. The difference in fixed costs with respect to productivity types,  $F(\theta_1) - F(\theta_2)$ , affects which ICCs and PCs are binding and thus the firms' informational rents. We have analyzed the case in which a difference in fixed costs with respect to productivity types,  $F(\theta_1) - F(\theta_2)$ , is sufficiently small and shown that the government obtains a larger payoff under the decentralized industry than under the integrated industry if  $q_{12}^I$  is sufficiently small. We have also examined the case in which the value of  $F(\theta_1) - F(\theta_2)$  is not small. In this case, we have shown that there are cases in which the government chooses the integrated industry. Thus, it is critically important to understand that optimal industrial structures depend on the difference in fixed costs with respect to the agents' types when we design regulatory policies under asymmetric information.

## Appendix A

### Proofs of Propositions 1 and 2

We distinguish the following five regimes depending on the magnitude of the difference  $F(\theta_1) - F(\theta_2)$ , which affects what constraints in (P-1) are binding. Due to the symmetry between the two agents in our model, we can follow the analysis in Kobayashi (2018) to find the optimal solutions to the contracting problem (P-1). For a complete analysis of the case of a single agent, the reader is referred to Kobayashi (2018).

#### Regime 1-1

First we consider the case in which the following constraints are binding:

$$\begin{aligned} \sum_{j=1}^2 p_{1j} [\tau_{1j}^A - \theta_1 q_{1j} - F(\theta_1)] &\geq \sum_{j=1}^2 p_{1j} [\tau_{2j}^A - \theta_1 q_{2j} - F(\theta_1)], \\ \sum_{i=1}^2 p_{i1} [\tau_{i1}^B - \theta_1 q_{i1} - F(\theta_1)] &\geq \sum_{i=1}^2 p_{i1} [\tau_{i2}^B - \theta_1 q_{i2} - F(\theta_1)], \\ \tau_{2j}^A - \theta_2 q_{2j} - F(\theta_2) &\geq 0, \quad j=1, 2, \\ \text{and } \tau_{i2}^B - \theta_2 q_{i2} - F(\theta_2) &\geq 0, \quad i=1, 2. \end{aligned}$$

From the binding conditions, monetary transfers to firms A and B are

$$\begin{aligned} \tau_{21}^A &= \theta_2 q_{21} + F(\theta_2), \\ \tau_{22}^A &= \theta_2 q_{22} + F(\theta_2), \\ \tau_{12}^B &= \theta_2 q_{12} + F(\theta_2), \\ \text{and } \tau_{22}^B &= \theta_2 q_{22} + F(\theta_2). \end{aligned}$$

It can be shown that these transfers satisfy the remaining ICCs and PCs. Substituting these transfers into (P-1) and taking the first order conditions with respect to  $q_{ij}$  yield

$$\begin{aligned} S_q(q_{11}^D) &= 2\theta_1, \\ S_q(q_{12}^D) &= S_q(q_{21}^D) = \theta_1 + \theta_2 + \frac{p}{1-p} (\theta_2 - \theta_1), \\ \text{and } S_q(q_{22}^D) &= 2\theta_2 + \frac{2p}{1-p} (\theta_2 - \theta_1), \end{aligned}$$

where  $S_q(\cdot)$  denotes the partial derivative with respect to  $q$  and  $q_{ij}^D$  an equilibrium output under decentralization  $D$ .

We note that  $F(\theta_1) - F(\theta_2) < (\theta_2 - \theta_1) q_{22}^D$  must be satisfied. Thus we have

$$q_{22}^D < q_{22}^{FB}, \quad q_{22}^D < q_{12}^D = q_{21}^D < q_{11}^D = q_{11}^{FB},$$

where  $q_{11}^{FB}$  is the first best output when both types are efficient. When one of two productivity types is efficient, output  $q_{12}^D$  and  $q_{21}^D$  are distorted and smaller than  $q_{11}^D$ . When both productivity types are inefficient, output  $q_{22}^D$  is distorted and smaller than  $q_{12}^D$  and  $q_{21}^D$ .

We get  $\tau_{11}^A = \theta_1 q_{11}^D + (\theta_2 - \theta_1) q_{21}^D + F(\theta_2)$  and  $\tau_{12}^A = \theta_1 q_{12}^D + (\theta_2 - \theta_1) q_{22}^D + F(\theta_2)$ . Similarly we have  $\tau_{11}^B = \theta_1 q_{11}^D + (\theta_2 - \theta_1) q_{12}^D + F(\theta_2)$  and  $\tau_{21}^B = \theta_1 q_{21}^D + (\theta_2 - \theta_1) q_{22}^D + F(\theta_2)$ .

The firms' payoffs are

$$u_{11}^A = u_{11}^B = (\theta_2 - \theta_1) \left( \frac{q_{21}^D + q_{12}^D}{2} \right) + F(\theta_2) - F(\theta_1) > 0,$$

$$\begin{aligned}
u_{21}^A &= u_{12}^B = 0, \\
u_{12}^A &= u_{21}^B = (\theta_2 - \theta_1)q_{22}^D + F(\theta_2) - F(\theta_1) > 0, \\
\text{and } u_{22}^A &= u_{22}^B = 0.
\end{aligned}$$

Regime 1-2

We suppose that the following constraints are binding:

$$\begin{aligned}
\sum_{j=1}^2 p_{1j} [\tau_{1j}^A - \theta_1 q_{1j} - F(\theta_1)] &\geq \sum_{j=1}^2 p_{1j} [\tau_{2j}^A - \theta_1 q_{2j} - F(\theta_1)], \\
\sum_{i=1}^2 p_{i1} [\tau_{i1}^B - \theta_1 q_{i1} - F(\theta_1)] &\geq \sum_{i=1}^2 p_{i1} [\tau_{i2}^B - \theta_1 q_{i2} - F(\theta_1)], \\
\tau_{12}^A - \theta_1 q_{12} - F(\theta_1) &\geq 0, \\
\tau_{21}^B - \theta_1 q_{21} - F(\theta_1) &\geq 0, \\
\tau_{2j}^A - \theta_2 q_{2j} - F(\theta_2) &\geq 0, \quad j=1, 2, \\
\text{and } \tau_{i2}^B - \theta_2 q_{i2} - F(\theta_2) &\geq 0, \quad i=1, 2.
\end{aligned}$$

From the binding conditions, monetary transfers to firm  $A$  are

$$\begin{aligned}
\tau_{12}^A &= \theta_1 q_{12} + F(\theta_1), \\
\tau_{21}^A &= \theta_2 q_{21} + F(\theta_2) \\
\text{and } \tau_{22}^A &= \theta_2 q_{22} + F(\theta_2).
\end{aligned}$$

The monetary transfers to firm  $B$  are

$$\begin{aligned}
\tau_{12}^B &= \theta_2 q_{12} + F(\theta_2), \\
\tau_{21}^B &= \theta_1 q_{21} + F(\theta_1), \\
\text{and } \tau_{22}^B &= \theta_2 q_{22} + F(\theta_2).
\end{aligned}$$

Substituting these transfers into (P-1) and taking the first order conditions with respect to  $q_{ij}$  yield

$$\begin{aligned}
S_q(q_{11}^D) &= 2\theta_1, \\
S_q(q_{12}^D) &= S_q(q_{21}^D) = \theta_1 + \theta_2 + \frac{p}{1-p} (\theta_2 - \theta_1), \\
\text{and } S_q(q_{22}^D) &= 2\theta_2 + \frac{2p}{1-p} (\theta_2 - \theta_1).
\end{aligned}$$

Note that  $F(\theta_1) - F(\theta_2) < \frac{pq_{21}^D + (1-p)q_{12}^D}{2}$  must be satisfied in this case.

The transfers  $\tau_{11}^A$  and  $\tau_{11}^B$  are

$$\tau_{11}^A = \tau_{11}^B = \theta_1 q_{11}^D + (\theta_2 - \theta_1) q_{21}^D + \frac{1-p}{p} (\theta_2 - \theta_1) q_{22}^D + \frac{F(\theta_2) - (1-p)F(\theta_1)}{2}.$$

The firms' payoffs are

$$\begin{aligned} u_{11}^A = u_{11}^B &= \frac{1}{p} \left[ (\theta_2 - \theta_1) \left\{ \frac{p q_{21}^D + (1-p) q_{12}^D}{2} \right\} + F(\theta_2) - F(\theta_1) \right] > 0, \\ u_{12}^A = u_{21}^B &= 0, \\ u_{21}^A = u_{12}^B &= 0, \\ \text{and } u_{22}^A = u_{22}^B &= 0. \end{aligned}$$

Regime 1-3

We consider the case in which the following constraints are binding:

$$\begin{aligned} \tau_{11}^A - \theta_1 q_{11} - F(\theta_1) &\geq \tau_{21}^A - \theta_1 q_{21} - F(\theta_1), \\ \tau_{11}^B - \theta_1 q_{i1} - F(\theta_1) &\geq \tau_{12}^B - \theta_1 q_{12} - F(\theta_1), \\ \tau_{1j}^A - \theta_1 q_{1j} - F(\theta_1) &\geq 0, \\ \tau_{i1}^B - \theta_1 q_{i1} - F(\theta_1) &\geq 0, \\ \tau_{2j}^A - \theta_2 q_{2j} - F(\theta_2) &\geq 0, \\ \text{and } \tau_{i2}^B - \theta_2 q_{i2} - F(\theta_2) &\geq 0, \quad i = 1, 2 \text{ and } j = 1, 2. \end{aligned}$$

From the binding conditions, monetary transfers to firm *A* are

$$\begin{aligned} \tau_{11}^A &= \theta_1 q_{11} + F(\theta_1), \\ \tau_{12}^A &= \theta_1 q_{12} + F(\theta_1), \\ \tau_{21}^A &= \theta_2 q_{21} + F(\theta_2), \\ \text{and } \tau_{22}^A &= \theta_2 q_{22} + F(\theta_2). \end{aligned}$$

The monetary transfers to firm *B* are

$$\begin{aligned} \tau_{11}^B &= \theta_1 q_{11} + F(\theta_1), \\ \tau_{12}^B &= \theta_2 q_{12} + F(\theta_2), \\ \tau_{21}^B &= \theta_1 q_{21} + F(\theta_1), \\ \text{and } \tau_{22}^B &= \theta_2 q_{22} + F(\theta_2). \end{aligned}$$

Substituting these transfers into (P-1) and taking the first order conditions with respect to  $q_{ij}$  yield

$$S_q(q_{11}^D) = 2\theta_1,$$

$$S_q(q_{12}^D) = S_q(q_{21}^D) = \theta_1 + \theta_2,$$

$$\text{and } S_q(q_{22}^D) = 2\theta_2.$$

We must have that  $(\theta_2 - \theta_1)(pq_{12}^D + (1-p)q_{12}^D) < F(\theta_1) - F(\theta_2)$  in this case. The firms' payoffs are

$$u_{11}^A = u_{11}^B = 0,$$

$$u_{12}^A = u_{21}^B = 0,$$

$$u_{21}^A = u_{12}^B = 0,$$

$$\text{and } u_{22}^A = u_{22}^B = 0.$$

Regime 1-4

We examine the case in which the following constraints are binding:

$$\sum_{j=1}^2 p_{2j} [\tau_{2j}^A - \theta_2 q_{2j} - F(\theta_2)] \geq \sum_{j=1}^2 p_{2j} [\tau_{1j}^A - \theta_2 q_{1j} - F(\theta_2)],$$

$$\sum_{i=1}^2 p_{i2} [\tau_{i2}^B - \theta_2 q_{i2} - F(\theta_2)] \geq \sum_{i=1}^2 p_{i2} [\tau_{i1}^B - \theta_2 q_{i1} - F(\theta_2)],$$

$$\tau_{1j}^A - \theta_1 q_{1j} - F(\theta_1) \geq 0,$$

$$\tau_{i1}^B - \theta_1 q_{i1} - F(\theta_1) \geq 0,$$

$$\tau_{21}^A - \theta_2 q_{21} - F(\theta_2) \geq 0,$$

$$\text{and } \tau_{12}^B - \theta_2 q_{12} - F(\theta_2) \geq 0, \quad i = 1, 2 \text{ and } j = 1, 2.$$

From the binding conditions, monetary transfers to the firm producing  $A$  are

$$\tau_{11}^A = \theta_1 q_{11} + F(\theta_1),$$

$$\tau_{12}^A = \theta_2 q_{12} + F(\theta_1),$$

$$\tau_{21}^A = \theta_2 q_{21} + F(\theta_2),$$

$$\text{and } \tau_{22}^A = \theta_2 q_{22} + (\theta_1 - \theta_2) q_{12} + F(\theta_1).$$

The monetary transfers to firm  $B$  are

$$\tau_{11}^B = \theta_1 q_{11} + F(\theta_1),$$

$$\tau_{12}^B = \theta_1 q_{12} + F(\theta_2),$$

$$\tau_{21}^B = \theta_1 q_{21} + F(\theta_1),$$

$$\text{and } \tau_{22}^B = \theta_2 q_{22} + (\theta_1 - \theta_2) q_{12} + F(\theta_1).$$

Substituting these transfers into (P-1) and taking the first order conditions with respect to  $q_{ij}$  yield

$$\begin{aligned}
S_q(q_{11}^{D,C}) &= 2\theta_1 - \frac{2p}{1-p}(\theta_2 - \theta_1), \\
S_q(q_{12}^{D,C}) &= S_q(q_{21}^{D,C}) = \theta_1 + \theta_2 - \frac{p}{1-p}(\theta_2 - \theta_1), \\
\text{and } S_q(q_{22}^D) &= 2\theta_2,
\end{aligned}$$

where  $q_{ij}^{D,C}$  denotes an optimal quantity with countervailing incentives.

We must have that  $(\theta_1 - \theta_2)\left(\frac{q_{21}^{D,C} + q_{12}^{D,C}}{2}\right) < F(\theta_1) - F(\theta_2)$  holds. We note that  $q_{11}^{D,C} > q_{11}^{FB}$  and  $q_{12}^{D,C} = q_{21}^{D,C} > q_{12}^{FB}$ .

The firms' payoffs are

$$\begin{aligned}
u_{11}^A &= u_{11}^B = 0, \\
u_{12}^A &= u_{21}^B = 0, \\
u_{21}^A &= u_{12}^B = 0, \\
\text{and } u_{22}^A &= u_{22}^B = (\theta_1 - \theta_2)\left(\frac{q_{21}^{CI} + q_{12}^{CI}}{2}\right) + F(\theta_1) - F(\theta_2) > 0.
\end{aligned}$$

Regime 1-5

We consider the case in which the following constraints are binding:

$$\begin{aligned}
\sum_{j=1}^2 p_{2j} [\tau_{2j}^A - \theta_2 q_{2j} - F(\theta_2)] &\geq \sum_{j=1}^2 p_{2j} [\tau_{1j}^A - \theta_2 q_{1j} - F(\theta_2)], \\
\sum_{i=1}^2 p_{i2} [\tau_{i2}^B - \theta_2 q_{i2} - F(\theta_2)] &\geq \sum_{i=1}^2 p_{i2} [\tau_{i1}^B - \theta_2 q_{i1} - F(\theta_2)], \\
\tau_{1j}^A - \theta_1 q_{1j} - F(\theta_1) &\geq 0, \\
\text{and } \tau_{i1}^B - \theta_1 q_{i1} - F(\theta_1) &\geq 0, \quad i = 1, 2 \text{ and } j = 1, 2.
\end{aligned}$$

From the binding conditions, monetary transfers to firm  $A$  are

$$\begin{aligned}
\tau_{11}^A &= \theta_1 q_{11} + F(\theta_1), \\
\tau_{12}^A &= \theta_1 q_{12} + F(\theta_1), \\
\tau_{21}^A &= \theta_2 q_{21} + (\theta_1 - \theta_2) q_{11} + F(\theta_1), \\
\text{and } \tau_{22}^A &= \theta_2 q_{22} + (\theta_1 - \theta_2) q_{12} + F(\theta_1).
\end{aligned}$$

Monetary transfers to firm  $B$  are

$$\begin{aligned}
\tau_{11}^B &= \theta_1 q_{11} + F(\theta_1), \\
\tau_{12}^B &= \theta_2 q_{12} + (\theta_1 - \theta_2) q_{11} + F(\theta_1),
\end{aligned}$$

$$\begin{aligned}\tau_{21}^B &= \theta_1 q_{21} + F(\theta_1), \\ \text{and } \tau_{22}^B &= \theta_2 q_{22} + (\theta_1 - \theta_2) q_{12} + F(\theta_1).\end{aligned}$$

Substituting these transfers into (P-1) and taking the first order conditions with respect to  $q_{ij}$  yield

$$\begin{aligned}S_q(q_{11}^{D,C}) &= 2\theta_1 - \frac{2(1-p)}{p} (\theta_2 - \theta_1), \\ S_q(q_{12}^{D,C}) &= S_q(q_{21}^{D,C}) = \theta_1 + \theta_2 - \frac{p}{1-p} (\theta_2 - \theta_1), \\ \text{and } S_q(q_{22}^{D,C}) &= 2\theta_2.\end{aligned}$$

In this case, we have that  $(\theta_2 - \theta_1) q_{11}^{D,C} < F(\theta_1) - F(\theta_2)$  holds. Monetary transfers to firm  $A$  are

$$\begin{aligned}\tau_{21}^A &= \theta_2 q_{21}^{D,C} + (\theta_1 - \theta_2) q_{11}^{D,C} + F(\theta_1), \\ \text{and } \tau_{22}^A &= \theta_2 q_{22}^{FB} + (\theta_1 - \theta_2) q_{12}^{D,C} + F(\theta_1).\end{aligned}$$

The monetary transfers to firm  $B$  are

$$\begin{aligned}\tau_{12}^B &= \theta_2 q_{12}^{D,C} + (\theta_1 - \theta_2) q_{11}^{D,C} + F(\theta_1) \\ \text{and } \tau_{22}^B &= \theta_2 q_{22}^{FB} + (\theta_1 - \theta_2) q_{21}^{D,C} + F(\theta_1).\end{aligned}$$

The firms' payoffs are

$$\begin{aligned}u_{11}^A &= u_{11}^B = 0, \\ u_{12}^A &= u_{21}^B = 0, \\ u_{21}^A &= u_{12}^B = (\theta_1 - \theta_2) q_{11}^{D,C} + F(\theta_1) - F(\theta_2) > 0, \\ \text{and } u_{22}^A &= u_{22}^B = (\theta_1 - \theta_2) \left( \frac{q_{21}^{D,C} + q_{12}^{D,C}}{2} \right) + F(\theta_1) - F(\theta_2) > 0.\end{aligned}$$

Regime 1-1 is summarized in Proposition 1. Regime 1-5 is summarized in Proposition 2. This completes the proofs.

## Appendix B

Proofs of Propositions 3 and 4

Regime 2-1

Suppose that the following constraints are binding:

$$\omega_{1j} - (\theta_1 + \theta_j) q_{1j} - F(\theta_1) - F(\theta_j) \geq \omega_{2j} - (\theta_1 + \theta_j) q_{2j} - F(\theta_1) - F(\theta_j),$$

$$\begin{aligned}\omega_{i1} - (\theta_i + \theta_1)q_{i1} - F(\theta_i) - F(\theta_1) &\geq \omega_{i2} - (\theta_i + \theta_1)q_{i2} - F(\theta_i) - F(\theta_1), \\ \text{and } \omega_{22} - 2\theta_2q_{22} - 2F(\theta_2) &\geq 0, \quad i = 1, 2 \text{ and } j = 1, 2.\end{aligned}$$

From the binding conditions, the monetary transfers are

$$\begin{aligned}\omega_{11} &= 2\theta_1q_{11} + (\theta_2 - \theta_1)\left(\frac{q_{21} + q_{12}}{2} + q_{22}\right) + 2F(\theta_2), \\ \omega_{12} = \tau_{21} &= (\theta_1 + \theta_2)\left(\frac{q_{21} + q_{12}}{2}\right) + (\theta_2 - \theta_1)q_{22} + 2F(\theta_2), \\ \text{and } \omega_{22} &= 2\theta_2q_{22} + 2F(\theta_2).\end{aligned}$$

These transfers satisfy the remaining ICCs and PCs. Substituting these transfers into (P-2) and taking the first order conditions with respect to  $q_{ij}$  yield

$$\begin{aligned}S_q(q_{11}^I) &= 2\theta_1, \\ S_q(q_{12}^I) = S_q(q_{21}^I) &= \theta_1 + \theta_2 + \frac{p}{2(1-p)}(\theta_2 - \theta_1), \\ \text{and } S_q(q_{22}^I) &= 2\theta_2 + \frac{p(2-p)}{(1-p)^2}(\theta_2 - \theta_1),\end{aligned}$$

where  $q_{ij}^I$  denotes an equilibrium output. In this case, we must have that  $F(\theta_1) - F(\theta_2) < (\theta_2 - \theta_1)q_{22}^I$  holds. Thus we have

$$q_{22}^I < q_{22}^{FB}, \quad q_{22}^I < q_{12}^I = q_{21}^I < q_{11}^I = q_{11}^{FB}.$$

Output  $q_{11}^I$  is at the first best level  $q_{11}^{FB}$ , and  $q_{12}^I$  and  $q_{21}^I$  are distorted and smaller than  $q_{11}^I$ . Output  $q_{22}^I$  is distorted and smaller than  $q_{12}^I$  and  $q_{21}^I$ .

The firms' payoffs are

$$\begin{aligned}u_{11}^I &= (\theta_2 - \theta_1)(q_{21}^I + q_{12}^I) + 2F(\theta_2) - 2F(\theta_1), \\ u_{12}^I = u_{21}^I &= (\theta_2 - \theta_1)q_{22}^I + F(\theta_2) - F(\theta_1), \\ \text{and } u_{22}^I &= 0.\end{aligned}$$

## Regime 2-2

Consider the case in which the following constraints are binding:

$$\begin{aligned}\omega_{1j} - (\theta_1 + \theta_j)q_{1j} - F(\theta_1) - F(\theta_j) &\geq \omega_{2j} - (\theta_1 + \theta_j)q_{2j} - F(\theta_1) - F(\theta_j), \\ \omega_{i1} - (\theta_i + \theta_1)q_{i1} - F(\theta_i) - F(\theta_1) &\geq \omega_{i2} - (\theta_i + \theta_1)q_{i2} - F(\theta_i) - F(\theta_1), \\ \omega_{12} - (\theta_1 + \theta_2)q_{12} - F(\theta_1) - F(\theta_2) &\geq 0, \\ \omega_{21} - (\theta_2 + \theta_1)q_{21} - F(\theta_2) - F(\theta_1) &\geq 0, \\ \text{and } \omega_{22} - 2\theta_2q_{22} - 2F(\theta_2) &\geq 0, \quad i = 1, 2 \text{ and } j = 1, 2.\end{aligned}$$



From the binding conditions, the monetary transfers are

$$\begin{aligned}\omega_{11} &= 2\theta_1 q_{11}^{FB} + (\theta_2 - \theta_1) \left( \frac{q_{21} + q_{12} + 2q_{22}}{2} \right) + 2F(\theta_2), \\ \omega_{12} &= \omega_{21} = (\theta_1 + \theta_2) q_{12} + (\theta_2 - \theta_1) q_{22} + 2F(\theta_2), \\ \text{and } \omega_{22} &= 2\theta_2 q_{22} + 2F(\theta_2).\end{aligned}$$

Substituting these transfers into (P-2) and taking the first order conditions with respect to  $q_{ij}$  yield

$$\begin{aligned}S_q(q'_{11}) &= 2\theta_1, \\ S_q(q'_{12}) &= S_q(q'_{21}) = \theta_1 + \theta_2 + \frac{p}{1-p} (\theta_2 - \theta_1), \\ \text{and } S_q(q'_{22}) &= 2\theta_2.\end{aligned}$$

We must have  $F(\theta_1) - F(\theta_2) < (\theta_2 - \theta_1) q'_{12}$ . The firms' payoffs are

$$\begin{aligned}u_{11} &= (\theta_2 - \theta_1) q'_{12} + F(\theta_2) - F(\theta_1) > 0, \\ u_{12} &= u_{21} = 0, \\ \text{and } u_{22} &= 0.\end{aligned}$$

### Regime 2-3

We consider the case in which the following constraints are binding:

$$\begin{aligned}\omega_{22} - 2\theta_2 q_{22} - 2F(\theta_2) &\geq \omega_{ij} - (\theta_i + \theta_j) q_{ij} - F(\theta_i) - F(\theta_j), \\ \omega_{11} - 2\theta_1 q_{11} - 2F(\theta_1) &\geq 0, \\ \omega_{12} - (\theta_1 + \theta_2) q_{12} - F(\theta_1) - F(\theta_2) &\geq 0, \\ \text{and } \omega_{21} - (\theta_1 + \theta_2) q_{21} - F(\theta_1) - F(\theta_2) &\geq 0, \quad i = 1, 2 \text{ and } j = 1, 2.\end{aligned}$$

From the binding conditions, the monetary transfers are

$$\begin{aligned}\omega_{11} &= 2\theta_1 q_{11} + 2F(\theta_1), \\ \omega_{12} &= (\theta_1 + \theta_2) q_{12} + (\theta_1 - \theta_2) q_{11} + 2F(\theta_1) \\ \omega_{21} &= (\theta_1 + \theta_2) q_{21} + (\theta_1 - \theta_2) q_{11} + 2F(\theta_1) \\ \text{and } \omega_{22} &= 2\theta_2 q_{22} + (\theta_1 - \theta_2) \left( \frac{q_{21} + q_{12}}{4} \right) + \frac{3F(\theta_1) + F(\theta_2)}{2}.\end{aligned}$$

Substituting these transfers into (P-2) and taking the first order conditions with respect to  $q_{ij}$  yield

$$\begin{aligned}
S_q(q_{11}^{LC}) &= 2\theta_1 - \frac{(1-p)^2}{p^2} (\theta_2 - \theta_1), \\
S_q(q_{12}^{LC}) &= S_q(q_{21}^I) = \theta_1 + \theta_2 - \left(\frac{1-p}{4p}\right) (\theta_2 - \theta_1), \\
\text{and } S_q(q_{22}^I) &= 2\theta_2.
\end{aligned}$$

In this case, we must have  $(\theta_2 - \theta_1)q_{11}^{FB} < F(\theta_1) - F(\theta_2) < (\theta_2 - \theta_1)q_{11}^{LC}$ .

The firms' payoffs are

$$\begin{aligned}
u_{11} &= 0, \\
u_{12} &= u_{21} = 0, \\
\text{and } u_{22} &= (\theta_2 - \theta_1) \left( \frac{q_{21}^{LC} + q_{12}^{LC}}{2} + q_{11}^{LC} \right) + \frac{3[F(\theta_1) - F(\theta_2)]}{2} > 0.
\end{aligned}$$

Regime 2-4

Suppose that the following constraints are binding:

$$\begin{aligned}
\omega_{22} - 2\theta_2 q_{22} - 2F(\theta_2) &\geq \omega_{12} - 2\theta_2 q_{12} - 2F(\theta_2), \\
\omega_{22} - 2\theta_2 q_{22} - 2F(\theta_2) &\geq \omega_{21} - 2\theta_2 q_{21} - 2F(\theta_2), \\
\omega_{12} - (\theta_1 + \theta_2) q_{12} - F(\theta_1) - F(\theta_2) &\geq \omega_{11} - (\theta_1 + \theta_2) q_{11} - F(\theta_1) - F(\theta_2), \\
\omega_{21} - (\theta_1 + \theta_2) q_{21} - F(\theta_1) - F(\theta_2) &\geq \omega_{11} - (\theta_1 + \theta_2) q_{11} - F(\theta_1) - F(\theta_2), \\
\text{and } \omega_{11} - 2\theta_1 q_{11} - 2F(\theta_1) &\geq 0.
\end{aligned}$$

From the binding conditions, the monetary transfers are

$$\begin{aligned}
\omega_{11} &= 2\theta_1 q_{11} + 2F(\theta_1), \\
\omega_{12} = \omega_{21} &= (\theta_1 + \theta_2) q_{12} + (\theta_1 - \theta_2) q_{11} + 2F(\theta_1), \\
\text{and } \omega_{22} &= 2\theta_2 q_{22} + (\theta_1 - \theta_2) \left( \frac{q_{12} + q_{21}}{2} + q_{11} \right) + 2F(\theta_1).
\end{aligned}$$

Substituting these transfers into (P-2) and taking the first order conditions with respect to  $q_{ij}$  yield

$$\begin{aligned}
S_q(q_{11}^{LC}) &= 2\theta_1 - \frac{(1-p^2)}{p^2} (\theta_2 - \theta_1), \\
S_q(q_{12}^{LC}) &= S_q(q_{21}^{LC}) = \theta_1 + \theta_2 - \frac{(1-p)}{2p} (\theta_2 - \theta_1), \\
\text{and } S_q(q_{22}^I) &= 2\theta_2.
\end{aligned}$$

We must have  $(\theta_2 - \theta_1)q_{11}^{LC} < F(\theta_1) - F(\theta_2)$ . The firms' payoffs are

$$\begin{aligned}
u_{11} &= 0, \\
u_{12} = u_{21} &= (\theta_1 - \theta_2) q_{11}^{LC} + F(\theta_1) - F(\theta_2) > 0 \\
\text{and } u_{22} &= (\theta_1 - \theta_2) \left( \frac{q_{21}^{LC} + q_{12}^{LC}}{2} + 2q_{11}^{LC} \right) + 2F(\theta_2) - 2F(\theta_1) > 0.
\end{aligned}$$

Regime 2-1 is summarized in Proposition 3. Regime 2-4 is summarized in Proposition 4. This completes the proofs.

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